Valentiner $\vartheta_l \propto$ (60k ϑ_l Smithe p. 119

Nuss 0.0025

Drude 0.001

Wien Ber. 106 [Boltzmann-Wind] Proc. R.S. 59 (1896) Roberts-Austen
105 [Jäger U. (4893) T. (2702) 57 (1895) Fitzgerald, Burbury
104 50 Kelvin
103
102
101
100
99

Nature 52 (Bosanquet, Culverwell, Bryan, Burbury) [78057] 53 ff.
51 (Culver, Burb.) T 18 ff. [3511]
46 (Watson, Burbury) Kroker U 12 ff
45 Kirchhoff Ray Tait Watson Cremon. U. 36 ff. [41.5.1]
44 Kelvin

Pr. Lond. math. soc. 27 (1896) Burbury 26 (1895) Burbury ~~Procee~~ (9-15) (78157)

Zittingsversl. K.A.W.A. 1896/7 (Lorentz)

Gött. Nachr. 1896 (Voigt) 10792 L.U. ~~XII~~ K3

Rep. B.A. 1894 (Bryan) 1891 (Culverwell)

Pr. Cambr. Ph. S. 8 (1895) (Bryan), 1893, 7 (1891) Larmor

Rev. gén. sciences (Poincaré) 5 (1894)

L.U. 4 37-42 (25006)
L.T 5, 13 ff. [3512]
Crem. 5, 25 - 28 [20, P. 2]
Wien T 3044
Phil Mag
T

Ph. M. 30 Burbury Culv.
33 Thomson Ray
32 Sutherland, Rayleigh
35 Boltzmann

Maxwell: Illustrations of the Dynamical Theory of Gases

Prop XIII: $\mu = \frac{1}{3}\rho l v = \frac{1}{3\sqrt{2}} \frac{Mv}{\pi s^2}$

" A remarkable result: ..... friction independent of density. Such a consequence ... very startling, and the only experiment I have met with on the subject does not seem to confirm it."

Prop. XIV. „In a system of moving particles whose density, velocity etc. are functions of x, to find the quantity of matter transferred across the plane of yz, due to the motion of agitation alone."

Quantity of matter, projected from stratum dx, crossing yz in + direction & striking other particles at distances between nl & (n+dn)l is:

$$\frac{MNv(x \mp nl)}{2nl^2}\, dx\, e^{-n} dn \qquad N, v, l \text{ are functions of } x$$

If U and r are functions (x) not vanishing with x whose variations are very small between the limits x = +r and x = −r:

$$\int_{-r}^{+r} \pm U x^m dx = \frac{2}{m+2} \frac{d}{dx}\left(U r^{m+2}\right)$$

$$\therefore \int_{-nl}^{+nl} \frac{MNvx}{2nl^2}\, dx = \frac{1}{3}\frac{d}{dx}\left(MNvn^2 l\right) \qquad \int_{-nl}^{+nl} \mp \frac{MNv}{2l}\, dx = -\frac{1}{2}\frac{d}{dx}\left(MNvn^2 l\right)$$

Now we have $\int_{+0}^{\infty} -\frac{1}{6}\frac{d}{dx}(MNvl)\, n^2 e^{-n} dn = \cdot \qquad \cdot q = -\frac{1}{3}\frac{d}{dx}(\rho v l)$

Prop. XXI „To find the amount of energy which crosses unit of area in unit of time when the velocity of agitation is greater on one side of the area than on the other"

Substituting $E = \frac{1}{2}\beta M v^2$ for M in Eq XIV: $\quad E = \rho \frac{\cancel{v}\,\theta}{N} = \rho v \frac{MNv^2}{2N}$

$$J_q = -\frac{1}{3}\frac{d}{dx}\left(\frac{1}{2}\beta M v^2 N v l\right)$$

$\beta = \cancel{M}\rho v . M$

Now: $MN = \rho \quad l = \frac{1}{A\rho} \quad \therefore MNl = \frac{1}{A}$

$$J_q = -\frac{1}{2}\frac{\beta v^2}{A}\frac{dv}{dx} \qquad \frac{2}{v}\frac{dv}{dx} = \frac{1}{T}\frac{dT}{dx}$$

$$\therefore \quad J_q = -\frac{3}{4}\beta \rho l v \frac{1}{T}\frac{dT}{dx}$$

[calculation: ~~a~~ conductivity of air 10^6 times less than ~~lead~~ copper faulty, see Clausius]

~~l is variable too! dependent of v~~

$$J_q = -\frac{\beta}{6}\frac{\partial}{\partial x}(\rho v^3 \lambda) = -\frac{\beta}{2}\rho\lambda v^2\frac{\partial v}{\partial x} = -\frac{\beta}{2}\rho\lambda v \; v\frac{\partial v}{\partial x}$$

$$v\frac{\partial v}{\partial x} = \frac{v^2}{2}\frac{1}{T}\frac{\partial T}{\partial x}$$

$= \cancel{\frac{3}{4}\ldots}$

$\beta = \frac{5}{3}$

$T = m\frac{v^2}{2} \qquad " \frac{1}{m}\frac{\partial T}{\partial x}$

$$q = \frac{3\beta\mu}{2}\frac{1}{m}\frac{\partial T}{\partial x} = \frac{3}{2}\beta\rho\frac{\partial T}{\partial x}$$

$k = \frac{5}{4}\mu$

$\beta\rho \qquad K = \frac{\beta}{2} 3\frac{\mu}{m}\frac{\partial T}{\partial x}$

$$J_q = \frac{3}{4}\beta\frac{p\lambda}{T}\frac{\partial T}{\partial x} \cdot \frac{3\mu}{\rho\lambda}$$

$\frac{p}{\rho} = T.n$

$\frac{\frac{nmc^2}{3}}{mn} = \frac{mc^2}{2}\frac{2}{3n}$

$$= \frac{9}{4}\beta\mu n\frac{\partial T}{\partial x}$$

$\lambda = \frac{1}{\pi n \sigma^2} \qquad \mu v = RT \qquad J_q = -\frac{3}{4}\beta\lambda R\frac{dT}{dx}$

$$q = \frac{3}{2}\beta\frac{\mu}{m}\frac{\partial T}{\partial x}$$

für einatomige Moleküle: $\beta = \frac{5}{3}$ somit $K = \frac{5}{2}$..

$\cancel{\ldots} =$

Boltzmann: $E_T = (1+\beta) E_{pr}$

$k\,\cancel{M} = \gamma_v\,\mu =$ || Maxwell II $k = \frac{5}{2}\gamma_v\,\mu$

Maxwell I: berücksichtigt wo θ gilt dort auch v grösser daher Factor $\frac{3}{2}$, aber [illegible]

Beibl. 21 p. 212 De Heen [illegible] CO_2 :
p. 623 " [illegible]

findet dass die Dichte der Flüssigkeit doppelt so gross wie die des Dampfes

Dagegen p. 579 Villard [illegible]

Amagat p. 209 [illegible]

	CO_2	C_2H_4	Äther	Luft
kr. temp.	31·35	8·8	195	−140·7
Druck	72·9	48·5	36·5	35·9
Dichte	0·464	0·212	0·253	0·344

daraus erklärt [illegible] das [illegible]

---

p. 957 ~~[illegible]~~ Leduc & Sacerdote [illegible]:

	θ	π	
HCl	51·5	96	Vincent Chappuis
	52·3	86	Dewar
	52	83	Led. S. S.
PH_3	52·8	64	"
H_2S	100	88·7	Olszewski
	100·2	92	Dewar
	100	90	Led. S. S.
$(CH_3)_2O$	129·6	—	Nadejdine
	129·6	57	L. S. S.

---

p. 955 Kuenen [illegible]

(Phil. Mag. 44 p. 174)

[illegible]

	ϑ	π	a	b
N_2O	309	71.9	0.00753	0.00197
C_2H_2	308.25	61.02	881	231
CO_2	304.1	73.26	714	190
C_2H_6	305	48.8	1078	286

p. 404 Lorenz [illegible] temp. [illegible]
Theorie der Lindi' [illegible]

p. 21 Kamerlingh Onnes [illegible] temp.,
Wasserstoff therm. [illegible] Thermoelement, genaue [illegible] Nernst [illegible] }

p. 718 $\frac{c}{c} = 1.26$ für Acetylen Maneuvrier [illegible] Fournier

$c_p = 0.1233$ Argon Dittenberger

p. 856 Moissan & Dewar [illegible] Fluors CR 124 p. 1202
[illegible] -185^0

p. 851 S. Young Pentan $\vartheta = 197.2$, $\pi = 25100$ mm, $\frac{v}{gr} = 4.303$ cm^2

p. 329 Dewar [illegible] Roy Inst. 1896 27/3

Beschreibg. [illegible] Apparats

O [illegible] ; NO [illegible]

O spec. [illegible] $= 1.1375$ (bei [illegible] 6.5 mm)

Wroblewski 1.168

Olszewski 1.124

[illegible] W[illegible] ; [illegible]

Luft [illegible] 10 [illegible] H [illegible] ??

Beibl. 22

p. 73 Ledue [illegible] 20

[illegible]:

CO_2	1·5287
N_2O	1·5301
HCl	1·2692
H_2S	1·1895
Cl_2	2·491
NH_3	0·5971
SO_2	2·2639

p. 94 Moissan & Dewar:

Neue Erfahrg. [illegible] Fluor CR 125 p. 505

Siedp. d. Fluor: −187° bei −210° [illegible]

Dichte = 1·14 kleinere Cap. const. [illegible] 0

keine Absorption, nicht magnetisch

p. 265 Lange Spec. W. des flüssigen Ammoniaks)

Sehr ausführl. Beobachtg. zwisch. −50 und +100°

p. 392 Kuenen Athan

Kuenen	Observati.	Haenlen
	−93 1	
τ = 32·0, π = 48·8	34 50·2	34·5 50

~~Flüssig. Dewar~~ Fortschr. ~~52 p. 510~~

Rudorf p. 522

Fleming & Dewar magn. Suszept. d. fl. O_2

bei $-182°$: $\mu = 1{\cdot}0041$ wenig abhängig von Feldstärke ([illegible])

~~[illegible]~~ [illegible] = 3·32 mal so gross als bei 25°

Wismuth Zunahme 16%

Es scheint dass für paramagn. [illegible] Susz. [illegible] $\frac{\text{Dichte}}{\text{abs. Temp.}}$

Siedetemp. d. flüss. Ozon CR 1898, p. 1751 (−119°) Troost

Rudorf p. 375

A. Schuster (Nature LVIII p. 199 30/6):

Auffallende Ähnlichkeit des Spectrums des Argons mit gewöhnl.

C Spectrum 8 Streifen [illegible] ; [illegible] Cyan [illegible]

Olszewski Wied. Ann. 69 184 Helium!

Fortschr. 52 p. 323 The new process for the liquef. of air etc Nature

53 p. 515

Schon vor Linde hat Hampson [Brin's Oxygen Works 21/3 1895]

[illegible] Appar. constr. [illegible]

Linde Wied. Ann. 57 p. 328

p. 510 Fleming Dewar Electr. [illegible] tief. Temp.

p. 33 Gladstone Specifische Refraction s periodisch

Nach Rayleigh: spec. [illegible] Argon = 0·159

[illegible] Atomgew. = 20 [illegible]

Lockyer the story of helium Nature 53, 319, 342 (1896)
245
526

Eder & Valenta Wie. Denkschr. 64 p. 39 Spectrum des Argons

Wszechświat p. 669:
Brit. Ass. Bristol: Crookes Monium (118)
Ramsay & Travers: Xenon najniższa temp. wrzenia pozostaje gdy inne opary gęstość jego większa niż innych. Widmo analogiczne z argonem
Neon: 19.2, jednoatomowy, w powietrzu 1:40.000

Fortschr. 97 p. 157 Berthelot (CR 124 p. 113,) Recherches sur l'hélium.
Fand chemische Verbindungen mit Benzol, CS_2 [illegible]
Ersteres [illegible] Spectrum = 2 artige [illegible]
Letzteres Kohleartig; [illegible] He [illegible] Spectr. [illegible]
Analysen nicht ausgeführt.

Wilde glaubt nicht an Berthelots Hyp. dass Argon = allotr. N
[illegible] N [illegible]

W. Ramsay Ein unentdecktes Gas
Glaubt dass He und Arg in eine Gruppe [illegible]; [illegible] 3tes Element v. Atomgew. 20 liegen [? At = 10] Suchen ohne Erfolg gemacht
Brit. Ass. Toronto 19/VIII 97

CR 126 p. 1610

W. Ramsay, M. Travers u. Miss Emily Aston

Dr Hampson liefert 750 cm³ flüssige Luft

Alles wurde [illegible] 10 cm³

[illegible] O_2 [illegible] Cu [illegible] N_2 [illegible] MgO u. CaO [Selectiv-Funken [illegible] O_2 u. N_2

[illegible] 26·2 cm³ [illegible] [2 [illegible] D_3 He [illegible]]

Dichte: 22·47 (O_2=16) $\frac{C}{c} = 1·666$ [illegible] Kundt ...
22·51

also weniger flüchtig als O_2 N_2 Ar, einatomig = Kr

Orthotol [illegible] (5567) [illegible] Nordlichter [illegible] Eosinum [illegible]

CR 126 p. 1762 Dieselben

18 lit. Argon 216

Dichte 613 ; [illegible] ; [illegible] Ar [illegible]

[illegible] 2 Fractionen à 75 cm³ [illegible]

[illegible] sehr brillantes orange Spectrum [illegible] (5849·6)

Also folgende Linien:

Na	D_1	5895·0
Na	D_2	·89·0
He	D_3	·75·9
Kr	D_4	66·5
Ne	D_5	·49·6

Neon

Dichte 14·67 [illegible]
10-11 * ?

Es wird von Alum. Electr. [illegible] absorb. [[illegible] Ar, He, Kr] [illegible] orange

$\frac{C}{c} = 1·66$

v 63 P Ni–Co

P[illegible] Fraction ~ [illegible] Compl. Spectr.

Dichte : 1.987 $\frac{C}{c} = 1.66$ Metargon

Es ist noch fest bei Siedetemp. während Kr [illegible]

daher kann man es sehr leicht rein erhalten

$D_5 = 5849.6$

Oct. 22 Dewar: Platinthermometer hat 0 Punkt von −265.2 Pt Graden

[illegible] H_2 [illegible] −256.8 [illegible]

Dichte von H_2 bei Siedep. = 0.07

Dichte des gasf. H_2 = 0.55

klar, farblos, kein Absorpt. Spectrum, also kein Metall

Oct. 22 p. 267 Ramsay & Travers

[illegible] homogen; [illegible] Ar 2

Oct. 22 p. 217 Ramsay & Travers [Proc. R.S. 62 p. 225 1898]

Brechungs Exp. d. Luft, O_2, N_2, Ar, H_2, He

mit d. Apparat von Rayleigh (siehe weiter)

n Luft = 1

H_2	0.4733
O_2	0.9243
N_2	1.0163
A_2	0.9596
CO_2	1.5316

Beibl. 20 p. 312

Rayleigh [Pr. 59, p. 198, Ztschr. f. ph. Ch. ~~[illegible]~~ 19 p. 364, 1896]

1). Außergewöhnlich reines Arg. Dichte = 19.940 $(O = 16)$

2). $\frac{n_{Ar}}{n_{Luft}} = 0.961$ $\frac{n_{He}}{n_{Luft}} = 0.146$ also die kleinste [illegible]

[illegible] N_3 [illegible]

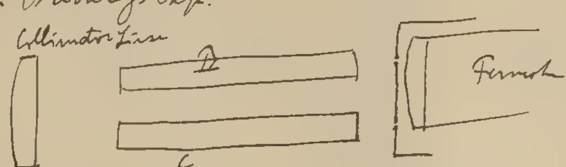
3). Capillare Methode ~~[illegible]~~ Reibungscoeff. $\eta_{Ar} = 1.21$ $(O_2 = 1)$

$\eta_{He} = 0.96$

Siehe besonders obiges Referat in Zeitschr.

Reinigung von Ar ausführlich beschrieben

Methode für Brechungsexp.



[illegible] C [illegible] Luft [illegible]

[illegible] ; [illegible]

[illegible] compens. ; [illegible]

[illegible] ~~[illegible]~~ [illegible] Cylinder [illegible]

[illegible] vertical ; [illegible] horiz. [illegible]

Es genügen Glasstäbe u. wassergefüllte Röhren

In Luft ist von He $< \frac{1}{10000}$ [illegible] Ar. u. He [illegible] V [illegible]

[illegible] = [illegible]

Fortschr. 96 p. 118

W. Ramsay (Nature 53 p. 598)

He aus Clevit 7·2 cm³ auf 1 gr

Dito: Bröggerit 1 cm³ "

Samarskit 0·6 " "

Fergusonit 1·1 " " } Analyse 54·7 % H_2

13·9 % CO_2

31·2 % He

Dichten: 2·18, 2·12, 2·14

---

p. 120 W. Ramsay The position of ar. and he among the elements

Atomgew. Ar = 39·88

He = 4·28

W. R. & Collie: [illegible] 38

Karl Olszewski Wied. Ann 69, p 184 [illegible]

Helium aus Clevit (v Ramsay) Dichte (H = 1) = 2·133

$\frac{C_p}{C_v} = 1{\cdot}652$

100 cm³ [illegible] 0·7 cm³ He

[illegible] -264° [illegible]

Fortschr. 95 Rayleigh Ramsay p. 124

sp. G. atmosph. N : 2·310 [illegible] 0·8 % [illegible]

chem. 2·299

Vol. [illegible] 13° [illegible] 4·0 Vol Ar

$\frac{C_p}{C_v} = 1{\cdot}645$

Siedepunkt − 186·9

krit. Punkt − 121° krit. Druck 50·6

Durch Olszewski auch feste Krystalle (weiß) [illegible] − 189·5° [illegible]

---

p. 132 Kellas:

100 cm³ Luft enthalten 0·937 cm³ Ar

also auf 100 cm N_2: 1·186 cm³ Ar

Litergewicht von N_2 (rein) = 1·2511

atmosph. = 1·2572

Ar = 1·782

---

p. 126 Berthelot C. R. 120

Ar [illegible] Benzoldämpfe [illegible]

---

p. 132 Ramsay: He:

[illegible] Clevéit (oder Uranpecherz) mit [illegible] H_2SO_4

[illegible] = 3·89 [illegible] D_3

---

p. 134 Ramsay, Collie, Travers

[illegible] U Y Th

Dichte = 2·13 (O = 16)

[illegible] 0·0073 Vol. He $\frac{C}{c}$ = 1·65 − 1·63

---

p. 782 Olszewski (1895)

	ϑ	π	Siedep.	Gefrierp.	Gefrierdruck mm	Dichte der Flüssigkeit beim Siedep.
H_2	220·0	20·0	?	?	?	?
N_2	146·0	35·0	194·4	−214·0	60	0·885
CO	139·5	35·5	190·0	207·0	100	?
A	121·0	50·6	187·0	189·6	?	1·5
O_2	118·8	50·8	182·7	?	?	1·124
NO	93·5	71·2	153·6	167·0	138	?
CH_4	81·8	54·9	164·0	185·8	80	0·415

250 in Vacuum

Archl. 22 p. 515 Dewar: Solidif. d. H_2: $-238°$; [illegible] ca $-223°$; [illegible] ca 15

Nature 58 p. 370 Crookes:

Helium lines distinctly visible in the more volatile portion of liquid air. Helium out from Bath Gas containing Neon.

Nature 58 p. 566

Xenon the heaviest of the three gases (A, Kr, Xe.)

~~At~~ last portions of liquefied A, ~~lighter [illegible]~~ remains behind

Spectr. analogous to A but differing entirely in the position of the lines.

p. 545

Baly [illegible] He [illegible] Ne [illegible] ; Ramsay & Travers [illegible]

~~He~~ Ne [illegible] 9.6 [at. 1' = 19.2] [illegible] Fl & Na

	ϑ	π	φ	δ	
CH_4	−81·8	54·9			Olszewski CR 100, p. 940 (1885)
	−95·5	50·0			Dewar } Phil. M. (5) 18 p. 210 (1884)
C_2H_6	35	45·2			Dewar

Dichte: Methan 0·4148 bei $\underbrace{-164·0}_{=\text{Siedep.}}$ Olszewski Bull. Crac. 14 (1886) Beibl. 1886

−185·8 = δ

t	p
−85·4	49·0
−93·3	40·0
105·8	26·3
110·6	21·4
126·8	11·0
138·5	6·2
153·8	2·24
185·8	0·105 = Gefrierpunkt = 80 mm Hg
201·5	0·066

Olsz. CR 100

Äthan 4° : 46 Atm. Cailletet CR 85 p. 851 1877

Äthan:

+34° (kr)	50·2 (kr)
29	46·7
23·5	40·4
0	23·8
−93 (Siedep.)	1
−151	Dp 6

Propan

+97°	44
49	18
43	15·7
30	11·1
20	8·8
10	7·4
0	5·0
−45° (Sied.)	1·0
−151	Dp 6

Olsz. Crac. 1889

Fortschr. 1894 p. 354

Hainlen: Liebig's Ann. 282, p. 229, (1894)   Fortschr. 1894 p. 354

Äthan: Siedp.: $-89{\cdot}5°$

kr. $+34{\cdot}5$ $\pi = 50$

$\delta = 0{\cdot}446$ bei $0°$

$0{\cdot}396$ bei $+10{\cdot}5°$

Propan: Siedp. $-37{\cdot}0$ kr. $+102°$ $\pi = 48{\cdot}5$

δ	bei
$0{\cdot}536$	$0°$
$0{\cdot}524$	$6{\cdot}2$
$0{\cdot}520$	$11{\cdot}2$
$0{\cdot}515$	$15{\cdot}9$

n-Butan Siedp. $+1$ $\delta = 0{\cdot}60$ bei $0°$

Propan:

t	p
−33	1·8
19	2·7
15	3·1
11	3·6
5	4·1
2	4·8
+1	5·1
+5·5	6·9
+12·5	7·1
22	9
53	17·0
85	35
+102	48·5

S. Young: Journ. Chem. Soc. 71 p. 446 (1897)
Siedp. von normalem Pentan bei 760 mm: $36{\cdot}3°$
sp. G. bei $0°$: $0{\cdot}64536$
kr. Temp. $197{\cdot}2°$ $\pi = 25100$ mm
Volum eines gramm bei kr. Temp. $4{\cdot}303$ cm³

Dampfdrucke Biot'schen Formel: $\log p = a + b\,\alpha^t + c\,\beta^t$

$a = 7{\cdot}62281$, $b = -4{\cdot}531970$, $\log b = 0{\cdot}656866$, $c = -1{\cdot}213285$, $\log c = 0{\cdot}0980498$,

$\log \alpha = \bar{1}.0{\cdot}99926637$, $\log \beta = \bar{1}.0{\cdot}99448608$, $t = t°C + 20$.

2510 : 76 = 33·03
230
2

Wied. Ann. 5 p. 405

A. Ritter [illegible]

[illegible]

[illegible]

[illegible]

$$d\varphi = c_p\, dT - A v\, dp = 0 \quad , \quad dp = -\gamma\, dz$$

$$0 = c_p\, dT + A\, dz$$

Für die ganze Höhe der Atm [illegible] $A \int_0^H dz = - c_p \int_{T_0}^{0} dT$

[illegible] für $T_0 = 273$

$H = 27491.1\,m$ [illegible] 3. 705 [illegible]

[[illegible] Goldberg [illegible] Temp. [illegible] vertic. [illegible] Atm [illegible] d. [illegible] 10/ , Met. [illegible] 1878]

[illegible]

Atm [illegible] 0°C. : $H = 348{,}952\,m$

[illegible] $H = 28\,705\,m$

$$d\varphi = c\,dT + T\, d\left(\frac{xr}{T}\right) = 0$$ x kg. [illegible] (1-x) kg [illegible] x = [illegible]

$$\frac{xr}{T} - \frac{x_0 r_0}{T_0} = c \log\left(\frac{T_0}{T}\right)$$

Clapeyron Clausius:

$$\frac{r}{T} = A u \frac{dp}{dT}$$ [u = Vol. [illegible]

$$A \underbrace{x u}_{\neq v} \frac{dp}{dT} = \frac{x_0 r_0}{T_0} + c \log \frac{T_0}{T}$$

$$A v \frac{dp}{dT} = \frac{x_0 r_0}{T_0} + \xi \log\left(\frac{T_0}{T}\right) \qquad x_0 = 1$$

$$A\, dz = -\left(\frac{r_0}{T_0} - \xi \log T_0\right) dT + \xi \log T\, dT$$

somit $A H = \xi T_0 + r_0$

[illegible] Schlüsse [illegible] 200 km

[illegible] 26 [illegible]

[illegible] R. Pictet [illegible] Cailletet [illegible] adiab. [illegible] O [illegible] N [illegible]

[illegible] p. 416

[illegible] O [illegible] N [illegible] Schneewolken [illegible]

[illegible] Schmelztemp., [illegible] 60 [illegible]

[illegible] H = 349 km

[illegible] −650 [illegible]

[illegible] O_2
[illegible] N_2
[illegible]

[illegible]

[illegible]

$= 0{\cdot}54/100\,m$

Transf

[illegible]

[illegible]

[illegible]

[illegible] Tōhoku [illegible] p. [illegible]

[illegible]

[illegible]

[illegible] (1912) p. [illegible]

Rudzki [illegible]

15 (1912) p. [illegible]

Tōhoku 49 (1914) p. 384

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] Tōhoku [illegible] (1912) p. 310

[illegible]

[illegible]

[illegible]

[illegible] Tōhoku [illegible]

[illegible]

[illegible]

[illegible]

$$\frac{\partial u}{\partial r} \ldots + \frac{\partial u}{\partial t} \ldots$$ [illegible]

$$k \nabla^2 T = \frac{1}{k}\left(\ldots \right)$$ [illegible]

[illegible]

[illegible]

[illegible] 26/12 [illegible]

— [illegible]	0°·720	[illegible]	$= \frac{0{\cdot}76}{100}$
[illegible]	[illegible]		
[illegible]	[illegible]		

[illegible] VIII [illegible]

[illegible]	[illegible]	
0·2	[illegible]	
1·0	[illegible]	$= \frac{0{\cdot}63}{100}$
[illegible]	[illegible]	
[illegible]	[illegible]	

[illegible] CR. (1874 [illegible] [illegible] 242

[illegible] $= -4.5°$

[illegible]

[illegible]

$c(t_2 - t_1) = A h$

P. Schreiber [illegible] Civiling. 39 (1893)
Z. f. Luftschiffahrt 13 (1894) } Wahrscheinlich Unsinn

Fortschr. 50 p. 454

Egon v. Oppolzer Zur Dynamik d. Atm. M. Z. 11 (1893) Fortschr. 50 p. 455
[illegible]

[illegible]

Ferrel Fortschr. 45 (1889) p. 337 Popular Treatise on Winds

[illegible]

[illegible]

p. 227 [illegible] : Sur la masse de l'atmosphère C.R. 114

[illegible]

[illegible]

$\frac{k}{2} = \frac{1}{2}$

[illegible]

Vogel [illegible]

Nature [illegible] 7 p. 115

Dog: In the best of [illegible] p. 5

[illegible] $\frac{1}{150000}$ [illegible]

[illegible]

[illegible] 1/8 [illegible]

[illegible]

Hutchins p. 75 $\frac{1}{184000}$ [illegible]

Langley [illegible]

[illegible]

Helmholtz [illegible]   Berl. ~~Ber.~~ 1888 p 647   Fortsch. 44 (1888) p. 217

M. Zs. 5, 329   Naturw. Rdsch. 3 p 465

[illegible]:

[illegible]:

$$-\frac{\partial P}{\partial x} - \frac{1}{\varepsilon}\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \dots - \frac{k^2}{\varepsilon}\left[\frac{\partial^2 u}{\partial x^2} + \dots\right]$$

$$-\frac{\partial \varepsilon}{\partial t} = \frac{\partial(\varepsilon u)}{\partial x} + \dots$$

Statt $x\ y\ z$ | $nx\ ny\ nz$ | bleibt gleich $P\ p\ \varepsilon\ u\ v\ w\ \theta$
t | nt
k^2 | nk^2 |

[illegible]

[illegible]

[illegible]

[illegible] 8026 m ; [illegible] 28 ; [illegible] 16

[illegible] 10.8 m. red. [illegible]

$\frac{65.5}{8026}$ [illegible] $= \frac{1}{2}$ [illegible]

[illegible] z [illegible] u [illegible] x

$$\frac{\partial u}{\partial t} - \frac{k^2}{2}\frac{\partial^2 u}{\partial z^2} = 0 \qquad [illegible]\ z=0 \quad [illegible] \quad \Big|_{z=0}^{u=0} \quad \Big|_{z=h}^{\frac{\partial u}{\partial z}=0}$$

$$u = A e^{-nt} \sin(\varphi x) \qquad \frac{du}{dt} = -n u \qquad e^{-nt} = \frac{1}{2}$$

$$n = \frac{k^2}{2}\varphi^2 \qquad \varphi = \frac{\pi}{2h} \qquad + nt \log e = + \log 2$$

$$= \frac{k^2}{2}\frac{\pi^2}{4h^2} \qquad n = \frac{\log 2 \quad \log 10}{42747 \cdot 365 \cdot 86400}$$

[illegible] $e^{-nt} = \frac{1}{2}$ [illegible] $t = 42747$ [illegible]

[illegible] 10 [illegible] $(2h)$ [illegible] k^2 [illegible]

[illegible]

[illegible]

[illegible]

[illegible] $\frac{1}{2}$ red. [illegible] 36164 [illegible]

„[illegible] Convection [illegible]

[illegible]"

[illegible] Circul. [illegible] 30° [illegible]

[[illegible] 30° [illegible] 133 m [illegible]]

[illegible]

Guldberg & Mohn in Christiania Z. Met. 1878 VIII p. 113

Fortschr. 34 (1878) p. 1014

[illegible]

[illegible] $\frac{p}{p_0} = \left(1 - \frac{g z}{\alpha m T_0}\right)^m$ $p = \alpha \rho$

$m = \frac{g}{\alpha_1 \alpha}$

[illegible]

[illegible] Dönitz 1888 Met. Z. p. 466

[illegible]

Bezold [illegible] Thermodynamik d. Atm. Met. Z. 6 p. 201, 287 Fortschr. p. 220

[illegible]

potentielle Temp. = Temp. [illegible] adiab. [illegible] Normaldr. [illegible]

Trockenstadium

Regen- "

Hagel "

Schnee "

Oberbeck [illegible] Atm. Met. Z. 5 p. -05 Fortschr. 225 (1888)

Licht [illegible] incompr. [illegible]

[[illegible] Circulation [illegible]!]

[illegible] Max. 45°

[illegible] 35°16' [illegible] Äq. 60° [illegible]

[illegible] Vertical [illegible] Rad.

[illegible]

2). [illegible] Rot. [illegible] Parallel [illegible] NW [illegible] Äq. [illegible] 35° [illegible]

[illegible] NO [illegible] Äq. [illegible] Pol. Max. 54°

André Fatel. 44 p. 238 Sur les mouvements verticaux de l'air

[illegible]

Linss [illegible] Mz. 5 p 77 (1888) Fatel p 239

[illegible] $v = 0.04$ [illegible] Liter, [illegible] $\frac{mm}{Stunde}$

[illegible] Cyklonen [illegible] $r = 2$ [illegible] 3 mm also [illegible] $v \geq \frac{10\,cm}{sec}$ [illegible]

[illegible] XIV [illegible]

[illegible] 170°

[illegible] −170°

Naturw. Rdsch. 1888 p. 370

Maurer [illegible] Ballfahrt am 3/X [illegible]

[illegible] Anticyklone [illegible]

[illegible] Cyklone

Zu Rosin. [illegible] 1500 – 7500 0.56° pro 100 m

0.62°

[illegible] 5000 m 0.80

p. 363 Brutstadt [illegible]

[illegible] 28/VII und 24/VII [illegible]

[illegible]

Rudzki

[illegible] Abbe Atmosph. Radiation of Heat Amer. J. of Science 1892 (Mai)

[illegible] Tratut: 1 kg. [illegible] 0·032 – 0·086 Cal.

Fröhlich Rep. f. Meteor. VI [illegible] temp. [illegible] ca −127° – 131°

Mendelejeff [illegible] C. Pet. 1884 (p. 269)

[illegible]

{ Oberbeck [illegible] Crelle $\boxed{\text{LXXXI}}$ 1876 p. 62
{ Lamb Motion of Fluids p. 225 [illegible]

[illegible]

Ritter's [illegible]: [illegible] adiab. [illegible]

[illegible] Centr. [illegible] (?)

Mascart [illegible] Journ. d. Physique 1892

Gromeka [illegible] temp. [illegible] = const. ([illegible])

Γρονεκα Kasan 1886 Niekotorye sluczaj ravnovesija
sviernennago gaza.
Thomson: Equilibrium of a gas under its own gravitation
Phil. Mag. 5 ser. 23 p. 287
Ritter Wied. Ann. 1882

Olszewski $-220°$ 4 mm [illegible] Handb. d. allg. u. [illegible] 1892

[illegible] (?)

Kin. [illegible] $> \sqrt{2ga}$ [illegible]

$= 11200$ m

[illegible] (!)

[illegible] Centrif [illegible]

[illegible]

[O_2 [illegible] : $\frac{1}{2\sqrt{\pi a}} \int_{11200}^{\infty} e \, dv \, e^{-\frac{v^2}{a^2}}$ [illegible] Krampow [illegible]].

$\alpha = 397$

~~[illegible]~~

[[illegible] 804 m [illegible] $\frac{219}{100000}$ [illegible] O_2]

[illegible]

[illegible]

Why e [illegible]

1). station [illegible]

2). [illegible]

3). [illegible]

*) $\frac{\partial u}{\partial t} - 2(v\zeta - w\eta) = -\frac{\partial P}{\partial r} - \frac{2\mu}{\rho}\left(\frac{1}{r}\frac{\partial\zeta}{\partial\theta} + \frac{\zeta\cot\theta}{r} - \frac{1}{r\sin\theta}\frac{d\eta}{d\varphi}\right) + \frac{\mu}{3\rho}\frac{\partial\delta}{\partial r}$

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + v\frac{\partial\rho}{r\partial\theta} + \frac{w}{r\sin\theta}\frac{\partial\rho}{\partial\varphi} + \rho\delta = 0$$

---

*) [illegible] Whitehead Sec. appr. to viscous fluid motion Quart. J. XXIII 189[illegible] p. 145

([illegible] $u=v=0$, [illegible] Russell [illegible])

[[illegible] Stokes M. ph. papers Camb. 1880 I p. 103]

(Edwardes: Steady motion of a viscous fluid in which .. Quart. J. 1892 p. 75

[illegible])

Hicks Recent progress in Hydrodyn. Rep. B. Ass. 1881 p. 80: [illegible]

[illegible]

$P = \frac{1}{2}(u^2+v^2) + \int\frac{dp}{\rho} - V$ [illegible] $u=v=0$, $\delta=0$ (incompr.)

$\zeta=0 \quad 2\zeta = \frac{1}{r}\frac{\partial w}{\partial\theta} + w\frac{\cot\theta}{r} \quad 2\eta = -\frac{\partial w}{\partial r} - \frac{w}{r}$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(wr) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(w\sin\theta)\right] = 0$$

[illegible]

$w = \frac{\omega_0 a^3}{r^2}\sin\theta$ $\qquad$ $\frac{a^3}{r^3}\sin\vartheta = 289$

$$\frac{\partial V}{\partial r} + \frac{a^6\omega^2}{r^5}\sin\vartheta = \frac{1}{\rho}\frac{\partial p}{\partial r} \qquad \frac{\partial V}{\partial r} = -\frac{g a^2}{r^2}$$

$$\frac{\partial V}{\partial\theta} + \frac{a^6\omega^2}{r^4}\sin\theta\cos\theta = \frac{1}{\rho}\frac{\partial p}{\partial\theta}$$

$\frac{\partial p}{\partial z}$ [illegible] < 0 [illegible] Laplace

ρ centrif. [illegible]

~~[illegible]~~ [illegible] $6\tfrac{1}{2}$ km [illegible] $2\,\frac{1\,\text{m}}{\text{m}}$ [illegible]

---

transl. [illegible] Crelle LXXXI 1876 p. 62

[illegible] [comp. Lamb motion of fluids p 225]

[illegible] 2 [illegible]

2 Aequator [illegible] $p = \text{const} + \frac{3}{2}\mu\frac{V}{a}\cos\varphi$ $\qquad$ $V = 1$ [illegible] (transl.)

a = Radius

$\mu = 0{\cdot}134\,\rho$

$\rho = \frac{760}{9{\cdot}81\,\text{m} \cdot 6370000\,\text{m}}$ $\qquad$ 10^{-14} mm. [illegible]

[illegible] incomp. [illegible]

[illegible] Idee [illegible]

G. G. Schmidt (Stuttg.) [Gilb. Ann. 62 (1819)]

[illegible]

[illegible] Nr. 1, [illegible] temp. [illegible]

[illegible] sp. [illegible] temp.

I). temp. Abnahme linear nach Humboldt 121.1 Toisen für 1° Reaumur

ergibt $h = 6.6$ g. M. (0°)

II [illegible] prop. d. temp.

Formel: $z = a \log\left(\frac{c+t}{c+x}\right)$ $z = ??$, x = temp.

ergibt 27.5 g. M.

Kelvin: Phil. Mag. (5) 23 p. 287

F. Neumann theor. Physik

Hicks B. A. R. 1881

Gustav Mie Entwurf einer allg. Th. d. Energieübertrag. !
Wien. Sitzgsber. 107 p. 1113 (1898)

Liénard Die theorie von Lorentz (L'éclairage électrique 14 p. 417, 456, 1898)

Poincaré [illegible] magn. Energie [illegible] Maxwell [illegible] Hertz (L'écl. électr. 18 p. 361 1899)
[illegible] $(c v_0 - b w_0 + a u_0)\, d\tau = \int \frac{[illegible]}{8\pi}\, d\tau$
[illegible] Energie [illegible] $\int \mu\, d\tau \frac{\alpha^2 + \beta^2 + \gamma^2}{8\pi}$

$u_0\ v_0\ w_0$ [illegible] Compon. [illegible]

Fabry & Perot CR. 128 p 1156 (1899) intensive monochrom. [illegible]
[illegible] Bogenlampe im Vacuum [illegible]

Planck Rem. Thi. Annl. 23 p. 702 (1899)

[illegible]

Demarçay [illegible]

Boccara [illegible]

N. Cim. [illegible]

1826 [illegible] Petersburg. Akad.: [illegible]

Fresnel geb. 1788 gest. 1827
1815 Erste Abh. ; vollst. 1818 Mém. sur la diffr. de la lum.
1821 polarisation double réfr.

Laplace, Poisson etc. [illegible]

Wollaston 1802, Brewster, Frankfurter 1821 [-30) [illegible]

Hamilton - Lloyd 1832

1858 Balfour Stewart The absorption of a plate equals the radiation and that for every description of heat.
(1855 Angström [illegible])

Wied. Ann. 67 p. 452
Jaumann[illegible] - Voigt [illegible]
[illegible] Stearin [illegible] Voigt Wied. Ann. 53 p. 43 (1894)
[illegible] Constante [illegible]
[illegible] 145 [illegible] 10 - 413 [illegible]
[illegible]

Ketteler [illegible] Wied. Ann. 49 p. 507 !

Drude 48. p. 542 ! Dielectric [illegible]

[illegible] van der Waals [illegible] Boltzmann – Mechanik [illegible] 68 p. 350

$$\left(pT + \frac{a}{v}\right)\left(v - \tfrac{1}{3}b\right) = 2T\left(v + \tfrac{2}{3}b\right)$$

[illegible] $\left(v - \frac{b}{3}\right)^{-1} \infty$ [illegible] $v = \frac{1}{3}b$

Amagat CO_2

Humphreys [illegible] p 219

[illegible]

1). [illegible]

2). [illegible] 3). [illegible] 4). [illegible] temp.

II 5). [illegible] 6). [illegible] 7). [illegible]

8). [illegible] [reducing [illegible]] [illegible]

9). [illegible] Licht. [illegible] magnetic [illegible] coeff. $\sqrt[3]{\text{at. vol.}}$ (? 1)

11). [illegible] $\frac{1}{2}$ [illegible] $\sqrt[3]{\text{at. vol.}}$

12). [illegible] (Atomgew.)

pag. 558 Ann. Bd. 22

Wilsing [illegible]

[illegible] Lommel's Absorpt. theorie [illegible] harm. [illegible] period. [illegible]

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + p^2 x + b\varepsilon x^2 + \cdots + f \sin qt = 0$$

Absorpt. Max. wenn $q = p$ [illegible]

[illegible] Period [illegible]

[illegible]

[illegible]

$$\frac{d^2x}{dt^2} + 2k_1\frac{dx}{dt} + p_1 x^2 + \cdots + f \sin qt$$

$$k_1 = \frac{k}{1+\gamma} \qquad f_1 = \frac{f}{1+\gamma} \qquad p_1^2 = \frac{p^2}{1+\gamma}$$

Hauptabs. für $q^2 = p_1^2$ [[illegible] $\gamma = 8.10^{-6}$]

$\frac{k}{p^2} <$ [illegible]

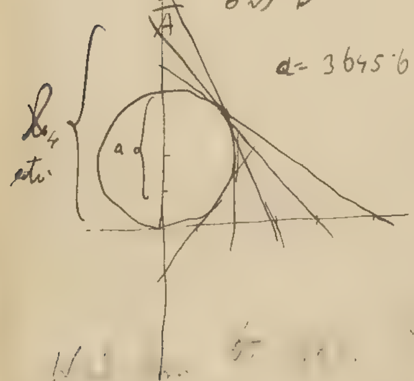
[[illegible] Spectra [illegible]]

Balmer W. Ann. 60 (1897) p. 380 ... Spectralen

$$\tau_n = A - \frac{B}{n^2} - \frac{C}{n^4} \quad \text{nach } KR.$$ (... ca $10^{-5}\lambda$)

$$\frac{1}{\lambda} = \tau_n = A - \frac{B}{(n+c)^2}$$ ... Balmer

$\frac{B}{A}$ ... = ... Rydberg



[illegible]

[illegible]

[illegible] 98 p. [illegible]

[illegible]

[illegible]

[illegible]

p. [illegible]

[illegible]

[illegible]

[illegible]

41 [illegible] 56

[illegible]

[illegible]

41 [illegible] 616, 270 [illegible]

[illegible]

42. p. 748 [illegible] Bemerkung [illegible]

p. 46 [illegible]

p. 674 [illegible]

[illegible] 22 [illegible]
24 [illegible]
22 [illegible]

[illegible]

[illegible]

43. [illegible] [illegible] 48 p. 3[illegible]

45 [illegible] 7?2

[illegible]
$1\,cm^2$ [illegible]

[illegible]

[illegible]

47 p. 2[illegible] [illegible]

47 p [illegible] [illegible]

48 p [illegible] [illegible] !!!
[illegible]

49. 3[illegible] } [illegible]
45. p [illegible] }

[illegible] p [illegible]

[illegible] p [illegible]

55 p [illegible]

p [illegible] [illegible]

[illegible]

[illegible] p [illegible]

51 p. 1

p. [illegible]

p. 44 [illegible]

52 p. 105

5[illegible] p [illegible]

52 p [illegible]

[illegible]

p [illegible] [illegible]

[illegible]

16 p. 7[illegible] [illegible]

p. [illegible] [illegible]

17 p. [illegible] [illegible]

5 ~~[illegible]~~

~~[illegible]~~

5[illegible] p [illegible]

[illegible] 52 p [illegible]

[illegible]

61 [illegible] p. [illegible]

62 [illegible] p. [illegible]

[illegible] [illegible] 30 p. [illegible]

~~[illegible] p. [illegible]~~

~~[illegible]~~

[illegible] 62 p. [illegible]

[illegible] p. [illegible]

[illegible] 1 p. [illegible]

[illegible] p. 280

[illegible] 62 p. [illegible]

[illegible] p. [illegible]

[illegible]

~~[illegible] no. 27 p. [illegible]~~

[illegible]

O E Meyer Pogg Ann 127 p. 253

$$\rho \frac{du}{dt} = \eta D_u^2 + \frac{1}{3}\eta \frac{\partial \Delta}{\partial x} - \frac{\partial p}{\partial x}$$

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$D_u^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

. . —

$$\rho u \frac{\partial u}{\partial x} = \eta\left[\frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] - \frac{\partial p}{\partial x}$$

$$0 = \frac{1}{3}\eta\frac{\partial^2 u}{\partial x \partial r}$$

$$0 = \frac{\partial(\rho u)}{\partial x}$$

Wenn [illegible]: p, ρ [illegible] $v = w = 0$

[illegible] $u, \rho, p = f(x)$ [illegible]

[illegible] $\underline{u=0}$ [illegible]

[illegible]

$u\frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 u}{\partial x \partial z}$ [illegible]

[illegible]

$$0 = \frac{1}{\rho}\frac{\partial \rho}{\partial x} + \frac{1}{u}\frac{\partial u}{\partial x}$$

$$\frac{dp}{dx} = \eta \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

$$0 = \frac{\partial \rho u}{\partial x}$$

$$p = k\rho(1+\alpha\vartheta)$$

u : p Nyrd x

$\frac{\partial u}{\partial r} = 0$ für $r=0$

Wied Ann. 61!
63!

Wied Ann. 64

p. 555

Wied Ann. 65

66

67

[illegible] p. 702 [illegible]

p. 80. [illegible]

p. [illegible]

[illegible]

[illegible]

[illegible]

Taylor [illegible] p. [illegible]

[illegible] {123 ; [illegible] 125 [illegible]

[illegible]

C. [illegible] [illegible]

[illegible] Ber. 105 p. 21 [illegible]

[illegible] Ber. 105 p. [illegible]

[illegible]

[illegible] p. [illegible]

[illegible] p. 516

[illegible] 6 p. [illegible]

Milner Phil. Mag. 43 p. 291 (1857)

20 [illegible] 57 / 50

[illegible]

[illegible] 2, [illegible]

[illegible] ([illegible]) !! p. 840

[illegible]

[illegible]

[illegible] p. 855

[illegible] 5, p. 457 [illegible]

[illegible] p. 224 [illegible] 21 p. 119

[illegible] N.A. 57, [illegible]

[illegible] 40 p. [illegible]

[illegible] 160 [illegible]

[illegible] p. 541 [illegible]

[illegible] 20 p. 461 [illegible] 17 p. 578 [illegible] 08 p. 335 [illegible]

[illegible] N.A. 50, [illegible]

~~[illegible] 26 p. 431~~ p. 325

[illegible] 41, [illegible]

[illegible] 50, p. 147 [illegible] p. 551

[illegible] 1895 [illegible]

p. 755

[illegible] 1096 [illegible]

-1093

1173, 1174, 1194

21 ~~[illegible]~~

[illegible] p. 73

[illegible]

[illegible] p. 110 [illegible] 1896)

V. d. Waals [illegible] p. 206

[illegible] p. 207

H. A. Lorentz [illegible] p. 400

[illegible] 1895/7 p. [illegible]

[illegible] 1896)

[illegible]

V. d. Waals [illegible] 2 p. 210

[illegible] p. 21 [illegible] 1897)

~~Voigt p. 854 Gött. Nachr. 1896 [illegible]~~ 22 p. 772

14 [illegible]

[illegible] p. 736 [illegible]

[illegible]

[illegible]

[illegible] 57 [illegible]

[illegible] 475, p. [illegible]

53 [illegible]

Planck [illegible]

[illegible]

[illegible] 51, p. 11, 321

[illegible] 52 [illegible]

[illegible]

[illegible] 2, p. [illegible]

[illegible] 57, p. [illegible] p. 736

[illegible] 56 [illegible] p. 30

[illegible] 57 p. 32 [illegible]

[illegible] 53 [illegible]

Poincaré [illegible] 5 p. [illegible]

[illegible] p. [illegible]

Sutherland Phil. Mag. 36 39 [illegible] 604

[illegible] p. 572

[illegible] CR 120, p. [illegible]

[illegible] p. 404

[illegible] Th. A. 52 p. [illegible]

p. [illegible]

[illegible] !! p. [illegible]

[illegible]

! [illegible] ! p. 97

~~[illegible]~~

~~[illegible]~~

Th. A. 22 p. 61

Dunbury [illegible] Th. 137 p. 578

[illegible]

Arch. [illegible] 26 p. [illegible]

[illegible]

[illegible] 13 p. [illegible]

[illegible] 11 p. [illegible]

[illegible]

Bryan Proc. Camb. Phil. Soc. 1894 $\frac{29}{11}$ 28

Boltzmann Phil. Mag. ~~XXXV~~ 35 p. 156 (1893) motion of ball upon billiard table

Dec. 17 p. 837

Maxwell Coll. Sc. Pap. $\frac{II}{p}$ 713 (1879)

Lord Kelvin Proc. R. S. 50 p. 85 (1891)

Rayleigh ~~Proc. R. S.~~ Phil. Mag. April (1892) p. 356

Kelvin Phil. Mag. May (1892) p. 466

~~Burbury Nature 46 p. 100 (1892)~~

Stoney Phil. Mag. 40 p. 362

On the Kinetic Th. of Gas regarded as ill. Nature

Not necessary reality but ideal mechanism

A = 3 motions of centre of gravity

B = intramolec. { Ba, Bb, Bc } much (little) short } influenced by A; Bc not influenced by A

~~aftereffect of~~ γ must depend on time

Effect of aether may be to link the B motions in a molecule with themselves

(See Fitzgerald) so that there will be much less degrees of freedom than without aeth.

electrons associated with Ba events will produce absorption spectrum

Bc - - - - gas transparent

Bb = $\boxed{\text{phosphorescence}}$ intermediate

p. 377

{ Bb_1 fast, Bb_2 slow } varying

It is possible that Bb motions influence very much the A or Ba motions, but in contrary are little influenced by them (?) p. 376

In fact transparent diatomic gases have γ = ca 1.4 smaller ~~and~~ coloured

5 freedom

[there may be 6 8 degrees more but linked by aether, therefore there appears but a small fraction]

What may be the effect of ~~...~~ perturbating forces not obeying the condition of Draw-Dotto?

Intermolec. bonds within a crystal ~~of those~~ of the same kind as interatomic in a molecule = = forces between electrons

In chemical actions: if electrons associated with Da events: colour effect
Db_2 " lightning "
Db_1 " both of them

~~the~~ temperature in Plücker tube need not to be high

Lewis Proc. R. Soc. March 1895 Radiant heat = first cause of chemical action ...

Rayleigh Phil. Mag. 45 p. 522 (1898) Pressure of Radiation !!

17 Heaviside p. 1000 Analogie Gravitation, Electromagn. Electrician 31 p. 281 (1893)
p. 702 Operatoren in d. math. Physik Tr. R. S. 52 p. 504 "

~~Sir W. Thomson Kinetische Theorie d. Energie Erstheny Ph. M. 33 p. 291~~ (1892)

~~...~~

[illegible]

[illegible] XI

[illegible] 80

152

[illegible] 32 [illegible]

[illegible] 32 [illegible] 1881

[illegible] 1881

p. 191 [illegible]

Tait p. 192 [illegible]

[illegible] p. 32 [illegible]

[illegible]

[illegible]

p. 95

Th. d. i. l. Sting d. Walze [illegible] ; [illegible] de pol. Th. von Voigt

Lit. 17 p. 726 (1882)

[illegible] 21 p. 44 76 (1880)

[illegible]

[[Wied. Ann. 68 p. 441 [illegible] Tornwall's (456) [illegible] Wetzstein]]!

Dieterici Wied. Ann. p. 826 Kin. Th. d. El.

[illegible]

[illegible] Ann. 99 p. 681, 861 [illegible]

Riecke W. Ann 50 p. 79 (1893) [illegible]

Voigt [illegible] p. 341 (1896), 19, 261 (1897) [illegible]

[illegible] Theorie

Milner Ph. Mag 43 p. 291 (1897)

A — a konz. [illegible]

O — inh. D.

J — ink. H.

i [illegible]

Annahmen:

1). [illegible] kons [illegible]

2). [illegible]

3). Abl. [illegible]

[illegible] $\frac{N_i}{2\sqrt{k_i m\pi}}$ [illegible] $\frac{N_a}{\sqrt{2 k_a m \pi}}$ $\left(= N_a \frac{\dots}{\dots} \int_0^\infty u\, e^{-k_a m u^2} du \right.$

[illegible] Schwellenwerth (beliebig)

$$n = N_i \sqrt{\frac{k_i m}{\pi}} \int_s^\infty \dots = \frac{N_i}{2\sqrt{k_i m\pi}}\, e^{-k_i m s^2} = \frac{N_a}{2\sqrt{k_a m\pi}}$$

wobei $\frac{1}{2} m c_a^2 = \frac{3}{4 k_a}$

also $N_i c_i e^{-h^2} = N_a c_a$ (3) [illegible]: $k_i m s^2 = h^2$

Energien:

$$\Sigma u\, \dots = N_i \sqrt{\frac{k_i m}{\pi}} \int_s^\infty \frac{1}{2} m u^2\, u\, e^{-k_i m u^2} du = \frac{N_i}{2\sqrt{k_i m\pi}}\, e^{-k_i m s^2} \left[\frac{m s^2}{2} + \frac{1}{2k_i}\right]$$

$$= n \left\{ \frac{1}{2} m s^2 + \frac{1}{2k_i} \right\}$$

davon ist die Durchbruchsarbeit $\frac{m s^2}{2} \cdot n$ unbestimmt es bleibt also $\frac{n}{2k_i}$ [illegible]

$$= \frac{N_a}{2\sqrt{k_a m\pi}} \frac{1}{2k_a}$$

(4. also $N_i m c_i^3 e^{-h^2} = N_a m c_a^3$

Somit aus (3): $c_i = c_a$

Dasselbe schon bei Voigt

Druck. $\int_{iA} = N_i \sqrt{\frac{k_i m}{\pi}} \int_0^\infty m u^2 e^{-k_i m(u^2/\cdot - s^2)} du = \frac{N_i}{4k_i} e^{-k_i m s^2} =$

$\int_{aA}$ $= \frac{N_a}{4k_a}$

(5) $N_i m c_i^2 e^{-h^2} = N_a m c_a^2$

Somit wieder $c_i = c_a$

Dies ist [illegible]

Ebenso Boltzm. Zeitsch. ph. Ch. 6 p. 474 (1890), 7 p. 88 (1891)

7 p. 36

Riecke „

Lorentz „ 6 p. 504

[illegible] Energie [illegible]

[illegible] $\frac{mc^2}{2}$ [illegible]

Die Gleichungen geben also: $N_i m e^{-h^2} = N_a m$

[illegible] vom spec. Vol.: $\frac{v_a}{v_i}$ (6) $\frac{v_a}{v_i} = e^{h^2}$ $h^2 = \frac{3}{2}\frac{ms^2}{mc^2}$

Physik. Bedeutung des [illegible] h^2 nach Voigt: $\frac{ms^2}{2} = T \cdot$ [illegible]

[illegible] (6) [illegible] $\frac{v_a}{v_i}$ [illegible]; [illegible]!

Man könnte (6) corrigiren durch Einführung der V. d. Waals'schen Correctionen:

$$\frac{v_a - b_a}{v_i - b_i} = e^{+h^2}$$

aber dann folgt für $v_i - b_i$ außerordentlich kleine Werthe, so dass dann [illegible]

[illegible] (Voigt)

Es muss also $\frac{ms^2}{2} = T\rho$ [illegible]; dies ist schon d. Kilmer [illegible], [illegible]

[illegible]:

Phil. Mag. 43 [illegible] p. 1[illegible]

[illegible] — [illegible]

[illegible]

[illegible] $N\,d\omega = [illegible]$ (1)

[illegible] $= d\varphi$

[illegible]

[illegible] $\frac{\lambda - s}{\lambda} = \frac{v - b}{v}$ (2)

[illegible]

[illegible]

Therefore [illegible]:

$$\tfrac{1}{2} m v_1^2 \cos^2\theta = [illegible] \quad (3)$$

[illegible]; $v_1 = \sqrt{[illegible] \; \frac{v-b}{v} \; \frac{1}{\cos\theta}}$

[illegible] law: $\frac{2N}{\sqrt{\pi \alpha^3}} \sin\theta \, [illegible]$ [illegible]

[illegible] $\times \frac{\lambda}{\lambda - s} = \frac{v}{v-b}$

Thus [illegible]

$N_1 = \frac{2N}{\sqrt{\pi}} \cdot$ [illegible]

$= \frac{N}{2\sqrt{\pi}}$ [illegible] (7)

[illegible] $N + dN$ [illegible]

$N_2 = \frac{2(N - dN)}{\sqrt{\pi}}$ [illegible]

$\therefore \quad N_2 = \frac{N - dN}{2\sqrt{\pi}}$ [illegible] (8)

$\therefore$ [illegible] e [illegible] $=$ [illegible]

$dL_i = \frac{RT}{[illegible]} \frac{v\,dv}{[illegible]}$

[illegible] : $L_i = \frac{RT}{[illegible]} \{ \log$ [illegible] $\}$ (9)

The [illegible] ~~[illegible]~~ [illegible]:

(10) $p(v - b) = \frac{RT}{[illegible]}$ [illegible]

[illegible]:

[illegible] $+ \frac{RT}{[illegible]} \frac{dv}{v - b} = 0$

[illegible] : $dp = p\,dT$

[illegible] $L' = \frac{1}{2}|V' - V|$

$V' - V = \frac{RT}{[illegible]} \int_v^{v'} \frac{[illegible]\,dv}{[illegible]} - TL_i$ [illegible]

[illegible] (9)

Sitzber. 99 Jäger p. 679 Zur Th. d. Dampfspannung

y v Mol. ~ 1,6 st. / b $\frac{mu^2}{2} > a$

$$Z = N\sqrt{\frac{hm}{\pi}}\, e^{-hmu^2} du = N\sqrt{\frac{3}{2\pi c^2}}\, e^{-\frac{3}{2}\left(\frac{u}{c}\right)^2} du = \frac{N}{\sqrt{\pi}}\, e^{-x^2} dx \qquad x = \frac{u}{c}\sqrt{\frac{3}{2}}$$

$$n = \frac{N}{\sqrt{\pi}} \int_{\sqrt{\frac{3a}{mc^2}}}^{\infty} e^{-x^2} dx \qquad \left.\begin{aligned} a &= a_0\left(1-\varepsilon \vartheta\right) \\ c^2 &= c_0^2\left(1+\alpha\vartheta\right)\end{aligned}\right\} \sqrt{\frac{3a}{mc^2}} = \sqrt{\frac{3a_0}{mc_0^2}\,\frac{1-\varepsilon t}{1+\alpha t}}$$

$$N = N_0\left(1+\gamma\vartheta\right)$$

$\underbrace{n = \text{prop.}}_{((a\,?))}$ e Dampfdichte d ?: $d = C\,(1+\gamma t)\displaystyle\int_{k\sqrt{\frac{1-\varepsilon t}{1+\alpha t}}}^{\infty} e^{-x^2}dx$

$$\int_x^\infty e^{-x^2}dx = \frac{e^{-x^2}}{2x}\Big[\underbrace{1 - \tfrac{1}{2}\,\frac{1}{1+x^2}}_{!} + \tfrac{1}{4}\,\frac{1}{(x^2+1)(x^2+2)} - \cdots\Big]$$

$\alpha = 0{\cdot}00366$

$\varepsilon = 0{\cdot}00230$

[illegible] $d_0 \quad d_{50} \quad d_{100}$:

$c = \cancel{3437000}$ 687600

$\gamma = -0{\cdot}00385$

$k = 3{\cdot}499$

[illegible] d_0 bis d_{100} [Abw. $< \frac{1}{2}\%$]

[Kein Wunder mit 3 Konst.!]

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pg 860 Die [illegible] des Flors mol.

[illegible] α 3 [illegible] Konst.

$c = 693600$

$\gamma = -0{\cdot}00390$

$k = 3{\cdot}500$

[illegible]

$$\frac{mu^2}{2} - \frac{mu'^2}{2} = a \qquad u' = \sqrt{u^2 - \frac{2a}{m}}$$

$$\int_0^\infty Z u' = [illegible] = N\sqrt{\frac{3}{2\pi c^2}}\int_{\sqrt{\frac{2a}{m}}}^\infty \sqrt{u^2-\frac{2a}{m}}\; e^{-\frac{3}{2}\left(\frac{u}{c}\right)^2} du =$$

$$= Nc\sqrt{\frac{2}{3\pi}}\int_{\sqrt{\frac{3a}{mc^2}}}^\infty \sqrt{x^2 - \frac{3a}{mc^2}}\; e^{-x^2}dx \qquad \left[\frac{3}{2}\frac{u^2}{c^2} = x^2\right]$$

Für [illegible] $a = 0$ $\therefore$ $N'c'\sqrt{\frac{2}{3\pi}}\int_0^\infty x e^{-x^2}dx = N'c'\sqrt{\frac{1}{6\pi}}$

[illegible] ; [illegible] $d = \frac{N'mc'^2}{3}$ (2)

also $N'c' = 2Nc\int_{\sqrt{\frac{3a}{mc^2}}}^\infty \sqrt{x^2 - \frac{3a}{mc^2}}\; e^{-x^2}dx$ (1)

$$= \frac{2}{3}Nmcc'\int_{\sqrt{\cdots}}^\infty \sqrt{\cdots}\; dx = \frac{2}{3}p_0(1-\beta t)(1+\alpha t)c_0c_0'\int_{k\sqrt{\frac{1-\cdots}{\cdots}}}\sqrt{x^2 - \cdots}$$

$N_m = \rho_0(1-\beta t)$ $\quad \beta =$ Ausdehnungskoeff. #

$c\,c' = c_0\,c_0'(1+\alpha t)$ weil [illegible]

Dies Integral angenähert (?) ausgewertet:

$$d = 0{\cdot}523\,\rho_0(1-\beta t)\,c_0 c_0'(1+\alpha t)\left[\frac{1}{\frac{3a}{mc^2}} + \frac{0{\cdot}726}{\left(\frac{3a}{mc^2}\right)^2}\right]e^{-\frac{3a}{mc^2}}$$

Aus d_0 und d_{100} ergibt sich $\quad 0{\cdot}523\,\rho_0 c_0 c_0' = 67.10^4$

$$\frac{3a_0}{mc_0^2} = 0{\cdot}66$$

Daraus berechnet sich d zwischen 0° und 100° mit bis zu 4% Abweichung

Nach Einsetzen von Dampfdichte $H_2O = 0{\cdot}623$, $\rho_0 = 1$ ergibt sich aus

$$(2):\quad \left.\begin{aligned} c_0' &= 61400 \text{ cm.} \\ c_0 &= 27800 \text{ cm} \end{aligned}\right\}\ (?!)$$

Auch noch auf andere Weise:

$$\frac{N}{N'} = \text{[illegible]} = \frac{6}{6'} \qquad \text{Somit [illegible] (1)}$$

$$c' = 2\frac{6}{6'}\,c\int^{\infty}\sqrt{x-\frac{3a}{mc^2}}\;e^{-x^2}\,dx$$

und bei Einsetzen des Wertes dieses Integrals

$$c_0 = 26700 \text{ cm}$$

[illegible] $\frac{a}{m}$ [illegible]

[illegible]

[illegible] !

$$\frac{a_0}{m_1} = 25.10^8\ (CGS) \qquad \text{[illegible] ca 10 mal [illegible]} !!$$

[illegible] incompr. [illegible] Stefan (Wied. Ann. 29 p 658) $\quad \rho A = p_2 - p_1$

$$A = \frac{a}{2m}$$

$p_2 - p_1 = 1235$ Atm. [illegible] (Fick 1221)

[illegible] $\frac{a}{2m}$

[illegible] $N\pi r^2$

$$\delta = \frac{3m\alpha}{a} \qquad \alpha = \text{Cap.} \;\; 76 \cdot 7 \, (\text{CS}_2)$$

$$= 2 \cdot 10^{-9} \text{ cm}$$

---

p. 1028 [illegible] 10^{-4} [illegible] temp.

$v\,dp = \int do$ = [illegible] Hydr.

$$\int_{v_1}^{v_2} v\,dp = A = \text{[illegible]} \; v_1 \; v_2$$

$$= \frac{a}{m}$$

$$p(v-b) = \frac{Nmc^2}{3} \quad (\text{Wüllner I. p. 954})\,(1)$$

[illegible] momentum [illegible] (!!)

[illegible] temp. [illegible] compressibel [illegible]

[illegible]:

$$c^2 = c'^2\left(1+\frac{\beta}{v}\right) \quad [\text{illegible}]$$

$$c^2 = c_0^2(1+\alpha t)\left(1+\frac{\beta}{v}\right)$$

$$p = \frac{c_0^2(1+\alpha t)\left(1+\frac{\beta}{v}\right)}{3(v-b)} \qquad dp = \cdots$$

$$v\,dp = \cdots$$

$$\int_{v_1}^{v_2} v\,dp = \cdots$$

$$\underbrace{\frac{\beta}{v_2-b} + \frac{\beta}{b}\lg\frac{v_2}{v_2-b}}_{\text{[illegible]}} + \lg\frac{v_1}{v_2} = \delta\,\frac{1-\gamma t}{1+\alpha t}$$

[illegible] v_1, v_2 [illegible]

[illegible] (Kein Wunder!!) 3 Monate!

[illegible]

[illegible]

---

100 p. 245 [illegible] Coop. [illegible]

[illegible]

[illegible] $\alpha = C(1+\gamma\vartheta)\int_{k\sqrt{\frac{1-\varepsilon t}{1+\alpha t}}}^{\infty} e^{-x^2}dx$ ([illegible])

[illegible]

[illegible] $v_{eff} \sim e^{\delta\frac{1-\varepsilon t}{1+\alpha t}}$

[illegible] $\alpha = \frac{2}{3}\rho_0(1-\beta t)\, g c_0^2 (1+\alpha t)\int_{k\sqrt{\frac{1}{\ldots}}}^{\infty}\sqrt{x^2-k^2}\ldots e^{-x^2}dx$

[illegible] c_0 [illegible]

[illegible]

$\delta =$ [illegible] $= 44\cdot 10^{-9}\sqrt[3]{\frac{\mu}{18 s}}$ $\mu =$ Mol. [illegible], $s =$ [illegible]

$\delta = \frac{3m\alpha_0}{a_0\rho_0}$ Coop. [illegible] $A = \frac{a_0\rho_0}{2m}$ [illegible] c_0 [illegible] $k^2 = \frac{3a_0}{mc_0^2}$

[illegible] c_0 [illegible] c_0^2 [illegible]
$(\frac{1}{2}$ bis $\frac{2}{3})$

Nature 45 (1892)

Rayleigh p. 80. On the Virial of a system of hard colliding bodies.

1). Van d. Waals 1873 by a peculiar way of argumenting $p(v-b) = \frac{1}{3}\Sigma m V^2$

2). Maxwell (Nature X p. 477, 1874) says: v is the volume of the vessel and not subject to correction. But by calculating the Virial he finds:

$$pv = \frac{1}{3}\Sigma m V^2\left\{1 - 2\,\mathrm{lg}\left(1 - 8\frac{\rho}{\sigma} + \frac{17\rho^2}{\sigma^2} - \dots\right)\right\} \quad \Bigg| \; \frac{\rho}{\sigma} = \frac{b}{4v} \; \Bigg| \; \begin{matrix}\rho = \text{mean density} \\ \sigma = \text{density of mol}\end{matrix}$$

$$= \text{approx. } \frac{1}{3}\Sigma m V^2\left(1 + \frac{4b}{v}\right)$$ which certainly is not ~~true~~ erroneous.

3). Lorentz Wied. Ann. 12 p. 127 and } arrive by calculating the virial

4). Tait Ed. Tr. 33 p. 90 1886 } according to Maxwell's suggestion to

V. d. W. Formula

with the same order of approximation

But Maxw. Eq., though numerically erroneous, very remarkable suggestion:

p must be proportional to $\underline{\Sigma m V^2}$ because all velocitys can be altered in ~~are~~ a constant ratio, without changing the manner of motion. Also if the molecules be bodies of any form.

$$\therefore \quad p = T\,\varphi(v)$$

If there are forces, besides, there must be added to p a term $\frac{a}{v^2}$, if the range of forces large in comparison with molecular distances.

$$p = T\,\varphi(v) - \frac{a}{v^2}$$ relation between pressure and temperature: linear verified by Ramsay – Young

$$p = T\,\varphi(v) + \chi(v)$$

~~$\chi(v)$ cannot be~~ Young - capillary
as long as there are only the two sorts of forces: repulsive and (Laplace Forces)
$\chi(v)$ must be $= -\frac{a}{v^2}$, other functions require assumption of intermediate kinds of forces.

If all bodies of similar shape, there can be only 2 constants: V. d W. law of corresponding --

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Korteweg p. 152 defending V. d W. way of calculation $\Big/ \; p = \frac{\frac{1}{3}\Sigma m v^2}{L - n d}$

~~for~~ example in one dimension

The same result: that only the $1-\frac{4b_1}{v}$ part of distance has to be travelled over by the centres of molecules [still to be corrected ~~by~~ on account of change in mean path in $\left(1-\frac{4b_1}{v}\frac{v}{v-4b_1}\right)$]

but under assumption that the encounters are independent whilst in reality "for a short time after every collision the possibilities of fresh collisions are considerably influenced by the proximity of the departing molecule".

In 1881 Lorentz by calculating the virial: $pv = \frac{1}{3}\Sigma m v^2 \left(1+\frac{4b}{v}\right)$

This has to be still corrected just as above: Mean free path according to Korteweg: Verslagen en Med. 2, Deel X, Archives Neerland. XII, and V. d. Waals is shortened in proportion $\frac{v}{v-4b_1}$

Polemics: Rayleigh 44: p. 499, 597

Tait 45 p. 199; 44 p. 546, 627

Nature 45 p. 512

Watson On the B. M. L. of P. of KE

the B.A Report quotes a paper of Prof Burnside where he proves that ~~it~~ it does not hold for a system of colliding spheres, where the centre of mass is at a small distance of the centre of figure.

But Watson shows that Burnside has omitted to take the frequency-factor of collisions ~~&~~ and says that by introducing it the law becomes confirmed. Calculates ~~after~~ the same example after Boltzmann method.

Burnside p. 533 : Boltzmann published in 1888 a criticism of the same paper in Wi. Sitzb. an objected to ~~the~~ Burnside's frequency factor and corrected his result

Bryan p. 533 : Shows more specially ~~also~~ in that Burnside's error consists.

Nature 46 p. 76 Culverwell against Lord K. Tait cases on the Maxwell B. Law (with A B C particles)

p. 29 ~~A~~ Watson

On a prop. in the Kin. Th. of G. Liouville's theorem

Ray. in Ph. M. objects to Maxwell's demonstration and ~~su~~ suggests the use of Hamilton's S function.

The same objection Boltzmann Ph. M. 1882. ~~the Watson's a~~ Watson used instead the S function with t independent (the same as Rayleigh)

But now he sees that this does not help for proving the particular case for which he was using it.

Nature 45 p. 277

History — An account of approximation in calculation b cannot be considered independent of v, because when greatest possible compression is reached we must have: $p_1(v - \mu b_1) = \frac{1}{3}\lambda \sum mu^2$ where $\mu b_1 = 3\sqrt{2}/\pi\ b_1 = 1.35 \ldots b_1$

Burbury – Watson p. 100

Exceptions of M.D.L. Lord Kelvin's case is a periodic motion and to these that law does not seem to apply.

Kelvin ~~does~~ prescribes a special direction of motions and other conditions.

Probably the law holds only for irregular heat motions.

Nature 44 p. 355 Kelvin On some test cases for the M.D. doctrine regarding distr. of E.

History: Maxwell Phil. Mag. 1860 On the collision of elastic spheres

" Phil. Tr. 1866 On the Foundation of KT of G.

Boltzmann Wien Ber. 1868 [illegible]

a large generalisation of it

Maxwell Camb. Ph. Soc. Trans. 1878, = Sc. papers II p. 713-741

still wider generalisation:

In the ultimate state of the system the average kinetic energy of two given portions of the system must be in the ratio of the number of degrees of freedom of those portions.

1 Motion of [illegible] particle in plane if $V = \frac{1}{2}(\alpha x^2 + \beta y^2 + c x^2 y^2)$

objects to ~~Maxwell's~~ applicability of Maxwells proof in this case

Ebert Wied. Ann 52 (1894) p 420

Anm: [illegible] Inv. Th. [illegible] CO_2 [illegible] 3 [illegible] 3 Invarianten [illegible]

[illegible] Functionaldeterminante [illegible]

$$\Delta = \begin{pmatrix} p_1 & p_2 & \cdots \\ q_1 & q_2 & \cdots \end{pmatrix}$$

[illegible]

[illegible] Inv. [illegible] Th. [illegible] $p_i = f_i(a_1 a_2 a_3 \ldots)$

Coord. [illegible]

$$q_i = \frac{\partial f_i}{\partial a_1} b_1 + \frac{\partial f_i}{\partial a_2} b_2 + \cdots \qquad \text{wo } b_k = \frac{da_k}{dt}$$ [illegible]

$$\begin{pmatrix} q_1 & q_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \begin{pmatrix} p_1 & p_2 & \cdots \\ a_1 & a_2 & \cdots \end{pmatrix} = D$$ [illegible] Th. [illegible] Multipl. [illegible] F.D.

$$\begin{pmatrix} p_1 & p_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \begin{pmatrix} p_1 & p_2 & \cdots \\ q_1 & q_2 & \cdots \end{pmatrix} \begin{pmatrix} q_1 & q_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \Delta . D \qquad \Delta - \text{Inv.}$$

[illegible] $(-1)^2$ (2 = [illegible]) mult Det [illegible] Transf [illegible]

[illegible] Th. [illegible]

Voigt-Jannasch. 67 p. 452

[illegible] : $570 \frac{gr.}{mm^2}$ [illegible] $= 10 \frac{gr}{mm^2}$ [illegible] 52 Atm $= 520 \frac{gr}{mm^2}$

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p. 871 Tammann 87 [illegible]

[illegible]; Heydweiller 64 p 730

Dissociation:

Boltzmann W.B. 88 (1883) p. 861, Wied. Ann. 22 (1884) p. 39

Natanson Wied. Ann. 38 (1889) p. 288

Jäger (Winkelmann)

Trombini Untersuchung von Flüssigkeiten 100 p. 1197

H.B. p. 231 etc. Influence of Dissociation on Viscosity; tries to explain Warburg-Babo CO_2 by Dissociation! No agreement.

100 p. 1122 [illegible]

[illegible]:

1). [illegible] $= pv = a_0(1+\alpha t)$

2). [illegible] $= b_0(1 + \varepsilon t)$

3). [illegible] Vol. [illegible] [illegible] [illegible]

$$c = c_0(1+\beta t)$$

$$r = r_0 + (\alpha a_0 - \varepsilon b_0 + \beta c_0)\, t$$

$r = A\, T\, u \frac{dp}{dT}$ [illegible]: $\frac{r'}{r} = \frac{u' \frac{dp'}{dT'}}{u \frac{dp}{dT}} \neq \frac{p \frac{dp'}{dT}}{p' \frac{dp}{dT}}$

$$p = (\text{II}, 678, \overset{100}{5}11) = C\, e^{-k \frac{1-\varepsilon t}{1+\gamma t}}$$

$\therefore \quad \frac{r'}{r} = \frac{k'(\varepsilon' + \gamma)}{k(\varepsilon+\gamma)}$ $\qquad k = \frac{3a}{mc^2} \quad k' = \frac{3a'}{mc'^2}$

$$a' = a(1+\varrho\kappa)$$

$$mc'^2 = mc^2(1+\varrho\lambda)$$

$$\therefore \frac{r'}{r} = 1 + \varrho(\kappa - \lambda)$$

„[illegible] Vol. [illegible]"

[illegible] ~~[illegible]~~ [illegible] A [illegible]

100. p. 793 [illegible]

[illegible] E, [illegible] $E + e + a'$, [illegible] $E + e + a$

[illegible] $\frac{a' - a}{m}$ [illegible] $= k(\alpha' - \alpha)$

[illegible]

„[illegible]"

[illegible]

[illegible]:
$p = p_0(1 + \gamma t)$ $\gamma =$ [illegible]

$\therefore\ k(\alpha' - \alpha) = p_0(1 + \gamma t)$

$\therefore\ \alpha' = \alpha + k_0 A i (1 + \gamma t) = \alpha_0'(1 - \varepsilon' t)$

$$\therefore\ \varepsilon' = \frac{\alpha_0 \varepsilon - 0{\cdot}56\, A i \gamma}{\alpha_0 + 0{\cdot}56 A i}$$

$$\neq\ \varepsilon - 0{\cdot}56\, A i\, \frac{\gamma + \varepsilon}{\alpha_0}$$

„[illegible]"

[illegible]: $d = $ [illegible] $= C e^{-\frac{3a}{mc^2}}$ $\left[c^2 = \right.$ [illegible]

$a = a_0(1 - \varepsilon t)$

$c^2 = c_0^2(1 + \gamma t)$

[illegible] $a >$ [illegible] $mc^2 <$

[illegible]: „[illegible]

[illegible]"

$\frac{mc'^2}{2} = \frac{mc^2}{2}(1 + \varrho\lambda)$ $\varrho =$ [illegible]

100 p. 1233 [illegible]

[Thomson's [illegible]]

[illegible] $= 2 =$ [illegible]

$$4\pi\,(2-\sqrt[3]{4})\,r^2\alpha = mu^2 \qquad \left(m = \frac{4\pi}{3}r^3\rho\right)$$

$$r = \frac{3(2-\sqrt[3]{4})\alpha}{\rho u^2} = \frac{1{\cdot}24\,\alpha}{\rho u^2}$$

[illegible]

---

p. 1182 [illegible]

[illegible] Natanson [illegible]!

---

101 p. 158 [illegible]

$r = a + b + c$ [illegible]:

$$\frac{d-d'}{d} = \varrho\,(\kappa-\lambda)\,k \qquad b = \frac{k-1}{k}\,r$$

$$A = \frac{b\rho}{2} \qquad \delta = \frac{3\alpha}{b\rho}$$ [illegible]

---

101 p. 400 Zur Stöchiometrie [illegible]

101 p. 553 [illegible]

Horrendum! [illegible] $\frac{1}{v}$

---

101 p. 920 Zur Th. [illegible]

[illegible] (wertlos)

$$pv = RT = R_1(1+\alpha t) = \frac{Nmu^2}{3}$$

$1\,[illegible]\quad R_1$

$1\,[illegible]\quad R_1'$ $\Big\}\quad \dfrac{R_1}{R_1'} = \dfrac{u_0^2}{u_0'^2}$ [illegible]

$p + P = \dfrac{RT}{v-b} = \dfrac{R_1(1+\alpha t)}{v-b}$ $\qquad P = [illegible] = \dfrac{a_0\,\rho}{2m}(1-\varepsilon t) = P_0(1-\varepsilon t)$

[illegible]:

$P_0(1-\varepsilon t) = \dfrac{R_1(1+\alpha t)}{v-b}$

$\rightarrow\quad v = b + \dfrac{R_1}{P_0}\,\dfrac{1+\alpha t}{1-\varepsilon t}$ [illegible] Vol. b [illegible] $\dfrac{R_1}{P_0}$ [illegible]

[illegible] R_1 [illegible] P_0 [illegible] [[illegible]]

[illegible] Vol. [illegible]

berechnet [illegible] k^2 [$d = e^{-k^2\frac{1-\varepsilon t}{1+\alpha t}}$] [illegible]

[illegible]

$v = v_0(1-\kappa p) = b + \dfrac{R_1}{P}(1+\alpha t)\left(1-\dfrac{p}{P}\right) = v_0\left[1 - \dfrac{R_1(1+\alpha t)}{v_0 P^2}\,p\right]$

$\kappa = \dfrac{R_1(1+\alpha t)}{v_0 P^2}$ [illegible] κ [illegible]

[illegible] Compr. [illegible] Stm. [illegible]

[illegible] P_0 [illegible]

[illegible] Verbindung (r, σ) $\dfrac{a}{v^2}$ [illegible]

Tammann [illegible] (II) Wied. Ann. 66 p. 495

Planck (Thermod. p. 18, 152), Ostwald Chemie p 373, 389 (1893),

Poynting Phil Mag 12 p 32 (1887) [illegible]

[illegible] continuirlich [illegible]

Tammann [illegible]

Voigt Gött. Nachr. 1897 p. 261 [illegible]

[illegible] Vol. [illegible]

[illegible]

[illegible] vol. [illegible]

[illegible]

1). [illegible]

2). Correction [illegible] Vol. [illegible]

[illegible]

Milner [illegible] Vol. [illegible]

[illegible] Correction

[illegible] v. d. W. [illegible]

$$dq = c_v\,dT + \frac{T}{A}\frac{dp}{dT}\,dv \qquad Q = \frac{RT}{A}\int_{v_1}^{v_2}\frac{dv}{v-b} = \frac{RT}{A}\log\left(\frac{v_2-b}{v_1-b}\right)$$

[illegible] [sv + n].

[illegible] [Mol. Rad.] [illegible] momentan

[illegible] $G =$ [illegible] , $c =$ [illegible]

$$c^2 = G^2 \lg \frac{v_a - b}{v_i - b}$$

[illegible] contin [illegible] ∞ [illegible]

$$\psi = \frac{1}{2} m G^2 \int_{v_i}^{v_a} \frac{dv}{v-b} \qquad c^2 = G^2 \int_{v_i}^{v_a} \frac{dv}{v-b}$$ [illegible] constant [illegible]

[illegible]

[illegible] $\frac{1}{2} c^2 dM$ [illegible] Mol. [illegible]

[illegible] compensiren [illegible]

[illegible] dM [illegible] $= \frac{1}{2} c^2 dM$

$$\therefore \varphi = \frac{G^2}{2A} \int_{v_i}^{v_a} \frac{dv}{v-b} = \frac{\Delta T}{A} \int_{v_i}^{v_a} \frac{dv}{v-b}$$

[illegible] [abs. constant] [illegible] [ca 20 Young für Centan, Hexan etc]

[illegible] Energie [illegible] Mol.

[illegible] V. d. Waals [illegible]

Naturw. Rdsch. XV p. 285 (1900) 9/VI

~~Spannung & Plastiz. [illegible]~~

? [illegible] etc. [illegible] Cryl [illegible] ?

[illegible] 1000 Atm

1. [illegible] [[illegible]]

[illegible] Calcination

2. [illegible] [[illegible]] [[illegible]]

[illegible]

Cu + Cu = [illegible] [illegible] + Zn + Cu = [illegible]

[illegible] Diffusion [illegible]

[illegible]

Zn [illegible] comp.

[illegible] [[illegible]] [illegible]

Au, Pt, Ag, Cu, Zn, [illegible], Bi [illegible]

[illegible] [[illegible]] ~~Pt 100~~ 3 – 12h

[illegible] Cu + Zn = 16h ¼ mm [illegible]

Cu – Pb [illegible] 36 mm [illegible]

[illegible] [Zn Pt, 2 Bi] [illegible]

[illegible] $BaSO_4$, Na_2CO_3 $BaCO_3$, Na_2SO_4

[illegible] Reaction

[illegible]

[illegible] ?

[illegible] Vol. [illegible]

[illegible] Vol. [illegible]
vD. 48S

[illegible] Vol. [illegible]

2 D. Cu Ca acetat
Sulfurhydrat

[illegible] allotr. Mod. [illegible] Vol. [illegible]

[illegible] prism. [illegible] octaedr.

amorph [illegible]

C [illegible] diamant [illegible] Fe

[illegible] Vol. [illegible] allotr. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(10000 Atm).

[illegible]

Tammann Ztschr. ph. Ch. 21 p. 17 (1896)

Wied. Ann. 62 p. 280

[illegible] Se [illegible] 50° amorph [illegible] [Hittorf P. Ann. 84 p. 217 (1851)

Betol Schm. 95° [illegible] Glas [illegible] −10° [illegible] −200 continuirliche [illegible] Viscos.

[illegible] amorph [illegible]

Lithofellinsäure (Wöhler)	205°	103 – 110°
Amygdalin	200°	125 – 130°
Rohrzucker	160°	90 – 100°

[illegible] zu amorph [illegible] Kryst. [illegible] keine [illegible]

[illegible] Unterkühlung

Zweiter [illegible] : O. Lehmann: Se [illegible] amorph Se [illegible] 50°.

[illegible] 90° [illegible] grau [illegible]

[illegible] 217° [illegible]

[illegible]

[illegible] amorph [illegible]

Chloroform (Pictet) Ztschr. ph. Ch. 16 p. 422 (1895) : [illegible] −68.5, [illegible] −81°

[illegible] 37 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] 24 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] $c = \frac{2\pi N}{V}$ [illegible]

[illegible] : N [illegible]

[illegible]

$\frac{1}{2} p V = N E \left(1 + \frac{b}{V}\right)$

[illegible]

[illegible]

[illegible]

Naturw. Rdsch. XV p. 221

[illegible] Dulong u. Petit Richarz

Wied. Ann. 48 p. 467

$= \frac{m}{2} u^2 = -\frac{1}{2} \Sigma (Xx + Yy + Zz)$

$\Delta = \frac{3}{2} p v + \frac{1}{2} \Sigma r f(r) - \frac{1}{2} \Sigma (\mathfrak{X} x + \mathfrak{Y} y + \mathfrak{Z} z) = \Delta_i + \Delta_a$ [illegible]

[illegible]

[illegible]

$\mathfrak{X}\ \mathfrak{Y}\ \mathfrak{Z}$ = [illegible]

$\Delta_i = \frac{1}{2} \Sigma r f(r)$ [[illegible] + A] [illegible]

[illegible]: $\Delta_a = -\frac{1}{2} \Sigma (\mathfrak{X} x + \mathfrak{Y} y + \mathfrak{Z} z) + \frac{3}{2} p$ [= A

[illegible] [[illegible]]

III [illegible] (endoth.) [illegible] CHN, C_2H_2 (Acetylen), Se H, Te H, O_3, Si_2Cl_3,

Si_2Cl_4?, Si F, PCl, Ag_2O

Si not [illegible] fluoride

[illegible]

Newton

$\overline{\Sigma\, r f(r)}$ $\overline{L_i}$

> 0

$L_i = \frac{1}{2} \rho f(\rho)$

$$\frac{dL_i}{dT} = \frac{1}{2}\left[f(\rho) + \rho f'(\rho)\right] \frac{d\rho}{dT}$$

IV.

pot. Energie:

$f(r) = \frac{c^2}{r^n}$ $\qquad L_i = \frac{c^2}{2} \frac{1}{r^{n-1}}$ $\qquad \varphi(r) = -\int_r^\infty f(r)\,dr = -\frac{c^2}{n-1}\frac{1}{r^{n-1}}$

$$\overline{\varphi(r)} = -\frac{c^2}{n-1}\,\overline{\frac{1}{r^{n-1}}} = -\frac{2}{n-1}\,\overline{L_i}$$

Energie: $\overline{E_i} = \overline{L_i} + \overline{\varphi(r)} = \frac{n-3}{n-1}\,\overline{L_i}$

$$\frac{d(\overline{E_i})}{dT} = \frac{n-3}{n-1}\,\frac{d(\overline{L})}{dT}$$ Temp.

$n - 3 > 0$

$\frac{1}{r^2}$ $n = 3$ stabile [Lipschitz] Crelle 66 p. 363 (1866)

$$\frac{d(E_i)}{dT} = \frac{d(\overline{L})}{dT} - \frac{2}{n-1}\,\frac{d(\overline{L})}{dT}$$ $\left[\ n = 7 \quad \frac{dL}{dT} - \frac{1}{3}\frac{dL}{dT}\right]$

[OEM. Scott. p. 99]

V. Boltzmann kin. Th. mehrat. Mol. Wi. Ber. 63 p. 417 (1871)

Atoms $=$ Temp. $-\frac{3}{2h}$ K Mol.

$\frac{c}{c} = 1{,}4$ [illegible] 0'81 [illegible]

[illegible]

[illegible] Atome [illegible]

[illegible]

VI [illegible] Statistik

[illegible] $\rho \ldots \rho + d\rho$

$$dW_\rho = \frac{e^{-h\chi}\rho^2 d\rho}{\int_0^R e^{-h\chi}\rho^2 d\rho}$$

[illegible]

$$e^{-h\chi}R^3 \log R = 0$$

$$e^{-h\chi} r^2 \log r \Big|_0 = 0$$

[illegible]

VII [illegible]

$$f(r) = \frac{d\chi}{dr}$$

$$\overline{rf} = \frac{\int_0^R r^3 \frac{d\chi}{dr} e^{-h\chi} dr}{\int_0^R e^{-h\chi} r^2 dr} = \frac{-\frac{1}{h} e^{-h\chi} r^3 \Big|_0^R + \frac{3}{h}\int e^{-h\chi} r^2 dr}{\int \cdots} = \frac{3}{h} = 2\bar{L}_i$$

$$\bar{L}_i = \frac{1}{2}\overline{rf}$$

[[illegible] Atome [illegible]
([illegible])]

[illegible]

$0°75$ [illegible] $\frac{N_3 U}{N_2 L_i}$ $= 25$ [illegible] $= p$

$u =$ [illegible] $= 25 \cdot 10^6 \frac{dyn}{cm}$ (NO₂)

$54 \cdot 10^6$ " (HJ) $\qquad p = 1{\cdot}01 \cdot 10^6 \frac{dyn}{cm^2}$

$483 \cdot 10^6$ -- (H_2) $\qquad$ [illegible]

Voigt p 674 [illegible]

$s =$ [illegible] $\quad s_2 =$ [illegible]

$$c = \frac{4s - s_2}{s_2(3s - s_2)}, \quad c_1 = \frac{s_2 - 2s}{s_2(3s - s_2)}, \quad c_2 = \frac{1}{s_2}$$

	$c \cdot 10^{-12}$	$c_1 \cdot 10^{-12}$	$c_2 \cdot 10^{-12}$
Al	0·811	0·307	0·252
Bo = [illegible]	1·41	0·61	0·40
Cd	2·33	1·85	0·24
Fe	1·46	0·43	0·51
Au	1·11	0·546	0·280
Cu	1·11	0·173	0·468
Mg	0·498	0·163	0·167
Ncb	1·08	0·355	0·36
Ni	2·69	1·16	0·76
Ag	1·08	0·499	0·29
Pt	2·47	0·89	0·79
Bi	0·41	0·17	0·12
Zn	1·49	0·73	0·38

[illegible] p. 708

[illegible] 63 p. 731 (1871) [illegible]

[illegible]

Boltzm. 53 p. 195 (1866), 63 p. 712 (1871), 90 p. 231 (1884)

Clausius Poggend. 142 p. 433 (1871)

Helmholtz Crelle 97 p. 111, 317 (1884) Sitzgsb. Berl. Ak. 1884 p. 159
p. 311

[illegible]

$$\varphi = \text{Pot.} = F_1 + F_2 + F_3 + \ldots \quad (\text{homog. Fn.})$$

$$X = -\frac{\partial \varphi}{\partial x} = -\frac{\partial F_1}{\partial x} - \frac{\partial F_2}{\partial x} - \ldots$$

$\begin{matrix} x \\ y \\ z \end{matrix} = 0$ [illegible] $X = Y = Z = 0$ [illegible] $\left.\frac{\partial F_1}{\partial x} = \frac{\partial F_1}{\partial y} = 0 \right\} F_1 = 0$

$$\varphi = F_2 + F_3 + \ldots$$

$$-\tfrac{1}{2}(Xx + Yy + Zz) = \frac{x}{2}\frac{\partial F_2}{\partial x} + \frac{y}{2}\frac{\partial F_2}{\partial y} + \frac{z}{2}\frac{\partial F_2}{\partial z} + \frac{x}{2}\frac{\partial F_3}{\partial x} + \ldots = F_2 + \tfrac{3}{2}F_3 + 2F_4 + \tfrac{5}{2}F_5 + \ldots$$

Nach Virialsatz: $\overline{\varphi} = \overline{F_2} + \overline{F_3} + \ldots$

$$\overline{Z} = \overline{F_2} + \tfrac{3}{2}\overline{F_3} + 2\overline{F_4} + \ldots$$

[illegible] $\overline{\varphi} =$ [illegible] Multiplum [illegible] $\overline{Z}$ [illegible]

[illegible] $\overline{F_2} = \overline{Z} = \overline{\varphi}$

[illegible]

$k = \frac{C_p}{C_v} =$

Ag	1.02
Fe	1.01
Cu	1.015
Pt	1.007
Au	1.016
Al	1.02
Pb	1.039
Zn	1.040

[illegible] $\bar{\varphi} + \bar{L}$ [illegible] prop. $\bar{T}$ [illegible] $= \bar{T}\,\varphi(T)$ [illegible]

[illegible]

1). [illegible] Atom [illegible] Atomvol. [illegible]

2). [illegible]

	Atomgew.	Atomvol.	Atomwärme
Li	7.0	11.9	6.6
Be	9.1	5.6	3.6–5.3
B	10.9	4.0	2.5–5.5
C	12.0	3.6	1.4–5.5
Na	23.0	23.7	6.7
Mg	23.9	13.8	5.9–6.0
Al	27.0	10.6	5.5–5.8
Si	28.0	11.2	4.6–5.7
P	31.0	13.5	5.3–5.9
S	32.0	15.7	5.2–5.7
K	39.0	45.4	6.5
Ca	39.9	25.4	6.8

[illegible]

Report of the meetings of the [illegible] [illegible] [illegible]
[illegible] [illegible]

[illegible] [I [illegible]]
[illegible] I ([illegible] I)
[illegible] I ([illegible])
[illegible] I [[illegible]]

Proceedings of the [illegible] [illegible]
[illegible] — 22 [illegible]
[illegible] I [illegible] [illegible] (I [illegible])

[illegible] [illegible] [illegible] [illegible]

[illegible] [illegible]

Proceedings of the [illegible] [illegible]

[illegible] I [illegible]

Proceedings of the [illegible] [illegible]

[illegible] [illegible] (I [illegible])

Abstract of the papers [illegible] Proceedings of the [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

[illegible] [illegible]. 3512 I

[illegible] XI [illegible]
[illegible] [illegible] [illegible]

X

Gullet Z ph Chemie 37 p. 385

Jackson Scient Proc. R. Dublin S. X 1903 part I, 8
tension in liquids without rupture

Ebert Ast. Nachr. 164 № 3917 8 p. [illegible]

<u>Riecke</u> Beiträge z. Hydrod. Gött Nachr 1888 p. 347
Wied. Ann 36 p. 322 (1889)

Opalescenz i. krit. Geb. d. Lösungen:

Rothmund Z. ph. Ch. 26 p. 446 (1898)

Friedländer 38 p. 385 (1901)

Guthrie Phil. Mag. (5) 18 p. 30, 503 (1884

Lehmann Molekularphysik I 2166 II

Tyndall's Versuch

$\angle\, \delta^2 =$

chciał się zaznajomić z własnościami materyi promieniotwórczej, to żadnej [illegible] nie miałbym odwagi polecić mu książeczki p. Muttermilcha.

W. Złobicki

Dr. St. Tołłoczko. Co to są elektrony. [illegible] z [illegible] rozszerzonych Wykładów Uniwersyteckich w Krakowie. Kraków; nakładem księgarni D.E. Friedleina. 1905. str. 81.

W miarę tego, jak z rozwojem fizyki zdobywano coraz to nowe tereny umiejętności fizycznych w dziedzinie elektryczności — w miarę tego zmieniały się kilkakrotnie pojęcia o istotnej naturze tego czynnika, który nazywamy elektrycznością. W wielu zjawiskach, na pozór odrębnych, zaczęto się dopatrywać jednej nici przewodniej. Snuła się ta nić, począwszy od praw elektrolizy Faradaya (1834) przez teoryę dyssocyacyi Svante Arrheniusa (1887), przez badania Hittorfa (1869) i Crookesa (1879) aż do

partyjny Blok
7 do 38 mandatów.
wski Blok Pracy Gos
mandatów.
lski Blok Obrony Chrz
zdobyła 1 do 2 mand
jalistyczna Lista Rob
yła 12 mandatów.

ie Nr 2 kandydował
ściśle z Bezpartyjnym Blo-
puszczać należy, że
ego Bloku w przyszłej Radzie
liczył 42 radnych na ogólną
czbę 64 radnych.

borów w poszczególnych okrę-
ące:
dzielnica I) lista Nr 1 uzyskała
mandaty, lista Nr 3 1069 głosów
Nr 4 820 głosów, bez mandatu.
kany w śródmieściu przez Karo-
Rostworowskiego uważać należy
obisty kandydata a nie za sukces
kiej, głosujący na tę listę bowiem
i zostali przedewszystkiem nazwi-
ołowego kandydata powołanego tak
wno przez rząd Rzeczypospolitej do Aka-
Literatury i ten wzgląd zapewnił liście
lat.

ta Nr 1 uzyskała 5171 głosów
ta Nr 3 707 głosów, bez man-
143 głosy i 1 mandat. W okrę-
ad którego wchodzą dzielnice
głosy socjalistyczne padały głów-
icy XII.
III (dzielnica IV) brak jeszcze
ych obliczeń głosów. Wedle prowi-
ch obliczeń lista Nr 1 uzyska 5 do 6
, lista Nr 4 uzyska 1 mandat a być
lista Nr 3 uzyska w tym okręgu 1

dzielnica V, lista Nr 1 uzy-
daty, lis

ch,
tami,
gram Be
przeszło

Nie można
tek głosów padł
nak już można, że tek ten pr
od 100 ogółu oddanych głosów, po
przy ostatnich wyborach do sejmu
partyjny (wraz z wyborcami żydows
rzy dziś nie głosowali na listę Nr 1) zy
spełna 50 proc. głosów. Przyrost cyfrowy więc
Bezpartyjnego Bloku jest bardzo znaczny
i wskazuje, że cofanie się wpływów partyj
politycznych w Krakowie postępuje coraz
szybciej.

2) Porażka socjalistyczna jest bardzo znacz-
na. Liczba głosów oddanych na kandydatów
socjalistycznych będzie z pewnością o wiele
niższa od cyfry głosów uzyskanej przy ostat-
nich wyborach sejmowych. Już w tej chwili
można mieć wątpliwość, czy ta cyfra głosów
wystarczyłaby socjalistom na uzyskanie 1
z 4 mandatów sejmowych Krakowa. Szczegól-
nie dotkliwą dla socjalistów jest ich porażka
na Grzegórzkach, w Dąbiu i Podgórzu a więc
w dzielnicach o ludności robotniczej. Socjali-
ści z krakowskiej PPS stracili prawo do okre-
ślania siebie jako reprezentacji robotniczej,
gdyż większość głosów robotniczych padła na
listę Nr 1. Niemniej wobec zupełnej klęski li-
sty endeckiej socjaliści są jedynymi liczeb-
niejszymi przedstawicielami opozycji w
szłej Radzie miejskiej. Tak jak wybory

sz
nie
5)
Pos
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spos
więk
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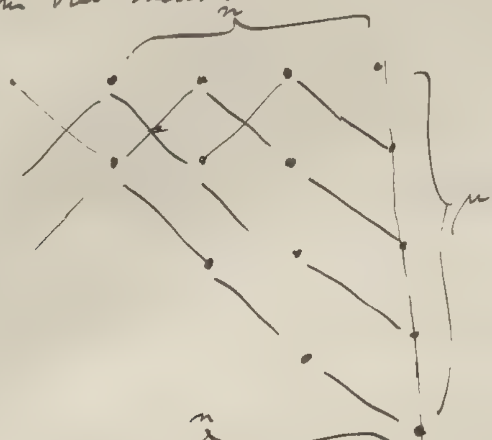
li mies
com,
na

55

Smoluchowskis Versuche sollten eigentlich berechnet werden auf Grund d. Formeln für „zum ersten Mal Erreichen“



$$a_{n\mu} = \cancel{\ldots} \binom{\mu}{\frac{\mu-n}{2}} \frac{n}{\mu \cdot 2^{\mu}}$$

$$A_\mu = \sum_{n=1}^{n=\mu} a_{n\mu}$$

oder:

$$A_\mu = \sum_{n=1,3,5\ldots}^{n=\mu} a_{n\mu}$$

oder

$$A_\mu = \sum_{n=2,4,6\ldots}^{\mu} a_{n\mu}$$

dasselbe wie ~~[illegible]~~ Wahrsch. „dass ein Teilchen eine vorher erlangte Elongation zur Zeit μ erlangt“

welche schon berechnet worden ist:

$$A_\mu = \frac{1}{2} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (\mu-2)}{2 \cdot 4 \cdot 6 \cdots \mu-1} \quad \text{(für ungerade } \mu\text{)}$$

$$= \frac{1}{2} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (\mu-1)}{2 \cdot 4 \cdot 6 \cdots \mu} \quad \text{(für gerade } \mu\text{)}$$

also auch hier $A_\mu = \frac{1}{2} \frac{1}{\sqrt{\frac{\mu}{2}\pi}}$ $\longleftarrow$

In Wirklichkeit Superposition der geraden und ungeraden Stellung, so wie obere Figur,

also: $\lim \sum A_\mu = \cancel{\ldots} \sqrt{\frac{\mu}{2\pi}}$ Somit jedenfalls Proportionalität mit $\sqrt{\text{Zeit}}$

Um den Zahlfaktor genauer zu erhalten, muss man d. Zusammenhang mit d. mittleren Elongation fest stellen.

Binomialcoefficienten $\alpha_{n\mu} = \binom{\mu}{\frac{\mu-n}{2}}$ $\sum_n \alpha_{n\mu} = 2^\mu$

Mittlere Elongation nach μ Intervallen

$$\overline{\varepsilon^2} = \frac{\sum_n n^2 \alpha_{n\mu}}{\sum \alpha_{n\mu}}$$ ~~für geradzahlige μ~~

$$\left(x+\frac{1}{x}\right)^\mu = \binom{\mu}{0} x^\mu + \binom{\mu}{1} x^{\mu-2} + \binom{\mu}{2} x^{\mu-4} + \cdots \binom{\mu}{\mu}\frac{1}{x^\mu}$$

$$\frac{d}{dx}\left[x \frac{d}{dx}\left(\left(x+\frac{1}{x}\right)^\mu\right)\right] = \underbrace{\mu^2\binom{\mu}{0} x^{\mu-1} + (\mu-2)^2\binom{\mu}{1} x^{\mu-3} + \cdots}_{}$$

für $x=1$ $= \sum n^2 \alpha_{n\mu}$

$= \mu\, 2^\mu$

also mittlere Elongation $\overline{\varepsilon^2} = \mu \frac{2^\mu}{2^\mu} = \mu$

$$\sqrt{\overline{\varepsilon^2}} = \sqrt{\mu}$$

Dagegen ist $\lim A_n = \frac{1}{\sqrt{2\mu\pi}}$

Genügt nicht, denn z. B. bei Br. B. unter Einfluss elastischer Kraft, wächst die durchschnittl. Absolut Elongation nur bis zu einer endlichen Grenze, während die durchschn. einseitige Maximal Elongation mit der Zeit beliebig gross werden muss.

für gerade μ

$$A_{\mu} = \frac{1}{2}\frac{1\cdot 3\cdot 5\cdot 7 \cdots (\mu-1)}{2\cdot 4\cdot 6 \cdots (\mu)} = \frac{1}{2}\frac{\mu!}{\left(2^{\frac{\mu}{2}}\right)^2\left[\left(\frac{\mu}{2}\right)!\right]^2} = \frac{1}{2}\frac{\mu!}{2^{\mu}\left(\frac{\mu}{2}!\right)^2}$$

$$\lim A_{\mu} = \frac{1}{2}\frac{\left(\frac{\mu}{e}\right)^{\mu}\sqrt{2\mu\pi}}{2^{\mu}\left(\frac{\mu}{2e}\right)^{\mu}\cdot 2\frac{\mu}{2}\pi} = \frac{1}{2}\sqrt{\frac{2}{\mu\pi}} = \frac{1}{\sqrt{2\mu\pi}}$$

durchschnittl. Maximal Abweichg : $\overline{E}_m = \sum_{m} \cancel{A_{\mu}}\, A_{\mu} = \int \frac{d\mu}{\sqrt{2\mu\pi}} = \sqrt{\frac{2\mu}{\pi}}$

ist also identisch mit der ~~...~~ durchschnittlichen absoluten ~~...~~ Elongation ! ?

Ist das möglich ?

Tatsächlich: wenn man die Wahrsch. für Erreichen einer Maximal Elongation n, innerhalb μ Intervallen, beachtet, so sieht man:

Durchschnittliche äusserste Maximal Elongation, innerhalb μ Intervallen:
(wobei negative Verschiebungen $=0$ betrachtet werden)

(für gerade μ)

$$\overline{E}_m = \frac{\sum}{2^m}\left[(1+2)\binom{\mu}{\frac{\mu}{2}-1} + (3+4)\binom{\mu}{\frac{\mu}{2}-2} + (5+6)\binom{\mu}{\frac{\mu}{2}-3} + \cdots (\mu-1+\mu)\binom{\mu}{0}\right]$$

Dagegen gewöhnliche „durchschnittliche Absolut Elongation", die am Schluss der μ Intervalle erreicht wird :

$$\overline{\Delta} = \frac{2\sum}{2^m}\left[2\binom{\mu}{\frac{\mu}{2}-1} + 4\binom{\mu}{\frac{\mu}{2}-2} + (6)\binom{\mu}{\frac{\mu}{2}-3} + \cdots \mu\binom{\mu}{0}\right]$$

Im Falle grosser Zahlen μ kommt das offenbar auf dasselbe hinaus

Sonst ist $\overline{\Delta} > \overline{E}_m$ $\boxed{\text{Differenz: } \overline{\Delta} - \overline{E}_m = \sum\left[\binom{\mu}{\frac{\mu}{2}-1} + \binom{\mu}{\frac{\mu}{2}-2} + \binom{\mu}{\frac{\mu}{2}-3} + \cdots \binom{\mu}{0}\right] = \frac{1}{2}}$

Genau ist somit $\overline{E}_m = \overline{\Delta} - \frac{1}{2}$

Also sind diese zwei Ausdrücke für grosse μ identisch !!

Gilt das vielleicht allgemein auch bei komplizierteren Systemen ?

Sedimentations Verteilung

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2} + \mu\frac{\partial n}{\partial x}$$

$$\boxed{\text{für } \lim \frac{\partial n}{\partial t} = 0 \qquad n = n_0 e^{-\frac{\mu}{D}x}}$$

für $x=0$ $\quad \frac{\partial n}{\partial x} = 0$ $\qquad x = \infty$ $\quad \frac{\partial n}{\partial x} = 0$

$$n = e^{-kt} f(x)$$

$$D\frac{d^2f}{dx^2} + \mu\frac{df}{dx} + kf = 0 \qquad D\alpha^2 + \mu\alpha + k = 0$$

$$\alpha^2 + \frac{\mu}{D}\alpha = -\frac{k}{D}$$

$$f = e^{\alpha x}$$

$$\alpha = -\frac{\mu}{2D} \pm \sqrt{\frac{\mu^2}{4D^2} - \frac{k}{D}}$$

$$= e^{-\mu x} \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}(\nu x)$$

$$= -\mu \pm i\nu$$

$$\left.\frac{\partial n}{\partial x}\right|_{x=0} = e^{-kt}\alpha = 0 \qquad \text{würde erfordern } \mu = \nu = 0$$

$$\frac{df}{dx} = -\mu e^{-\mu x}\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\nu x \mp \nu e^{-\mu x}\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\nu x \qquad \Big|^{x=0} = \frac{-\mu}{+\nu} \rightarrow \text{würde fordern } k = \frac{\mu^2}{4D}$$

Vielleicht geht es aber, wenn man annimmt $\left.\begin{matrix}\alpha_1\\ \alpha_2\end{matrix}\right\}$ reell < 0

$$f = Ae^{\alpha_1 x} + Be^{\alpha_2 x}$$

$$= e^{-\frac{\mu}{2D}x}\left[A e^{\sqrt{\cdots}\,x} + B e^{-\sqrt{\cdots}}\right]$$

$$\left.\frac{df}{dx}\right|_{x=0} = \alpha_1 A + \alpha_2 B = 0 \qquad A = \alpha_2 C$$

$$B = -\frac{\alpha_1}{\alpha_2}A \qquad B = -\alpha_1 C$$

$$f = \cancel{e^{-\frac{\mu x}{2D}} \sqrt{\mu^2 - 4kD} \left(-\mu + \sqrt{\mu^2 - 4kD}\right) e^{x\frac{\sqrt{\mu^2-4kD}}{2D}}}$$

$$f = C\left[\alpha_2 e^{\alpha_1 x} - \alpha_1 e^{\alpha_2 x}\right]$$

Dabei würde immer noch die Konstante k unbestimmt bleiben

Sie bestimmt sich tatsächlich daraus, dass auch am oberen Ende die Emulsion abgeschlossen

sein muss, denn wirklich ein stationärer stetig Zustand stattfinden soll, kann also nicht

$x=\infty$
$\frac{\partial n}{\partial x}=0$ genügen, sondern man muss haben

$\frac{\partial n}{\partial x}=0$ für $x=h$

$$\frac{\partial n}{\partial x} = e^{-kt}\, C\,\alpha_1\alpha_2\left[e^{\alpha_1 h} - e^{\alpha_2 h}\right] = 0$$ was nicht möglich ist, ausser dass $\alpha_1=\alpha_2$

in diesem Falle ~~…~~ müsste $k=\frac{\mu^2}{4D}$ sein

Also wäre dann:

$$n = e^{-\frac{\mu^2 t}{4D} - \frac{\mu}{2D}x} \qquad -\frac{\mu^2}{4D} = D\frac{\mu^2}{4D^2} - \frac{\mu}{2D}\mu$$

was offenbar nur ein spezielles partik. Integral ist, welches hier nicht brauchbar

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~~Vielleicht~~ Grenzbedingungen sind falsch!

denn im stationären Endzustand ist offenbar $\frac{\partial n}{\partial x}\Big|_{x=0} = -\frac{\mu}{D}n_0$

Vielleicht sind Grenzbedingungen ersetzbar durch

$$\frac{\partial}{\partial t}\int_0^h n\,dx = 0 = \int_0^h \frac{\partial n}{\partial t}\,dx = D\frac{\partial n}{\partial x}\Big| + \mu n\Big| = 0$$

$$D\frac{\partial n}{\partial x}\Big|_0 + \mu n\Big|_0 = D\frac{\partial n}{\partial x}\Big|_h + \mu n_h \quad = \text{annähernd} = 0$$

Also wäre (annähernd) $\frac{\partial n}{\partial x}\Big|_0 = -\frac{\mu}{D}n_0$!

Vielleicht nicht annähernd, sondern genau!

Auch könnte man rationeller so vorgehen:

$$n = n_0 e^{-\frac{\mu}{D}x} + \sum e^{-kt} f(x)$$

Denn Bedingung dass nichts durch die Wand durchgeht ist ja jetzt zu ändern! Jetzt ist der Diffusionsstrom nicht $D\frac{\partial n}{\partial x}$ sondern $D\frac{\partial n}{\partial x}+\mu n$

Also lautet die Aufgabe folgendermassen: $\underbrace{\qquad\qquad\qquad}^{\text{Grenzbedingungen}}$

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial u}{\partial x}$$

für $x=0$: $D\frac{\partial u}{\partial x} + \mu u = 0$

für $x=h$: $D\frac{\partial u}{\partial x} + \mu u = 0$

$$u = A e^{-\frac{\mu}{D}x} + \sum_k e^{-kt} f(x)$$

Versuch: $f(x) = e^{\alpha x}$ $\qquad \alpha = -\frac{\mu}{2D} \pm i\sqrt{\frac{k}{D} - \frac{\mu^2}{4D^2}}$

$$= e^{-\mu x}(A\cos\nu x + B\sin\nu x) \quad = -\mu + i\nu$$

$x=0$. $x=h$

$$u = A e^{-\frac{\mu}{D}x} + z$$

$D\frac{\partial z}{\partial x} + \mu z = 0$

$$\frac{\partial z}{\partial t} = D\left[+A\left(\frac{\mu}{D}\right)^2 e^{-\frac{\mu}{D}x} + \frac{\partial^2 z}{\partial x^2}\right] + \mu\left[-A\frac{\mu}{D}e^{-\frac{\mu}{D}x} + \frac{\partial z}{\partial x}\right]$$

$$\therefore \quad \frac{\partial z}{\partial t} = D\frac{\partial^2 z}{\partial x^2} + \mu\frac{\partial z}{\partial x}$$

Kondensation von Nebel an Kernen

$$\frac{\partial n}{\partial t} = D\,\Delta^2 n$$

$$n = \frac{1}{\sqrt{t^3}}\, e^{-\frac{r^2}{4Dt}}$$

$$n = e^{-kt} f(x\,y\,z)$$

$$-kf = D\,\nabla^2 f$$

$$\nabla u + \underbrace{\frac{k}{D}}_{K^2} u = 0$$

$t = 0 \qquad n = \text{const} = n_0$

$r = r_1 \qquad r_2 = r_2 \dots \qquad n = 0$

Im Falle sphärischer Symmetrie

$k = D\sqrt{K}$

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + K^2 u = 0 \qquad u = \frac{\cos Kr}{Kr} \qquad \frac{\sin Kr}{Kr}$$

$$\frac{du}{dr} = -\frac{\sin Kr}{r} - \frac{\cos Kr}{Kr^2}$$

$$\frac{d^2u}{dr^2} = -\frac{K\cos Kr}{r} + \frac{2\sin Kr}{r^2} + \frac{2\cos Kr}{Kr^3}$$

$$-\frac{2\sin Kr}{r^2} - \frac{2\cos Kr}{Kr^3} + \frac{K\cos Kr}{r} = 0$$

$$n = e^{-kt}\frac{\cos Kr}{r}$$

$r = a$

$u = 0$

$\cos Ka = 0 \qquad \sin Ka = 0$

$Ka = (2m+1)\frac{\pi}{2} \qquad Ka = m\pi$

$$K = \frac{(2m+1)}{a}\frac{\pi}{2}$$

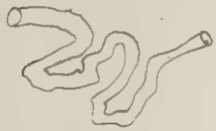
$$\cancel{n = \sum \left[A_k \cos\frac{(2m+1)\pi r}{2a} + B_k \sin\frac{m\pi r}{a}\right]}$$

$$n = \sum{}' \frac{1}{r}\left[A_k\, e^{-D\left(\frac{(2m+1)\pi}{2a}\right)^2 t} \cos\frac{(2m+1)\pi r}{2a} + B_k\, e^{-D\left(\frac{m\pi}{a}\right)^2 t} \sin\frac{m\pi r}{a}\right]$$

Bestimmung der Koeffizienten A_k B_k:

$$\sum\left\{A_k \cos\left(\frac{(2m+1)\pi}{2}\frac{r}{a}\right) + B_k \sin\left(m\pi\frac{r}{a}\right)\right\} = \cancel{\text{const}}\ r c$$

Ein kleineres Korn durchstreicht infolge der Brown'schen Bewegung längere Strecken als ein grosses aber ob ein grösseres Volum ?



[illegible]:

in welcher Zeit verschiebt sich eine Kugel im Mittel um ihren eigenen Durchmesser ?

$$2D\tau = 4a^2 \qquad D = \frac{H\theta}{N}\frac{1}{6\pi\mu a}$$

$$\tau = \frac{2a^2}{D}$$

also durchstrichenes Volumen pro Zeiteinheit $= Vol = \frac{a^2\pi \cdot 2a}{\tau} = 2a^3\pi \cdot \frac{D}{2a^2} = a\pi D$

unabhängig vom Radius! $= \frac{H\theta}{N}\frac{1}{6\mu} = \frac{8{,}3 \cdot 10^7 \cdot 300}{6 \cdot 10^{23} \cdot 6 \cdot 2 \cdot 10^{-4}}$

$$= \frac{1}{3}10^{-10} = 30\mu^3$$

Falls es sich aber um kleine Kugeln in Gas handelt, ist dagegen

$$D = \frac{H\theta}{N}\frac{1}{\cdots a^2 c} \quad \text{also} \quad Vol \sim \frac{H\theta}{N}\frac{1}{a c\rho_0} \;!$$

Geschw. $= \frac{2a}{\tau} = \frac{D}{a}$

$= \frac{H\theta}{N}\frac{1}{6\pi\mu a^2}$

$\sim \frac{H\theta}{N}\frac{1}{a^3 c\rho_0}$

somit grösser für kleine Kugeln als grosse

III Wenn aber noch dazu die Schwerkraft kommt, setzt sich das resultierende Volum zusammen:

$$v = \frac{\frac{4}{3}\pi a^3\rho g}{a^2\pi c\rho_0} = a\frac{\rho}{c\rho_0} = \frac{\frac{4}{3}\pi a^3\rho g}{a^2\varepsilon}$$

$$\text{Volum} = a^2\pi\sqrt{a^2\frac{\rho^2}{c\rho_0^2} + \frac{H\theta^2}{N^2}\frac{1}{a^6 c\rho_0^2}} = \frac{a^2\pi}{\varepsilon}\sqrt{\left(\frac{4}{3}\pi\rho g\right)^2 a^2 + \left(\frac{H\theta}{N}\right)^2\frac{1}{a^6}}$$

$$Vol^2 = A a^6 + \frac{B}{a^2}$$

$$\frac{d}{da} = 6a^5 A - \frac{2B}{a^3} = 0$$

$$a^8 = \frac{B}{3A}$$

~~Gibt es ein Maximum~~ ?

Also gibt es dann eine gewisse Radiusgrösse, für welche das durchstrichene Volumen maximal ist und zwar beträgt sie:

$$a^8 = \left(\frac{H\theta}{N}\right)^2 \frac{1}{\left(\frac{4}{3}\pi\rho\right)^2} \qquad a = \sqrt[4]{\frac{H\theta}{N}\,\frac{1}{\frac{4}{3}\pi\rho g}} \qquad H = 8.3.10^7 \quad N = 6.10^{23}$$

$$= \sqrt[4]{\frac{8.3.10^7.300}{6.10^{23}.4.10^3}} = \sqrt[4]{\frac{10^9}{10^{26}}} = \sqrt[4]{10^{-17}} = \frac{1}{\sqrt[4]{10}}\,10^{-4}$$

$$= \frac{10^{-4}}{2}\ \text{cm}$$

somit wäre dieser Radius noch ~~merklich~~ 5 mal grösser als d

also ist im ganzen Geltungsbereich der molekularen Reibungsformel das durchstrichene Volumen desto grösser je kleiner das Teilchen

Die Brown'sche Bewegung solcher Teilchen sieht man an den Wilson'schen Photographien für Röntgen-

Sobald aber Übersättigung herrscht, muss die Grösse d. durchstrichenen Volumens auf die Anwachsgeschwindigkeit d. Teilchens von Einfluss sein

[Aber macht das etwas merkliches aus im Vergleich zur molekularen Diffusion?]

Man müsste berechnen, wie gross d. durchstrichene Volumen ist, im Vergleich zu dem mittleren Kondensvolumen pro Teilchen, in der Zeit wo die halbe Diffusionskondensation erfolgt.

Bei sehr dichten Nebeln ~~kann~~ d. durchstrichene Volumen überwiegen, also ist dann aus obigem ersichtlich dass dann Tendenz zur ~~Gleich~~ Verteilung gleich grosser Tropfen besteht

dagegen muss bei dünnen Nebeln d. letztere überwiegen, dann besteht Tendenz zur Ungleichförmigkeit d. Tropfen denn Fallgeschw. $v = \frac{2}{9}\underline{a^2\rho g}$

also $\frac{\Delta v}{v} = 2\frac{\Delta a}{a}$ dadurch wird ein desto grösseres Volumen durchstrichen und stärkere Kondensation und das bewirkt ~~das~~ Zusammenstösse mit d. übrigen langsameren Tropfen, wodurch noch stärkeres Wachstum erfolgt

Wie rasch wächst der Radius eines kugelförmigen, in übersättigter Luft sinkenden Tropfens (unter Einfluss der Diffusion und Kondensation)?

Man kann sich die Kugel ruhend vorstellen, aber das Medium in Stokes'scher Weise strömend. Diffusionsgleichungen für bewegte Medien?

$$\frac{\partial}{\partial x}\left(cu - D\frac{\partial c}{\partial x}\right) + \cdots = \frac{\partial c}{\partial t}$$

Analoges Beispiel: wie rasch löst sich eine Salzkugel in strömendem Wasser?

$$\frac{\partial c}{\partial t} = D\cdot\nabla^2 c + \frac{\partial(cu)}{\partial x} + \frac{\partial(cv)}{\partial y} + \frac{\partial(cw)}{\partial z}$$

Die pro Zeiteinheit sich kondensierende Menge muss offenbar proportional sein $D^{?}$ überdies abhängen von a und v

also $\frac{m}{t} = D\, f.(a,v)$ Dimensionen von D: $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$

$D = \frac{x^2}{t}$ $m = D\frac{\partial c}{\partial x}\, a^2 t$ $= x^3$

Das ist nicht möglich, es muss allgemein sein

$$\frac{m}{t} = f.(D, a, v) = f.\left(\frac{x^2}{t}, a, \frac{x}{t}\right)$$

Weiter ist klar: falls verschieden grosse Kugeln dasselbe v haben, welche ihrem Radius proportional sind und falls $D \sim a^2$, ist der Bewegungsvorgang ähnlich, es werden ähnlich situierte Punkte in denselben ~~es erfüllt dieselben Konstante~~ Zeiten erreicht

Gradienten sind $\sim \frac{1}{a}$, Oberflächen $\sim a^2$, also Menge $m \sim a$

$$\varepsilon\frac{m}{t} = f(D\varepsilon^2, \varepsilon a, \varepsilon v)$$

$$\varepsilon\frac{m}{t} = D^\alpha a^\beta v^\gamma . \varepsilon = D^\alpha a^\beta v^\gamma \varepsilon^{\beta+\gamma+2\alpha}$$

somit $\beta+\gamma+2\alpha = 1$

$$\frac{m}{t} = D^\alpha \left(\frac{a}{v}\right)^{\beta} \ldots$$

$$\frac{x^3}{t} = \left(\frac{x^2}{t}\right)^\alpha \cdot t^\beta \cdot \frac{x}{t} \qquad \alpha = \beta = 1$$

unmöglich

Analoges Beispiel: Abkühlung einer Kugel durch eine strömende Flüssigkeit.

$$\frac{m}{t} = D^{\alpha}\, a^{\beta}\, v^{1-2\alpha-\beta}$$

$$\frac{x^3}{t} = \left(\frac{x^2}{t}\right)^{\alpha} x^{\beta} \left(\frac{x}{t}\right)^{1-2\alpha-2\beta}$$

$1 = \alpha + 1 - 2\alpha - 2\beta \qquad \alpha = -2\beta$

$3 = \cancel{2\alpha} + \beta + 1 - \cancel{2\alpha} - 2\beta \qquad 2 = -\beta$

$1 = -2\alpha$

$\alpha = 4$

$$\frac{m}{t} = D^{4}\, a^{-2}\, v^{1}$$

Ist Alles Unsinn! $\frac{m}{t}$ kann gar nicht <u>einer Potenz von v proportional sein</u>, da es für $\lim v = 0$ einen endlichen Wert besitzt!

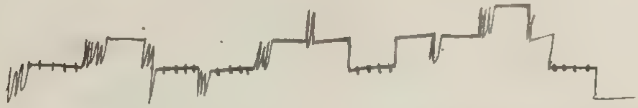
für $\lim v = 0$ haben wir $\frac{m}{t} \sim aD$

Es könnte also z.B. sein:

$$\boxed{\frac{m}{t} = aD . fc. \left(\frac{D}{av}\right)} \qquad m = aD\left(1 + \frac{av}{D}\right)$$

Grenzwerte von $P_n(\pm 1)$ für grosse n

$P_n(+1)$ ist natürlich Funktion der Intervallzeit



es definiert also die Wahrsch. (bezogen auf alle Punkte des n Zustands) dass zur Zeit t bereits ein $(n+1)$ herrscht

~~Somit erhält man die mittlere Dauer des n Zustandes indem man~~

~~$\int_0^\infty t \cdot P_n(\pm 1)_t \, dt$ bildet insofern nur Zuwachs in Betracht kommt~~

$P_n(+1) + P_n(-1)$ = Wahrsch., dass zur Zeit t ein $(n+1)$ oder $(n-1)$ herrscht

$P_n(\pm 0)$ definiert die Wahrsch., dass zur Zeit t ~~(noch der unveränderte)~~ derselbe Zustand n herrscht

Also wenn man von allen möglichen n Punkten sich t Intervalle abschneidet, so wird davon der Anteil $P_n(\pm 0)$ noch auf dieselbe n Linie fallen ; $\frac{dP_n(\pm 0)}{dt}$ würde also die Anzahl derjenigen welche aufhören sich in n zu befinden

Also erhält man mittlere Dauer des n Zustands $= \int t \frac{d}{dt} P_n(0) \, dt$

Das wäre unrichtig denn es werden in dem Falle auch die Werte t) mitgezählt, wo bereits eine Unterbrechung durch einen $\frac{n+1}{n-1}$ ten Zustand eingetreten ist.

Also wenn man die Wahrsch. wissen will, dass während Zeit t der Zustand n unverändert andauert ist von $P_n(0)$ noch der Anteil abzuziehen, der die Wahrsch. angibt, dass ~~bis zur~~ innerhalb der Zeit t bereits ein $\frac{(n+1)}{(n-1)}$ erschienen ist, (und dann wieder n geworden ist.

also: $P(n_0 (n \pm 1)_t, n_t)$

Ist das der $P_n(\pm 1)$?

wohl nicht, denn $P_n(\pm 1)$ umfasst nur die Zustände, wo zur Zeit t ein $(n+1)$ herrscht, aber nicht jene ~~Zustände, wo ein $(n+1)$ war erschienen~~, aber wieder einer andern Zahl Platz gemacht hat.

~~Also ist $P_n(r+1)$ zu vermehren um die Anzahl der Zustände wo zuerst n war, sodann (n+1) dann was anders $(n+2)$~~

Somit braucht man doch die ~~totalen~~ Kenntnis der Anzahl von Triplets etc

Vielleicht gelingt aber die (genäherte) Berechnung für grosse n?

Vielleicht könnte man sich so helfen, dass man $P_n(0) - T_\infty(n)$ verwendet, was allerdings auch noch zu gross ist — ~~aber~~

$$P_n(0) = e^{-\nu P}\Big[\overbrace{\binom{n}{0}(1-P)^n + \binom{n}{1}(1-P)^{n-1}P\frac{\nu P}{1!} + \binom{n}{2}(1-P)^{n-2}P^2\frac{(\nu P)^2}{2!} + \cdots}^{S}\;\binom{n}{n}(1-P)^0 P^n\frac{(\nu P)^n}{n!}\Big]$$

$$\binom{n}{0}(1-P)^n x^n + \binom{n}{1}(1-P)^{n-1}x^{n-1}P + \binom{n}{2}(1-P)^{n-2}x^{n-2}P^2 + \cdots \binom{n}{n}(1-P)^0 x^0 P^n = [x(1-P)+P]^n = \Phi(x)$$

$$1 + \frac{\nu P}{1!}x + \frac{(\nu P)^2}{2!}x^2 + \cdot \frac{(\nu P)^n}{n!}x^n + \cdots = F(x) = e^{\nu P x}$$

$$\Phi(x)\,F(x) = A + Bx^1 + Cx^2 + \cdots + Sx^n + Tx^{n+1} + \cdots$$

$$\frac{d^n(\overbrace{\Phi F}^{X_n})}{dx^n}\Big|_{x=0} = n!\,S$$

$$\frac{d}{dx}\left\{e^{\nu P x}[x(1-P)+P]^n\right\} = e^{\nu P x}\left\{\nu P[x(1-P)+P]^n + n(1-P)[x(1-P)+P]^{n-1}\right\}$$

$$= \nu P\,X_n + n(1-P)X_{n-1}$$

$$\frac{d^2}{dx^2} = (\nu P)^2 X_n + 2\nu P\,n(1-P)X_{n-1} + [n(1-P)]^2 X_{n-2}$$

$$\frac{d^3}{dx^3} = (\nu P)^3 X_n + 3(\nu P)^2 n(1-P)X_{n-1} + 3\nu P[n(1-P)]^2 X_{n-2} + [n(1-P)]^3 X_{n-3}$$

für $x=0$: $X_n = P^n$ stimmt $(\nu P)^n P^n + \binom{n}{1}(\nu P)^{n-1} n(1-P)P^{n-1} + \binom{n}{2}(\nu P)^{n-2}P^{n-2}[n(1-P)]^2$

aber das nützt nichts!

$$\lim \binom{n}{k}(1-P)^{n-k}P^k\frac{(\nu P)^k}{k!} = \left(\frac{\nu P^2}{1-P}\right)^k\frac{n!}{(n-k)!\,(k!)^2} = \left(\frac{P^2}{1-P}\right)^k\frac{\left(\frac{n}{e}\right)^n\nu^k\sqrt{2n\pi}}{\left(\frac{n-k}{e}\right)^{n-k}\left(\frac{k}{e}\right)^k\left(\frac{k}{e}\right)^k\sqrt{2(n-k)\pi}\cdot 2k\pi}$$

$$= \left[\frac{\nu P^2 e}{(1-P)k^2}\right]^k\frac{1}{\left(1-\frac{k}{n}\right)^{(n-k)}}\cdot\frac{1}{2k\pi}\sqrt{\frac{n}{n-k}}$$

$$\underline{e^{-\nu P}} \left[\frac{\nu P^2 e}{(1-P)k^2}\right]^k \frac{(n-k)^k}{\left(1-\frac{k}{n}\right)^n} \frac{1}{2\pi k}\sqrt{\frac{n}{n-k}}\,(1-P)^n$$

$$\cancel{\lim \left(1-\tfrac{k}{n}\right)^n = e} \qquad \frac{n^n}{(n-k)^{n-k}}$$

Anders getrennt:

für kurze Zeiten, wo P sehr klein ist

$$P(0) = e^{-\nu P}\left[(1-P)^n + \binom{n}{1}(1-P)^{n-1}\frac{\nu P^2}{1} + \binom{n}{2}(1-P)^{n-2}\frac{(\nu P^2)^2}{2!} + \dots \right]$$

$$= e^{-\nu P}\Big[1 - \binom{n}{1}P$$

$$+\binom{n}{2}P^2 + \binom{n}{1}\nu P^2$$

$$-\binom{n}{3}P^3 - \binom{n}{1}\binom{n-1}{1}\nu P^3$$

$$+\binom{n}{4}P^4 + \binom{n}{1}\binom{n-1}{2}\nu P^4 + \binom{n}{2}\frac{\nu^2 P^4}{2!}$$

$$= \cancel{e^{-\nu P}}\Big[1 - nP$$

$$+ \frac{n(n-1)+2n\nu}{2}P^2$$

$$- \frac{n(n-1)(n-2) - 6n(n-1)\nu}{6}P^3$$

$$+ \frac{n(n-1)(n-2)(n-3) + 12n(n-1)(n-2)\nu + 6n(n-1)\nu^2}{24}P^4 \Big]\left[1-\nu P+\frac{\nu^2 P^2}{2}-\dots\right.$$

$$= 1-(n+\nu)P$$

$$+ \frac{n(n-1)+2n\nu+\nu^2+2n\nu}{2}P^2 \dots$$

$$= 1-(n+\nu)P + \frac{n(n-1)+4n\nu+\nu^2}{2}P^2 + \dots$$

für lange Ketten, wo P beinahe gleich 1:

$$P_n(0) = e^{-\nu P}\frac{(\nu P^2)^n}{n!}\left[1 + \binom{n}{1}\frac{1-P}{\nu P^2} + \binom{n}{2}\left(\frac{1-P}{\nu P^2}\right)^2 + \binom{n}{3}\left(\frac{1-P}{\nu P^2}\right)^3 + \dots\right]$$

$$= e^{-\nu P}\frac{(\nu P^2)^n}{n!}\left[1 + \binom{n}{1}^2\frac{1-P}{\nu P^2} + \binom{n}{2}^2 2!\left(\frac{1-P}{\nu P^2}\right)^2 + \binom{n}{3}^2 3!\left(\frac{1-P}{\nu P^2}\right)^3 + \dots\right]$$

$$\neq e^{-\nu P}\frac{(\nu P^2)^n}{n!}\underbrace{\left[1 + n^2\frac{1-P}{\nu P^2} + \frac{n^4}{2!}\left(\frac{1-P}{\nu P^2}\right)^2 + \frac{n^6}{3!}\left(\frac{1-P}{\nu P^2}\right)^3 \dots\right]}_{e^{\frac{n^2(1-P)}{\nu P^2}}}$$

$$\neq e^{-\nu P + \frac{n^2(1-P)}{\nu P^2}}\frac{(\nu P^2)^n}{n!} =$$

$$= e^{n-\nu+\frac{n^2}{\nu}(1-P)}\left(\frac{\nu}{n}\right)^n\frac{1}{\sqrt{2\pi n}} = \frac{e^{-\nu}\nu^n}{n!}\cdot e^{\frac{n^2}{\nu}(1-P)}$$

für genügend lange Ketten muss sich die Exponentielle entwickeln lassen:

$$\lim P_n(0) = \frac{e^{-\nu}\nu^n}{n!}\left[1 - \frac{n^2}{\nu}(1-P) + \dots\right]$$

Dann ist also:

$$F = \frac{e^{-\nu}\nu^n}{n!} - P_n(0) \neq \frac{e^{-\nu}\nu^n}{n!}\cdot\frac{n^2}{\nu}(1-P)$$

Frage ob $\int^\infty F\, t\, dt$ endlich ist?

Es scheint nämlich evident zu sein, dass der Anfangszustand eine Nachwirkung hinterlässt, dass daher bei dieser Berechnungsart, wo $P_n(0) - W_\infty(n)$ verwendet wird, ein zu grosses Resultat erscheint. Wenn also dieses endlich ist, so ist das wirkliche F a fortiori endlich.

$$\int (1-P)\, t\, dt =$$

$$\int^\infty t\, dt\left[\frac{2}{\sqrt{\pi}}\int_0^{\frac{1}{2\sqrt{Dt}}} e^{-y^2}dy - \frac{1}{\beta\sqrt{\pi}}\left[1-e^{-\beta^2}\right]\right] = \int^\infty t\, dt\left\{\frac{\dots}{\sqrt{Dt}}\right\} = \infty \;!!$$

$$\frac{2}{\sqrt{\pi}}\int\left(1 - y^2 + \frac{y^4}{2}\right)\dots = \frac{2}{\sqrt{\pi}}\left[\beta - \frac{\beta^3}{3} + \frac{\beta^5}{10}\right] - \frac{1}{\sqrt{\pi}}\left[\beta - \frac{\beta^3}{2}\right] = \frac{1}{\sqrt{\pi}}\beta$$

$$P = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\rho} e^{-y^2} dy + \frac{1}{\rho\sqrt{\pi}} \left[1 - e^{-\rho^2}\right] \qquad \rho = \frac{h}{2\sqrt{Dt}}$$

$$\frac{dP}{dt} = \frac{dP}{d\rho}\frac{d\rho}{dt} = \left[-\frac{2}{\sqrt{\pi}} e^{-\rho^2} - \frac{1}{\rho^2\sqrt{\pi}}\left[1-e^{-\rho^2}\right] + \frac{2\rho}{\rho\sqrt{\pi}} e^{-\rho^2}\right]\frac{d\rho}{dt}$$

$$= -\frac{1-e^{-\rho^2}}{\rho^2\sqrt{\pi}}\frac{d\rho}{dt}$$

für kleine ρ:

$$\lim \frac{dP}{dt} = -\frac{1-\left(1-\rho^2+\frac{\rho^4}{2!}-\dots\right)}{\rho^2\sqrt{\pi}}\frac{d\rho}{dt} = -\frac{1}{\sqrt{\pi}}\left[1-\frac{\rho^2}{2}\right]\frac{d\rho}{dt}$$

$$= +\frac{1}{\sqrt{\pi}}\left[1-\frac{h^2}{4Dt}\right]\frac{h}{4\sqrt{Dt^3}}$$

$$\int t\,\frac{d(1-P)}{dt}\,dt = \int^{\infty} \frac{dt}{\sqrt{t}} = \infty \;!!$$

Also ist diese Rechnungsweise nicht brauchbar

Es handelt sich also um die Wahrscheinlichkeit, dass ein „n-Zustand" noch während der Zeit t unverändert weiter andauert.

Das ist die Wahrsch., dass keines der n Moleküle innerhalb dieser Zeit austrete und kein neues hereinkomme. Denn eine Compensation der ein und austretenden ist da äusserst unwahrscheinlich, da sie nur dann gelten würde, falls der Ein und Austritt in genau denselben Zeitmomente ~~statt~~finden. Dies ist ~~ff~~ natürlich ganz unwahrscheinlich.

Nun gibt P die Wahrsch. an, dass ein (anfangs) irgendwo im betrachteten Volum befindliches Molekül zur Zeit t sich ausserhalb jenes Volumens befinde, aber die Wahrsch. dass es schon einmal ausgetreten (eventuell wieder zurückgekommen sei) ist ganz anders, bedeutend grösser.

Würde es sich nur um die Möglichkeit einseitigen Austrittes handeln, so wäre die Frage schon ~~ert~~ erledigt durch frühere Überlegungen (Formel $\frac{n}{\mu 2^\mu}\binom{\mu}{\frac{\mu-n}{2}}$)

Hier ist aber die Komplikation, dass der Austritt beiderseitig stattfinden kann (oder auch einseitig, wenn dazu noch Reflexion an der anderen festen Wandfläche angenommen wird).

Wenn man die Sache makroskopisch behandelt, so entspricht dies der Diffusionsverteilung in einer Schichte, welche beiderseits constant auf der Konzentration 0 erhalten wird.

~~Als Probe nehmen wir noch die einseitige Tatsache vor.~~

$a_{n\mu}$ gibt die Wahrsch. an dass ein Teilchen aus einer Lage, welche um n Schritten von der Grenzfläche entfernt ist, die Grenzfläche zum ersten Male in der Zeit (μ Intervalle) erreicht

Die Wahrsch., dass ein Teilchen, aus einer beliebigen Anfangslage links von der Grenzfläche, dieselbe zum ersten Male in der Zeit μ erreicht, ist also (wenn ein Teilchen auf ~~[illegible]~~ 2 Distanzen entfällt) $= \sum a_n = A_\mu$

dessen Grenzwert im Falle grosser μ berechnet wird $\lim = \frac{1}{\sqrt{2\mu\pi}}$

dies ist also die relative Anzahl der in ~~[illegible]~~ 2 Intervallen durchtretenden Teilchen

Andererseits würde die übliche Diffusionstheorie liefern

$$c = c_0 + (c_1 - c_0)\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{Dt}}} e^{-y^2}\,dy$$

$$\left.\frac{\partial c}{\partial x}\right|_{x=0} = \frac{2(c_1-c_0)}{\sqrt{\pi}}\,\frac{1}{2\sqrt{Dt}}\, e^{-\frac{x^2}{4Dt}}\Big|_{x=0} = \frac{c_1-c_0}{\sqrt{\pi D t}}$$

Dabei ist jedoch der Unterschied, dass für $t=0$ die übliche Theorie ∞ geben würde, während der wirkliche Wert endlich bleibt

Durchtretende Menge $= \frac{c}{\sqrt{\pi D t}}$

Am einfachsten mittels Reihen entwickeln

$$\int^{l}\left[\cos(k+m)\frac{\pi x}{l}-\cos(k-m)\frac{\pi x}{l}\right]dx=0$$

$$\frac{\partial u}{\partial t}=D\frac{\partial^2 u}{\partial x^2}$$

$$\int_0^{l}\sin^2\frac{k\pi x}{h}\,dx=\frac{1}{2}\int\left(1-\cos\frac{2k\pi x}{h}\right)dx$$

$$=\frac{l}{2}$$

$$u=\sum A_k e^{-\frac{Dk^2\pi^2}{h^2}t}\sin\left(\frac{k\pi x}{h}\right)$$

$$\int_0^{l}\sin\frac{k\pi x}{l}\,dx=-\cos\frac{k\pi x}{l}\Big|_0^{l}\cdot\frac{l}{k\pi}$$

$$=0 \quad (k=2m)$$

$$\sum A_k \sin\frac{k\pi x}{l}=1$$

$$=\frac{2l}{k\pi} \quad (k=2m+1)$$

$$A_k=0 \quad (k=2m)$$

$$A_k=\frac{4}{k\pi} \quad (k=2m+1)$$

$$\left(\frac{\partial u}{\partial x}\Big|_0=\frac{4}{h}\sum e^{-\frac{Dk^2\pi^2}{h^2}t}\right.$$

$$u=\frac{4}{\pi}\sum_{k=1,3,5..}^{\infty}\frac{1}{k}e^{-\frac{Dk^2\pi^2}{h^2}t}\sin\frac{k\pi x}{h}$$

~~in derselben Weise vermindert sich~~ Gesammt Inhalt:

$$\int_0^{h}u\,dx=\;\ldots\int_0^{h}\sin\frac{k\pi x}{h}\,dx=\frac{h}{k\pi}\cos\frac{k\pi x}{h}\Big|_0^{h}=\frac{2h}{k\pi} \quad k \text{ ungerade}$$

$$=0 \quad k \text{ gerade}$$

$$\overline{U}=\frac{8}{\pi^2}\sum_{k=1,3,5..}^{\infty}\frac{1}{k^2}e^{-\frac{Dk^2\pi^2}{h^2}t}$$

(also nämlich $1=\frac{8}{\pi^2}\sum\frac{1}{k^2}$)

in derselben Weise nimmt die Dichte ab, dass ein Teilchen (die Grenzfläche noch nicht erreicht zur Zeit t)

$$\int_0^{\infty}t\frac{\partial\overline{U}}{\partial t}dt=\int_0^{\infty}t\cdot\frac{8}{\pi^2}\sum\frac{D\pi^2}{h^2}e^{-\frac{Dk^2\pi^2}{h^2}t}=\frac{8D}{h^2}\int_0^{\infty}\sum t\,e^{-\frac{Dk^2\pi^2}{h^2}t}dt$$

$$\int_0^{\infty}x\,e^{-\alpha x}dx=\frac{1}{\alpha^2}$$

$$T = \cancel{\;} 8D \sum \left(\frac{h^2}{Dk^2\pi^2}\right)^2 = \cancel{\sum_{k=1,3,5\ldots} \frac{8h^2}{k^2\pi^2} = \frac{8h^2}{\pi^2}\left(\frac{1}{1} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right)}$$

$$= \frac{8}{D}\frac{h^2}{\pi^4}\left[1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots\right]$$

Könnte die Rechnung nicht auch so gehen:

$$\left.\frac{\partial u}{\partial x}\right|_h = \frac{4}{h} \sum_{k=1,3,5\ldots}' e^{-D\frac{k^2\pi^2}{h^2}t} \cos\frac{k\pi x}{h}\Big|_h = \sum' \frac{4}{h}(-1)^k e^{-D\frac{k^2\pi^2}{h^2}t}$$

~~Menge~~ ausgetretene Gesamtmenge:

$$M = \left[-\left.\frac{\partial u}{\partial x}\right|_0 + \left.\frac{\partial u}{\partial x}\right|_h\right] D = D \sum_{k=1,3,5\ldots}' \frac{8}{h} e^{-D\frac{k^2\pi^2}{h^2}t}$$

$$\int_0^t M\,dt = \sum' \frac{8D}{h}\cdot\frac{h^2}{Dk^2\pi^2}\left(1 - e^{-\frac{Dk^2\pi^2}{h^2}t}\right) = \frac{8h}{\pi^2}\left\{\sum' \frac{1}{k^2} - \sum' \frac{1}{k^2} e^{-\frac{Dk^2\pi^2}{h^2}t}\right\}$$

$$= h - \frac{8h}{\pi^2}\sum' \frac{1}{k^2} e^{-\frac{Dk^2\pi^2}{h^2}t}$$

und übrig bleibender ~~Menge~~ Bruchteil $= \frac{8}{\pi^2}\sum' \frac{1}{k^2} e^{-\frac{Dk^2\pi^2}{h^2}t}$ so wie früher!

Wenn aber n Mol. in dem betrachteten Raume enthalten sind, so ist die Wahrsch., dass irgend eines davon austritt, n mal grösser

Wahrsch., dass ~~irgend eines~~ bis zur Zeit t ~~sch~~ noch keines ausgetreten sei, ist dann die combinierte ~~Wahrsch.~~ Wahrsch. für das gleichzeitige Eintreten von n Ereignissen: dass weder das erste, noch das zweite, noch das dritte u.s.w. ausgetreten sei

$$= \left[\bar{U}\right]^n$$

$$T = n\int_0^\infty t\left[\bar{U}\right]^{n-1}\frac{dU}{dt}\,dt = \cancel{t\left[\bar{U}\right]^n\Big|_0^\infty} - \int_0^\infty \left[\bar{U}\right]^n dt$$

das ist aber jedenfalls desto kleiner je grösser n

$$dW = -W_2(1-W)\,dt$$

Man könnte meinen, es sei $\overset{?}{}$

$W = 1 - n\,(1-\bar{U})$ $\overset{?}{}$ das wird aber nur dann sein, wenn der Fall wo 2 eingetreten

---

Nun ist noch die Möglichkeit des Eintrittes zu berücksichtigen

~~Die~~ Wahrsch. dass ein Teilchen (durch die linke Grenzfläche eintrete ist (aus beliebiger Anfangslage) in der Zeit t -- $t+dt$ (falls die Teilchendichtigkeit ν beträgt) $= \dfrac{\nu}{\sqrt{\pi D t}}\,dt$

Wahrsch. dass ein solches (durch die linke oder rechte Grenzfläche trete $= \dfrac{2\nu}{\sqrt{\pi D t}}\,dt$

Bei jener Berechnungsart sind also implicite alle möglichen Molekular-Anordnungen (mit der n Zahl) berücksichtigt, und zwar jede mit solcher Häufigkeit, als es ihrem Vorkommen im stationären Zustand entspricht, also entspricht dies dem Ausdrucke

$$\boxed{T = \tau\,\frac{N_1 + (1+2)N_2 + (1+2+3)\,N_3 + \cdots}{N_1 + 2\,N_2 + 3N_3}}$$

~~Man~~ Man kann nämlich ~~zweierlei~~ Wahrsch. für die Gesamt Dauer eines Zustands entwickeln:

~~In~~ Wenn wir wissen, dass einer gewissen molekularen Konstellation (für die n Zahl) eine Dauer t entspricht

~~Also~~ so handelt es sich noch um die Wahrsch. jener Konstellation, welcher die Dauer t nach sich zieht.

Jene Wahrsch. kann entweder gemeint sein als:

1). relative Häufigkeit jener Konstellation, in Bezug auf die Gesamtzahl aller Konstellation

$$= \frac{n_k}{n_1 + n_2 + n_3 \cdots}$$

oder als

2). relative Häufigkeit, in Bezug auf das Vorkommen im stationären Zustand, gemessen danach, wie wahrsch. man eine solche Konstellation im station. Zustand antreffen dürfte, wobei dann sämmtliche innerhalb der Zeit t ablaufenden Zustände nur einmal gezählt werden

$$= \frac{N_k}{N_1 + N_2 + N_3 + \cdots N_k + \cdots}$$

4)

~~$W(k) = \frac{N_1 + 2N_2 + 3N_3 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots} \quad \frac{N_1 + 2N_2 + 3N_3 + \ldots}{} = \frac{T_3}{T_4}$~~

~~Im ersten Falle erhält man als mittlere Wahrscheinlichkeit:~~

$$T = \frac{N_1 + 2N_2 + 3N_3 + \cdots}{N_1 + N_2 + N_3 \cdots} \tau$$

~~Im zweiten Falle die mittlere Dauer:~~

$$T = \frac{N_1 + 2^2 N_2 + 3^2 N_3 + 4^2 N_4 + \cdots}{N_1 + 2N_2 + 3N_3 + 4N_4 + \cdots} \tau$$

Früher hatten wir ~~[illegible]~~ noch eine dritte Auffassungsmöglichkeit erörtert

3). relative Häufigkeit in Bezug auf d. Vorkommen im stationären Zustand berücksichtigt wird dass bei längerem t der betreffende Zustand häufiger ~~[illegible]~~ anzutreffen ist wobei alle innerhalb t ablaufenden Zustände berücksichtigt werden

$$W(k) = \frac{kN_k}{N_1 + 2N_2 + 3N_3 + 4N_4 \cdots}$$

Daraus könnte man bilden

$$T_2 = \frac{N_1 + 2N_2 + 3N_3 \cdots}{N_1 + N_2 + N_3} = \frac{1}{\sum \frac{1}{k} W(k)} = \frac{1}{\int \frac{1}{t} W(t)\, dt}$$

4). Dieser letztere Ausdruck ist nahe verwandt mit der „Restdauer" eines Zustandes oder der Erwartungszeit für d. Eintritt d. ~~entgegengesetzten~~ Zustandes bei Gleichberechtigung sämtlicher Ausgangspunkte im stationären Zustand

Dann ~~[illegible]~~ 3) gilt

$$= \frac{\sum k W(k)}{\sum W(k)} = \sum k W(k)$$

$$T_3 = \frac{N_1 + 2^2 N_2 + 3^2 N_3 + \cdots}{N_1 + 2N_2 + 3N_3} \tau \qquad = \int t\, W(t)\, dt = \frac{\int t^2 f(t)\, dt}{\int t f(t)\, dt}$$

$$T_4 = \frac{N_1 + (1+2)N_2 + (1+2+3)N_3}{N_1 + 2N_2 + 3N_3} \tau \qquad = \frac{1}{2} \frac{\int t^2 f(t)\, dt}{\int t f(t)\, dt}$$

also im Falle ~~[illegible]~~ kurzer Intervalle τ und dem entsprechend grosser Zahlen k gilt:

$$T_3 = \tfrac{1}{2} T_4$$

$$\sum (1 + 2 + 3 + \cdots n) = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \qquad = \frac{1}{2}\left\{\frac{N_1 + 2^2 N_2 + 3^2 N_3}{\cdots} + \frac{N_1 + 2N_2 + 3N_3}{\cdots}\right\}$$

Genau aber gilt:

$$T_4 = \tfrac{1}{2}(1 + T_3)$$

Die vorher berechneten $\overline{U}$ sind nun wohl identisch mit den $W(t)$-Werten

Damit stimmt, dass wir haben

$$\int_0^\infty \frac{1}{t} W(t)\,dt = \infty$$

also $T_2 = 0$ wie schon von vornherein zu erwarten

Berechnet wurde die mittlere Dauer T_3 (oder T_4 ?)

Nun kann man aber nicht setzen

$$\Theta_3 = \frac{T_3}{\overline{W(n)}}$$

denn $$\Theta_3 \dot{\overline{W}}(n) = \frac{M_1+(1+2)M_2+(1+2+3)M_3+}{M_1+2M_2+3M_3+\dots} \cdot \frac{N_1+2N_2+3N_3}{M_1+2M_2+3M_3} \dots$$

$$T_3 = \frac{N_1+(1+2)N_2+(1+2+3)N_3+\dots}{N_1+2N_2+3N_3+\dots}$$

Es wäre nur dann richtig, falls gelten würde:

$$\frac{M_1+(1+2)M_2+(1+2+3)M_3}{(M_1+2M_2+3M_3+\dots)^2} = \frac{N_1+(1+2)N_2+(1+2+3)N_3}{(N_1+2N_2+3N_3+\dots)^2}$$

~~[illegible]~~ was wohl unrichtig ist

Statistik der 0 bei Svedberg

N_1	N_2	N_3	N_4	N_5
38	18	8	2	1

38
36
24
8
5
111

111 : 67 = 1.66
44
4

1 : 0.679 = 1.47 bzw.
321
49

Statistik der 1

1:0.646 = 1.55
354
31

	N_1	N_2	N_3	N_4
16	4 2 3 2 5	11111	1111	11111
11	1 5 3 2	111111	1111	
8	11 111111	1111111111		
12	11111111111	1111		
27	7 4 3 3 2 4 4			
74		25	8	5

74
50
24
20
168 : 112 = 1.5
56

2

1 : 0.722 = 1.38
278
61

	N_1	N_2	N_3	N_4
12	7, 2, 1, 1 1	1 4 1 1 1	11111	1
10	10	11111111		
15	2, 5 6 2	111111		
18	4 2 4 6 2			
12	2 8 2			
67		22	5	1

67
44
15
4
130 : 95 = 1.37
35
65

3

1 : 0.815 = 1.23
185
22

	N_1	N_2	N_3
20	3 7 2 5 3	1111	11
25	2 2 4 5 9 3	1121	
45		9	2

45
18
6
69 : 56 = 1.23
13
1

4

1: 0.889 = 1.12
111
22

	N_1	N_2	N_3	N_4
	10	111		1
	5, 7			
	22	3	0	1

22
6
4
32 : 26 = 1.23
6

Es handelt sich also bei Bestimmung der $\overset{T_1}{T_s}$ Dauer für Wiederkehr um die Wahrscheinlichkeit, dass ein „Nicht n-Zustand" innerhalb des Zeitraumes t Intervalls zum ersten Male einem „n Zustand" Platz macht.

Wir kennen zwar die Wahrsch., dass ein „n Zustand" ... einem „Nicht n Zustand" Platz macht aber von vornherein ist da kein Zusammenhang ersichtlich?

Sollte nicht wegen der Stationarität eine solche Beziehung herrschen, dass: die Häufigkeit der Fälle, wo innerhalb der Zeit t (... $t+dt$) ein „n-Zustand" zum ersten Male in „Nicht n Zustand" übergeht, ist gleich gross wie umgekehrten Vorgang?

In diesem Falle müsste die Beziehung gelten:

Wahrsch. (eines n Zustands) × Wahrsch (Übergang n → Nicht n) = Wahrsch (Nicht n) × Wahrsch (Nicht n → n)

Sicher ist das richtig, wenn man die Worte „zum ersten Male" weglässt – aber darauf kommt es eben an! (das ist einfach die Behauptung von der Umkehrbarkeit der Zeitfolge)

Unter dieser Hypothese hätte man also:

$$W(m\to n)_t\,dt = W(n\to m)_t\,dt\,\frac{W(n)}{W(m)} = \frac{e^{-\nu}\nu^n}{n!}\,\frac{1}{1-\frac{e^{-\nu}\nu^n}{n!}}\,W(n\to m)_t\,dt$$

~~und daraus wäre ... vorliegen~~ ~~Die mittlere Dauer des n Zustandes ist nun~~

~~$\int_0^\infty t\,W(m\to n)_t\,dt$~~

~~Da nun $W(n)$, $W(m)$ von der Zeit unabhängig sind, würde das die Folgerung nach sich ziehen, dass die mittlere Dauer des selteneren Zustandes grösser wäre als die des häufigen Zustandes.~~

Nun muss aber auf jeden Fall: $\int_0^\infty W(n\to m)_t\,dt = 1$ sein und ebenso: $\int_0^\infty W(m\to n)_t\,dt = 1$

das ist also unmöglich!

~~Es spielt also die Häufigkeit des Übergangs~~.

Es ist eben nur im Allgemeinen gültig: Häufigkeit $m \to n$ = Häufigkeit $n \to m$

~~darnach~~ Übergang zum ersten, zweiten, dritten Mal (innerhalb t ...) ..

$$\frac{d}{dt}\left[P_n(0)\right] dt = \text{„Wahrsch. dass der } n \text{ Zustand in der Zeit } t \ldots t+dt \text{ in } n \text{ übergeht"}$$

$$= dt \sum (W_1 + W_2 + W_3 + \cdots)$$

zum ersten, zweiten, dritten, ... Male innerhalb der Zeit t

aber die einzelnen $W_k(m \to n)$ und $W_k(n \to m)$ sind ganz verschieden.

---

z.B. Wahrsch., dass $\overset{m}{7}$ zum ersten Male in $\overset{n}{2}$ übergeht,

das heisst, dass zur Zeit $t=0$ eine 7 anwesend war und dass innerhalb $t \ldots t+dt$ eine 2 erschienen ist, aber vor t ~~...~~ noch niemals 2 erschienen ist.

das kann nur in der Weise geschehen, dass um 5 mehr austreten als eintreten.

also z.B. 6 rote austreten, 1 weisse eintritt, und zwar muss 1 weisse irgendwann eintreten und zum ersten Mal

~~...~~ in der Zeit $t+dt$ 6 rote austreten

Es können aber auch solche Vorgänge stattgefunden haben:
Dann sind 7 kanzeptirt und zum ersten Male
~~Im Allgemeinen ... irgendwann innerhalb t~~

7r
6r + 1w
5r + 2w
...
0r + 7w
0r + 6w
0r + 5w
⋮
0r + 3w
0r + 2w

Im Allgemeinen kann also Eintritt „des ersten Males" geschehen ~~ist~~ entweder durch die Bewegung der roten oder der weissen Teilchen

also m geht zum ersten Mal in n über: entweder so dass irgendwann innerhalb $t \ldots t+dt$ .. die Zahl $n-m+k$ wenn k rote austreten, und innerhalb $t \ldots t+dt$ die Zahl der ausgetretenen kleiner war als $n-m+k$ eintritt, während bis t immer die Zahl der ausgetretenen kleiner war als $n-m+k$

Das genügt aber nicht, denn es konnte 2 bereits erreicht werden, als bloss 5 rote ausgetreten ~~...~~ und $n-m+k-2$ eingetreten waren.

Während, dass $n = (n-1)$ zum ersten Mal in n übergeht, wobei in der Zeit t alle möglichen Kleinen ...

Nur so, dass zu den $(n-1)$ten ein weiteres dazukommt, und zwar erst im Zeitpunkt t – $(n-1)z + w$

oder auch $(n-1)^z \to (n-2)z \to +1w \to +1w$

oder $(n-1)z - z - z + w + w + w$

oder $(n-1)z - z + w - z + w + w$

aber $(n-1)z - z + w + w - z + w$ gilt nicht

ebenso $(n-1)z + w - z - z + w - z$ nicht etc.

Vielleicht bekommt man neue Relationen durch Heranziehung der $P_n(0)$ für $t = 2, 3, \dots \tau$

$$W(n) = \frac{N_1 + 2N_2 + 3N_3}{N_1 + 2N_2 + 3N_3 \dots}$$

$$P_n(0)_\tau = \frac{N_2 + 2N_3 + 3N_4 + 4N_5}{N_1 + 2N_2 + 3N_3 + \dots}$$

$$P_n(0)_{2\tau} = \frac{N_3 + 2N_4 + 3N_5 + \dots}{N_1 + 2N_2 + 3N_3 + \dots}$$

$$P_n(0)_{3\tau} = \frac{N_4 + 2N_5 + 3N_6 + \dots}{N_1 + 2N_2 + \dots}$$

$$\varepsilon_1 = \frac{N_1 + N_2 + N_3 + \dots}{N_1 + 2N_2 + 3N_3 + \dots} = \frac{\tau}{T_2} = 1 - P_n(0)_\tau$$

$$\varepsilon_2 = \frac{N_2 + N_3 + N_4 + \dots}{N_1 + 2N_2 + 3N_3 + 4N_4} = P_n(0)_\tau - P_n(0)_{2\tau}$$

$$\varepsilon_3 = \frac{N_3 + N_4 + N_5 \dots}{N_1 + 2N_2 + 3N_3 + \dots} = P_n(0)_{2\tau} - P_n(0)_{3\tau}$$

Nun ist aber:

$$T_2 = \frac{N_1 + 2N_2 + 3N_3 + \dots}{N_1 + N_2 + N_3}$$

$$= \frac{1}{1 - P_n(0)_\tau}$$

$$T_4 = \frac{N_1 + (1+2)N_2 + (1+2+3)N_3 + \dots}{N_1 + 2N_2 + 3N_3 + \dots}$$

$$T_4 = \varepsilon_1 + 2\varepsilon_2 + 3\varepsilon_3 + 4\varepsilon_4 + \dots$$

$$= \sum_k k\,\varepsilon_k$$

$$= 1 - P_n(0)_\tau + 2\left[P_n(0)_\tau - P_n(0)_{2\tau}\right] + 3$$

$$+ 3\left[P_n(0)_{2\tau} - P_n(0)_{3\tau}\right] + 4\left[P_n(0)_{3\tau} - P_n(0)_{4\tau}\right] + \dots$$

$$\boxed{\frac{T_4}{\tau} = 1 + P_n(0)_\tau + P_n(0)_{2\tau} + P_n(0)_{3\tau} + P_n(0)_{4\tau} + \dots}$$

Im Falle unendlich kurzer Intervalle τ :

$$t = \frac{h^2}{4D\beta^2}$$

$$I_1 = \int_0^\infty P_1(0)_t \, dt$$

$$\beta = \frac{h}{2\sqrt{Dt}}$$

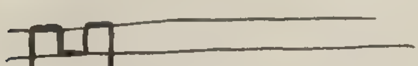
Beispielsweise ist: $P_0(0) = e^{-\nu P}$

Also $$I_1(0) = \int_0^\infty e^{-\nu P} dt \qquad P = 1 - \frac{2}{\sqrt{\pi}}\int_0^\beta e^{-y^2} dy + \frac{1}{\beta\sqrt{\pi}}\left[1 - e^{-\beta^2}\right]$$

$$= \frac{h^2}{2D}\int_0^\infty \frac{e^{-\nu P}}{\beta^3} d\beta \qquad = \infty!$$

Das ist aber falsch! ! Denn $P_n(0)_{2\tau} > \frac{N_3 + 2N_4 + \dots}{N_1 + 2N_2 \dots}$

weil dabei auch solche Fälle mitgezählt werden, wo unterdessen eine andere Zahl war und dann erst wieder n gekommen ist! usw.



Jedes M_1 bewirkt einen solchen Zuwachs

$(n\ m\ n)$

also sollte sein: $$P_n(0)_{2\tau} = P(n\,n\,n) + P(n\,m\,n)$$

$$= \frac{N_3 + 2N_4 + 3N_5 + \dots + M_1}{N_1 + 2N_2 + 3N_3}$$

ebenso $$P_n(0)_{3\tau} = P(n\,n\,n\,n) + P(n\,m\,n\,n) + P(n\,n\,m\,n) + P(n\,m\,m\,n)$$

Nun sind aber dabei ~~die~~ nur diejenigen M_1 einzustellen, welche nach N_1 und vor N_2 oder nach N_2 und vor N_1 entfallen aber nicht jene welche z.B. vor N_1 und nach N_1 entfallen. Da aber die Häufigkeit der M_1 von der umgebenden Konstellation abhängt, lässt sich dies nicht weiter in Rechnung stellen!

$\frac{1}{2^m}\binom{m}{\frac{m-2}{2}}$ $\cancel{\frac{16}{16}}$ $\frac{64}{16} = 4 = m$

$$x_n^2 = m\,\delta^2 = \cancel{2D t_n}$$

$$\cancel{\frac{dc}{dt}} = D\,\frac{\partial^2 c}{\partial x^2}$$

25
45
10
80 . 2
160 : 32 = 5 = m

$$x_t^2 = \frac{t}{\tau}\,\delta^2 = 2Dt$$

$$D = \frac{\delta^2}{2\tau}$$

$$\sqrt{\frac{2\tau}{t\pi}} = \delta\sqrt{\frac{1}{Dt\pi}} = \qquad \frac{1}{\delta}\sqrt{\frac{1}{2\pi t\tau}}$$

$$\frac{1}{\tau}\sqrt{\frac{2}{m\pi}} = \sqrt{\frac{2}{t\tau\pi}}$$

$$\int_0^\infty e^{-\alpha t}\,dt = \frac{1}{\alpha}$$

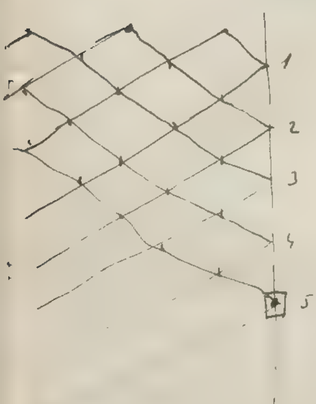
$$\sum \frac{8D}{h}\,\frac{h^2}{Dk^2\pi^2} = \frac{8h}{\pi^2}\sum\frac{1}{k^2}$$

$$\cancel{1 + \alpha + \alpha^2 + \cdots = \frac{1}{1-\alpha}} \qquad 1 + \frac{1}{9} + \frac{1}{25} + \cdots$$

Wahrsch. dass erst beim n ten Wurf zum ersten Mal ein Teilchen (oder mehr) durch die Grenzfläche durchgeht ist gleich dem Produkt der Wahrsch. dass

$= \sum$

III. Anordnung



für $n = 5$

$$\frac{7}{256}\left[1-\frac{1}{8}-\frac{3}{32}-\frac{5}{128}\right]\cancel{+\frac{7}{128}}\left[1-\frac{1}{32}-\frac{5}{128}\right]\left[1-\frac{1}{128}\right]$$

$$+\frac{7}{128}\left[1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}-\frac{5}{128}\right]\left[1-\frac{1}{32}-\frac{5}{128}\right]\left[1-\frac{1}{128}\right]$$

$$+\frac{5}{128}\left[1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}-\frac{5}{128}\right]\left[1-\frac{1}{8}-\frac{3}{32}-\frac{9}{128}\right]\left[1-\frac{1}{128}\right]$$

$$+\frac{7}{4\cdot 128}\left[\quad\cdot\quad\right]\left[\quad\cdot\quad\right]\left[1-\frac{1}{32}-\frac{5}{128}\right]$$

$$+\frac{1}{512}\left[\quad\cdot\quad\right]\left[\quad\cdot\quad\right]\left[\quad\cdot\quad\right]\left[1-\frac{1}{128}\right]$$

---

für $n = 1$: $\frac{1}{2}$

$n = 2$: $\frac{1}{8}+\frac{1}{8}\left[1-\frac{1}{2}\right] = \frac{3}{16}$

$n = 3$: $\frac{1}{16}\left[1-\frac{1}{32}\right]+\frac{3}{32}\left[1-\frac{1}{2}-\frac{1}{8}\right]+\frac{1}{32}\left[1-\frac{1}{2}-\frac{1}{8}\right]\left[1-\frac{1}{32}\right]$

$= \frac{1}{32\cdot 16}\left[31+3\cdot 6+6\cdot\frac{31}{32}\right] = \frac{1}{32\cdot\cancel{16}\,8}\left[31\cdot\frac{\cancel{38}\,19}{32}+\frac{9}{\cancel{16}}\right]$

$$\begin{array}{r} 31 \\ 279 \\ 288 \\ \hline 877 \end{array}$$

$= \frac{877}{8\cdot 32\cdot 32}$

$W(t)\,dt = n \frac{dP}{dt}\,dt\ P^{n-1} = \frac{d}{dt}(P^n)\,dt$

$$\rho = \frac{h}{2\sqrt{Dt}}$$

$$\lim_{\substack{h=0 \\ \frac{n}{h}=c}} \frac{d}{dt}(P^n)$$

$$P = 1-\varepsilon$$

$$P^n = (1-\varepsilon)^n = (1-\varepsilon)^{\frac{n\varepsilon}{\varepsilon}} = e^{-n\varepsilon}$$

$$\lim_{\rho=\infty} P = 1 - \frac{1}{\rho\sqrt{\pi}} \qquad \varepsilon = \frac{1}{\rho\sqrt{\pi}} \qquad P^n = e^{-\frac{n}{\rho\sqrt{\pi}}} = e^{-\frac{2n\sqrt{Dt}}{h\sqrt{\pi}}} =$$

$$\varphi = \lim P^n = e^{-2\frac{\nu}{h}\sqrt{\frac{Dt}{\pi}}} = e^{-\frac{\nu}{\rho\sqrt{\pi}}}$$

$$n\frac{dP}{dt}\,dt\,P^{n-1}\cdot\varphi + P^n \frac{d\varphi}{dt}\,dt = \frac{d}{dt}\left[P^n\cdot\varphi\right]dt$$

$$\int_0^\infty t\,\frac{d}{dt}(P^n\varphi)\,dt = \cancel{tP^n\varphi} - \int_0^\infty P^n\varphi\,dt \quad = \text{unklar}$$

$\sqrt{t} = x$

$dt = 2x\,dx$

denn schon $\displaystyle\int_0^\infty \varphi\,dt = \int e^{-\alpha\sqrt{t}}\,dt = 2\int_0^\infty e^{-\alpha x}\,x\,dx = +\frac{2}{\alpha^2}$

$$= \frac{\cancel{4}\,h^2\pi}{2\cancel{4}\,\nu^2 D}$$

eigentlich, da wir zusammenstoßen:

$$T_3 = \int_0^\infty t\,\frac{d}{dt}\left[P^n\varphi^2\right]dt$$

$$= \int_0^\infty P^n\varphi^2\,dt = \int_0^\infty e^{-\frac{2n\sqrt{Dt}}{h\sqrt{\pi}} - 4\frac{\nu}{h}\sqrt{\frac{Dt}{\pi}}}\,dt = \int_0^\infty e^{-\frac{(n+2\nu)}{\rho\sqrt{\pi}}}\,dt$$

$$= \frac{h^2\pi}{2(n+2\nu)^2 D}$$

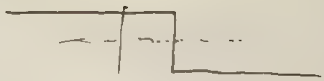
Pierwotna argumentacja:

$\overline{\Delta n} = (\nu-n)P = 1$

$$P = \frac{1}{\nu-n} = \frac{2\sqrt{Dt}}{h\sqrt{\pi}}$$

$$t = \frac{h^2}{4(\nu-n)^2}\frac{\pi}{D}$$

w razie $n \gg \nu$: $T_3 = \frac{t}{2}$



$$\left[\frac{8}{\pi^2}\sum \frac{1}{k^2} e^{-\frac{Dk^2\pi^2}{h^2}t}\right]^n \qquad \neq \left[\cancel{8} e^{-\frac{D\pi^2 t}{h^2}}\right]^n = e^{-\frac{D\pi^2 t}{h^2}n}$$

$$\cancel{= \frac{8}{\pi^2}\sum \frac{1}{k^2}\left(1 - \frac{Dk^2\pi^2}{h^2}t \ldots\right)} = 1 - 8\sum \frac{D}{h^2}t$$

$$= \frac{8}{\pi^2}\left[e^{-\frac{D\pi^2}{h^2}t} + \frac{1}{9}e^{-9\frac{D\pi^2}{h^2}t} + \frac{1}{25}e^{-25\frac{D\pi^2}{h^2}t} \ldots\right.$$

$$= \frac{8}{\pi^2}e^{-\frac{D\pi^2 t}{h^2}}\left[1 + \frac{1}{9}e^{-8\frac{D\pi^2 t}{h^2}} + \frac{1}{25}e^{-24\frac{D\pi^2 t}{h^2}} \ldots\right.$$

$$I_3 = \int_0^\infty t\,\frac{d}{dt}\left(e^{-\frac{D\pi^2 t}{h^2}(n+\nu)}\right)dt = \int_0^\infty e^{-\frac{D\pi^2 t(n+\nu)}{h^2}}dt = \frac{h^2}{D\pi^2(n+\nu)}$$

1 : 49 = 0.0204
1/81 = .012345
37
1:121 =

1.1111
400
2041
12345
12

$$p + 2p^2 + 3p^3 + \ldots = p\frac{\partial}{\partial p}(1 + p + p^2 + \ldots)$$

$$= p\frac{\partial}{\partial p}\frac{1}{1-p} = \frac{p}{(1-p)^2}$$

$$\lim = \frac{1}{m2^m}\binom{m}{\frac{m-1}{2}} = \frac{1}{2^m}\frac{m(m-1)(m-2)\ldots\left(\frac{m}{2}+\frac{1}{2}+1\right)}{\left(1.2.3\ldots\frac{m-1}{2}\right)^2\frac{m+1}{2}} \Bigg| \frac{\left(\frac{m}{2}+\frac{1}{2}\right)!}{} = \frac{1}{2}\frac{\left(\frac{m}{e}\right)^m\sqrt{2m\pi}}{\left(\frac{m-1}{e}\right)^{m-1}\sqrt{\pi(m-1)}\,\pi\frac{m+1}{e}}$$

$$= \frac{m(m-2)(m-4)\ldots}{m(m+1)(m-1)(m-3)\ldots}$$

$$= \frac{m-1}{m+1}\left(\frac{m}{m-1}\right)^m$$

$$= \left(\frac{m}{m-1}\right)^m \frac{\sqrt{m}}{m+1}\frac{1}{\sqrt{\pi}\,e}$$

$$= \ldots = \frac{1}{\sqrt{m\pi}}$$

$n! = \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$

$\lg \bar{w} = -\nu + \cancel{\nu(1+\delta)\lg\nu} + \nu(1+\delta) - \cancel{\nu(1+\delta)\lg\nu} - \nu(1+\delta)\lg(1+\delta)$
$\quad - \lg\sqrt{2\pi n}$

$= \nu\delta - \nu(1+\delta)\left(\delta - \frac{\delta^2}{2} + \frac{\delta^3}{3}\right) \quad - \lg\sqrt{\ } -$

$= \nu\delta - \left[\nu\delta - \nu\frac{\delta^2}{2} + \nu\frac{\delta^3}{3} + \nu\delta^2 - \nu\frac{\delta^3}{2}\right.$

$= -\nu\frac{\delta^2}{2}$

0·3247 · ~~[illegible]~~ 9/8
29232
3654 · 9/16
~~32886~~
16443
6166 9/8
56
69368 9/32
53
216
1953 9/40
14647 3/8.62
144
24
87

00	·3247 ·	359
01	·3654	404
02	·2055	227
03	·07707	8·525
04	·02170	2·40
05	·048825	0·54
06	·00915	0·10

7708
7708
962
8525

217
217
1302
240

488
488
29
5397

915
915
55
1012

216.512
1080
216
43
1106

3247
325
~~79~~
~~3960~~
3591

3654
3654
219
4043

2055
2055
123
2273

324.512
1620
32
6
1658

$\nu P e^{-\nu P} \cdot e^{-\nu}$

$P e^{-\nu P} \cdot e^{-\nu}\nu$

216 · 31/2
108
324
335 · 31/4
1005
10385 :4 = 2596 · 31/6
7788
80476 :6 = 1341

1341 · 31/8
4023
41571 :8 = 5196

52 · 31/10
156
161

161
483
499 :12
416

416
1248
129 :14 =
3 | 92

3247
85:7
22729
649
162
26
2357
8561
2357
1414
118
18
39'1

2357
8527
16999
471
118
18
17100

1710
8527
11977
342
86
13
12418

1242
8527
8094
278
62
10
9014

7258.9
053

2596.
215
12980
260
52
132'92

1341
215
6705
134
27
68'66

52.512
2560
1024
26.6

805
101
325
824

2080
416
83
2'13

512. 324
1536
1024
204
465(9)

512. 335
1536
1536
250
1715

2357
5171
2357
1649
24
11
40'41

1711
9231
1711
513
34
15
22'73

1242
786
745
99
8
8'52

901. 266
2394

2'4

6'5. 8'2
520
1
0'53

0
:14=
192

161
83
499:12
16

171'5

(11) 354
2478
35
17
6'54

(21) 133
322
900
497
42'8

(31) 087
280
1374
850
10'24

(41) 266
2'37
~~25'3~~
26'3.24
526
105
6'3

(51) 8'2
197
16

(22)
278
894
83
37'0

(32)
280
1374
6181
19'92

(42)
287
24'3.28
546
2184
7'64

(52)
274
2192
55
2'25

185
(33) 687
5496
343
12'71

214
532
27
11
5'7

(53) 234

1872
46
1'99

266 90
2394
264.24
528
105
63

264.29
528
2376
7'65

212.268
424
1272
17
5'7

266
266
26
2'85

266
1064
798
27
1147

111
266
27
3
2'96
02

0'6'8

139
1112
28
1'14

75

$n=$	0	1	2	3	4	5	
$n=0$	45 39.9	35 40.4	19 22.7	7 8.5	5 2.4	0 0.5	3.1 | 110.5
1	40 40.4	55 65.4	40 42.8	17 19.2	10 6.3	1 1.6	175.7
2	19 22.7	42 42.8	35 37.0	24 19.9	6 7.6	2 2.2	132.2
3	6 8.5	23 19.2	22 19.9	13 12.7	5 5.7	0 1.9	67.9
4	2 2.4	8 6.3	10 7.6	4 5.7	6 2.9	2 1.1	26.0
5	0 0.5	1 1.6	2 2.2	2 1.9	0 1.1	0 0.7	8.0
0							

0 1 2 3 4 5

216 335 260 134 052 016 004

168 : 518 = 324
126
23

$n=17$
$\nu=1.55$ } 15.45

0.4343.
2172
~~19~~ 6
6711

$$W(17) = \frac{e^{-\nu}\nu^n}{n!} = \frac{e^{-\nu}\nu^n}{\left(\frac{n}{e}\right)^n\sqrt{2n\pi}} = e^{n-\nu}\left(\frac{\nu}{n}\right)^n\frac{1}{\sqrt{2n\pi}}$$

0.19033
1.23045
(0.95988 − 2) . 17
671916
1.6218 − 34
6.711.
8.3428 − 34
− 2.0287
6.3141 − 34 = $\sqrt{27.6859}$

~~0.0151~~
1.2305
.3010
.4972
2.0287

16.318
6.711
23.029 − 34
− 2.029
21 − 34 = 13.

~~86400.39~~
.365. 24.60.39
= 36.24.6.4.10^3
= 9.29.10^4 = 2.10^7

10^{13} y = 5.10^5 Jh

Kann man die Diffusion annehmen:

Wiederkehrzeit eines Zustandes n = Zeit, in welcher die durchsch. maximal ~~max~~ Elongation Abweichung n beträgt

~~Ab~~ Wäre

Ornstein-Hasenöhrl   wahrscheinlichste ~~...~~ Maxim.-Abw. $\frac{x}{(\xi)} = \sqrt{2\lg n} = 2.15\sqrt{{}^{10}\lg n}$

$$n = e^{\frac{x^2}{2\xi^2}}$$

$$dW = \frac{2}{\sqrt{\pi}}\, e^{-\frac{x^2}{2}}\, \frac{dx}{\sqrt{2}}$$

Wieviel diffundiert innerhalb $\lim \tau = 0$ kurzen Zeiten aus der h Schichte nach aussen?

falls Zeit so kurz, dass die Wege als geradlinig betrachtet werden:

$\lim_{t=0} P = ?$

$$P = \frac{v_x t}{\sqrt{\pi} h} = \frac{v t}{2 h \sqrt{6\pi}}$$

Stosszahl pro cm² und sek

$$\lim T_1 = \frac{h}{W(n)} \cdot \frac{\tau}{2v\tau} = \frac{h}{2v} \frac{1}{W(n)}$$

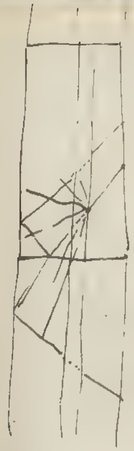
Unsere Verwechslung von P und $P_n(0)$!

$$\lim \Theta_1 = \frac{T_1}{W(n)}$$

$$\lim W_n(0) \;(32) = 1-(\nu+n)P = 1-(\nu+n)\frac{v t\,(2)}{h\sqrt{6\pi}} \qquad \lim T_1 = \frac{h\sqrt{6\pi}}{2(\nu+n)v}$$

lim T_1 muss allerdings von v oder m und nicht nur von D abhängen; denn D ist gleich gross für verschieden schwere Teilchen von gl. Radius; dieselben haben aber sehr verschiedene Geschwindigkeiten und die rascheren müssen häufiger hin und her schwanken als die langsameren, also müssen die rascheren ein kürzeres T_1 verursachen!

Verdünntes Gas



Falls Strahlung gegeben ist aber alle Richtungen gleich möglich, wird ~~[illegible]~~ die mittlere ununterbrochene Verweilungsdauer in der Schichte a unendlich gross

$$t = \frac{a}{v\cos\varphi}$$

$$\bar{t} = \frac{1}{2\pi}\int_0^{\frac{\pi}{2}} 2\pi \sin\varphi \, d\varphi \, \frac{a}{v\cos\varphi} = \frac{a}{v} \log\cos\varphi \Big|_{\frac{\pi}{2}}^{0} = \infty$$

Dabei war vorausgesetzt, dass die Wände regelmässig reflektieren.

Wenn aber die Richtung der reflektierten Bahn ganz zufällig ist (~~[illegible]~~ rauhe Wände)

Sobald nur die beiden parallel-Wände rauh sind, ändert sich nichts

Dagegen ändert sich die Richtung, falls das Gefäss auch seitlich durch reflectierende Wände begrenzt ist

$$\Rightarrow \overbrace{\frac{a}{\sqrt{a^2+b^2}}}^{c}\left(\frac{b}{2}+\frac{a}{\sqrt{a^2+b^2}}\left(\frac{b}{2}\right.\right. \cdots \quad c+c^2+c^3\cdots = \frac{c}{1-c}$$

Es wird $\bar{t} = -\frac{a}{v}\log\frac{a}{\sqrt{a^2+b^2}} + \varepsilon$ entsprechend dem gestrichenen Raume

für $\lim a = 0$ wird $\bar{t} = 0$ (?), das Verhältnis $\lim \frac{\bar{t}}{a} = \infty$

aber $\sum_a \bar{t}$ über das ganze Gefäss $= \infty$

Dasselbe gilt auch für die M.B., denn immer lässt sich die Zeit so kurz wählen, dass im ersten Moment alle Geschw. Richtungen gleich wahrscheinlich sind und alle Bahnstücke als Gerade betrachtet werden können.

Somit nähert sich das Verhältnis $\frac{t}{a}$ keinem bestimmten endlichen Grenzwert; es wächst im Laufe der Zeit fortwährend an.

(innerhalb gewisser Zeit erreichten)

Klar ist der Begriff d. durchschn. Maximalelongation aus der Anfangslage
verschiebung

aber nur für astatische Systeme, 𝔛

sonst muss auch noch d. Verhältnis d. Anfangs- zur Normallage bekannt sein.

Kann man für statische Systeme eine „mittlere durchschn. Maximalelongation" einführen?
M oder

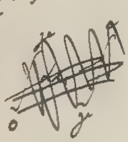
Allerdings, indem man ~~die~~ die Anfangslagen nach d. Wahrscheinl. Funktion verteilt und das Mittel der betreffenden Max.El. nimmt

Das ist eben das was ich (Götz Vol.) mit der Svedberg'schen Zahlenreihe gemacht habe

$M = f_n(t)$ so dass zugleich $M = 0$ $t = 0$

Dagegen ist für die Verdünnungszeiten endlich für alle Argumente ...

$$\left(\sqrt{\frac{\pi}{\alpha}}\right)^n \cdot e^{-\alpha x_1^2}\, dx_1\; e^{-\alpha(x_2-x_1)^2} dx_2\; e^{-\alpha(x_3-x_2)^2} dx_3 \;\ldots\ldots\; e^{-\alpha(x_n-x_{n-1})^2} dx_n = W(x_1 \ldots x_2, x_3 \ldots x_n)$$



$$\int_{y'}^{\infty} dx_1 \int_{-\infty}^{y} dx_2 \int_{-\infty}^{y} dx_3 \;\cdots\; \int_{-\infty}^{y} dx_n + \int_{-\infty}^{y} dx_1 \int_{y}^{\infty} dx_2 \int_{-\infty}^{y} dx_3 \int_{-\infty}^{y} dx_n \;+\; \cdots$$

Meine Rechnung Götz V. würde sich auf folgenden Fall beziehen: wenn wiederholt

M.B. beobachtet in Sekunden Intervallen, dann 2 Sek I., 3 Sek Interv., 4 Sek Interv. etc

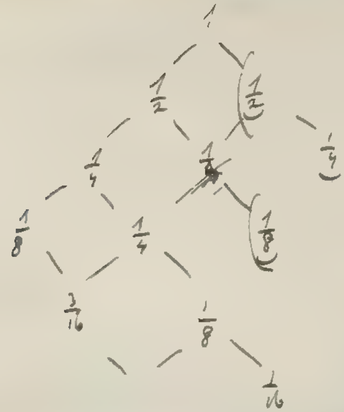
sodann Maximum gesucht zwischen:
aufeinander

δ_1	δ_2	δ_3	δ_4
1·1	1·5	1·7	1·9
0·9	1·6	1·8	1·7
1·4	1·4	1·6	2·1
0·8	1·5	1·7	2·3
1·0	1·5	1·9	1·8
1·0	1·6	...	

max = 1·9
1·8
2·1
2·3
1·9

Mittel ...

1



$$\frac{m+3}{2}\cdot\frac{m+1}{2} \qquad \frac{m-1}{2}\cdots 3\cdot 2\cdot 1$$

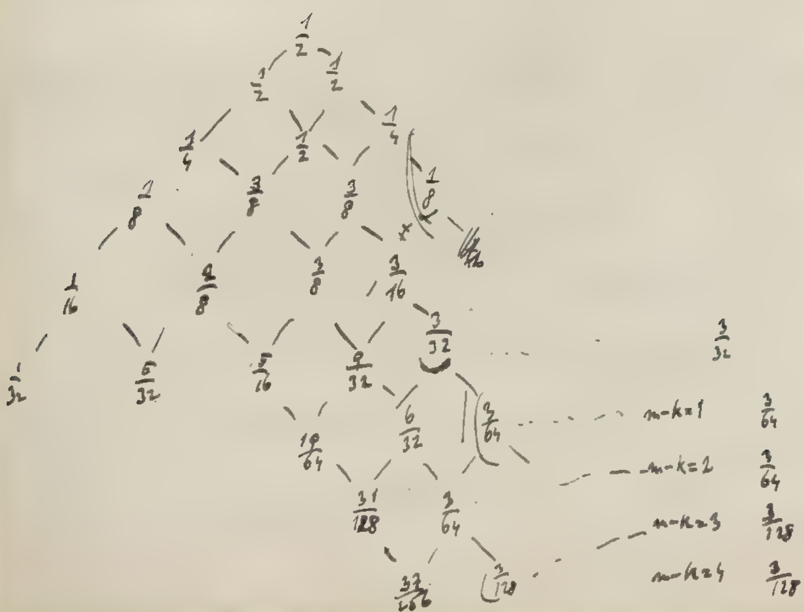
$$\frac{1}{m\,2^m}\binom{m}{\frac{m-1}{2}} = \frac{1}{m}\;\frac{m(m-1)(m-2)\cdots\underbrace{\cancel{\left(m-\frac{m-1}{2}+1\right)}}}{\left(1\cdot 2\cdot 3\cdots\frac{m-1}{2}\right)^2\cdot\frac{m+1}{2}}$$

$2\cdot 2\cdot 2\cdots 2 \cdot 2$

$$= \frac{m!}{m\left[2\cdot 4\cdot 6\cdots(m-1)\right]^2(m+1)} = \frac{(m-1)!}{\left[2\cdot 4\cdot 6\cdots(m-1)\right]^2}\,\frac{1}{m+1}$$

$\frac{3}{16} = \frac{1}{4}\,\frac{1\cdot 3}{2\cdot 4}$

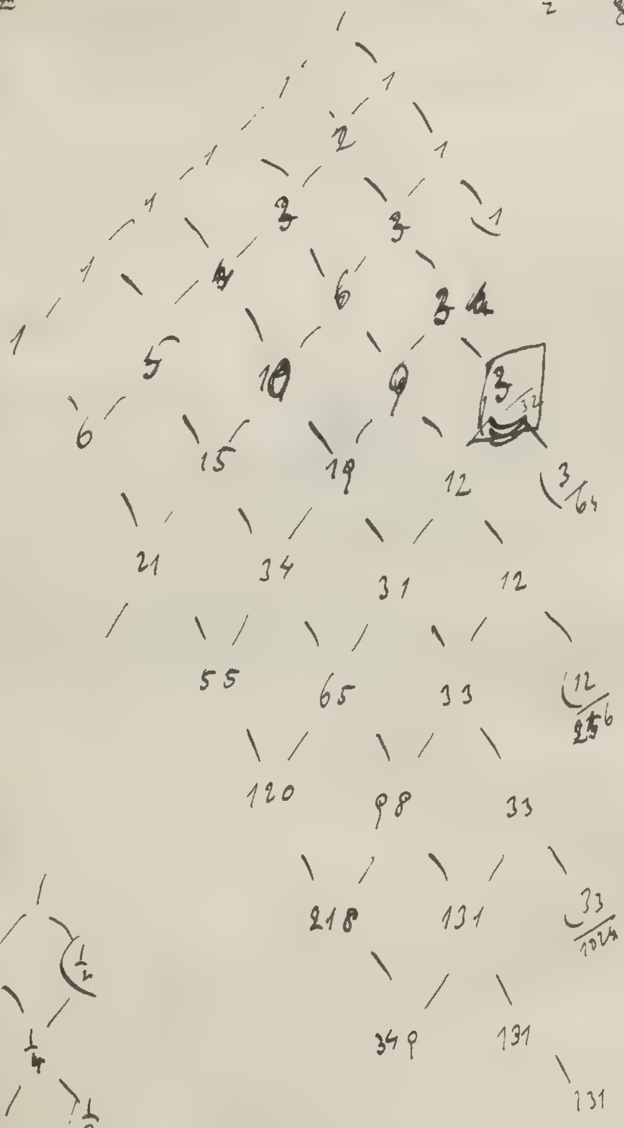
$$= \frac{1\cdot 3\cdot 5\cdots(m-2)}{2\cdot 4\cdot 6\cdots(m-1)}\,\frac{1}{m+1}$$

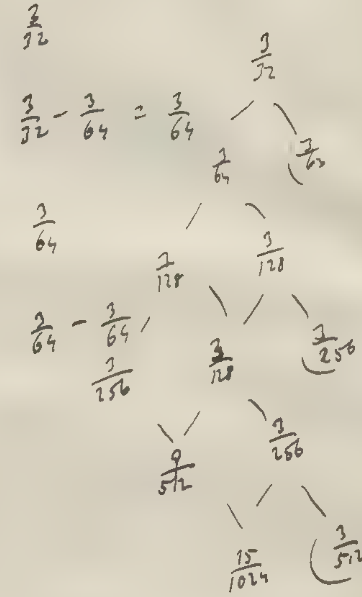


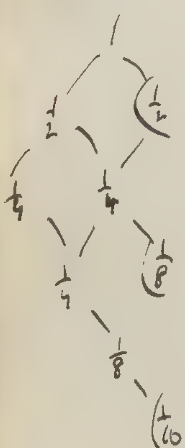
$\frac{3}{128} - \frac{37}{1024}$

$m =$

$\frac{1}{2}$ $\frac{1}{8}$







$n = \underline{1}$ 2 3 4 5 6

x $\frac{1}{2}\frac{1}{4}$ $\frac{1.3}{2.4.6}$

$\frac{3}{8}$ $\frac{1}{16}$

$\frac{1}{2}\frac{1}{4}$

$n=3$ $m=7$

$\frac{1}{8}\left[\left[\frac{1}{2}-\frac{1}{8}\right]+\frac{3}{32}\frac{1}{2}+\frac{9}{128}\right.$ $\binom{7}{2}\frac{1}{2^7}$

$\frac{3}{64}+\frac{3}{64}+\frac{9}{128}= \frac{21}{128}$ $\frac{\cancel{7.6}}{\cancel{1.2}}\cdot\frac{1}{2^7}$

21 8.16

$\frac{1}{16}\frac{3}{8}+\frac{1}{16}\frac{1}{2}=\frac{1}{16}\frac{7}{8}=\frac{7}{128}$ $n=4$ $m=7$ $\frac{\partial}{\partial\alpha}\left(\sqrt{\frac{\pi}{\alpha}}\right)=\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$

$n=3)$ $5\frac{1}{8}+1\frac{3}{8}=1$

$$\sqrt{\overline{\Delta^2}}=\frac{\int_{-\infty}^{\infty}x^2e^{-\alpha x^2}dx}{\int_{-\infty}^{\infty}e^{-\alpha x^2}dx}=\sqrt{\frac{\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}}{\sqrt{\frac{\pi}{\alpha}}}}=\frac{1}{\sqrt{2\alpha}}$$

$2\left[\binom{3}{1}\frac{1}{2^3}+3\cdot\frac{1}{2^3}=\frac{6}{8}\right]=\frac{3}{2}$

$e^{z}dz$

$\alpha x^2 = z$

$2\alpha x dx = dz$

$2\left[\frac{4}{2^4}\cdot+\frac{2}{2^4}\binom{4}{1}\right]=\frac{3}{2}$

$$\overline{|\Delta|}=\frac{\int_0^{\infty}x\,e^{-\alpha x^2}dx}{\int_0^{\infty}e^{-\alpha x^2}dx}=\frac{\frac{1}{2\alpha}}{\frac{\sqrt{\pi}}{2\sqrt{\alpha}}}=\frac{1}{\sqrt{\alpha\pi}}$$

$\cancel{\int e^{-\alpha x^2}dx}$

$$\overline{|\Delta|}=\sqrt{\frac{2}{\pi}}\sqrt{\overline{\Delta^2}}$$

$\frac{1}{t}\sqrt{\frac{1}{2\pi}\frac{\sigma}{t}}$

$2\frac{1}{8}\sqrt{\frac{8t}{2\pi}\frac{1}{t}n}$

$$\frac{1\cdot 3\cdot 5\cdot 7 \ldots n-1}{2\cdot 4\cdot 6 \ldots n} = \frac{n!}{2^n\left(\frac{n}{2}!\right)^2} = \frac{\left(\frac{n}{e}\right)^n\sqrt{2n\pi}}{\left(\frac{n}{2e}\right)^n \sqrt{n\pi}^{\,2}\, 2^n} = \frac{\sqrt{2n\pi}}{\sqrt{n\pi}^{\,2}} = \sqrt{\frac{2}{n\pi}}$$

$$\frac{\partial c}{\partial x} = \frac{c_0}{\sqrt{\pi}}\, e^{-\frac{x^2}{4Dt}}\, \frac{1}{\sqrt{Dt}}$$

$$e^{-\frac{\beta(x-x_0e^{-\beta t})^2}{2D(1-e^{-2\beta t})}}\; e^{-\frac{\beta x_0^2}{2D}} = e^{-\frac{\beta}{2D}\frac{\overbrace{(x-x_0e^{-\beta t})^2+x_0^2(1-e^{-2\beta t})}^{x^2-2xx_0e^{-\beta t}+x_0^2}}{1-e^{-2\beta t}}}$$

$F = f(R, \eta,$

↗ daraus würde die ~~mittlere~~ durchschnittl. Dauer eines n Zustandes folgen $= \frac{h\sqrt{6\pi}}{2(n\nu)C}$

Wie viel Zahlenwechsel finden pro Sek statt?

Zu berechnen wie die Stosszahl in Gastheorie für die ganze Oberfläche

Von jenen Zahlenwechseln ist der Prozentsatz $W(n)$ ein günstiger

Also nach Analogie mit Scintillationen wäre die durchschn. Erwartungszeit für die Zahl n: oder durchschn. Dauer von einem zum andern Sz. $= \frac{h\sqrt{6\pi}}{\nu C}\frac{1}{W(n)}$

Falls die Schicht so dünn wäre dass ein merklicher Bruchteil der auf der einen Seite hereinfallenden auf der andern hinausgehen würde wäre dies vielleicht nicht unbedenklich(?)

Einseitige Bewegung gibt Stosszahl: $\frac{\nu C}{h\sqrt{6\pi}}$

man muss natürlich beide Bewegungsrichtungen und beide Flächen berücksichtigen, deshalb

$$\frac{4\nu C}{h\sqrt{6\pi}}$$

Im Falle vollständiger Unabhängigkeit der Elementar-Ereignisse:

$M_1\ M_2 \ldots$ = Wahrsch. für ein einfaches, doppeltes ... Zustandl. „(Nichten)"

$$M_1 = \cancel{A}\, e^{-k\tau} = \alpha$$

$$M_2 = \cancel{A}\, e^{-2k\tau} = M_1^{\,2} = \alpha^2$$

$$M_3 = \cancel{A}\, e^{-3k\tau} \ldots = M_1^{\,3} = \alpha^3$$

$$\Theta_1 = \tau\,\frac{\alpha + 2\alpha^2 + 3\alpha^3}{\alpha+\alpha^2+\alpha^3} = \tau\left[1 + \alpha\,\frac{\alpha + 2\alpha^2 + 3\alpha^3}{\alpha+\alpha^2+\alpha^3 \ldots}\right] = \tau\left[1+\alpha\left(1+\alpha\left(1+ -\ \right)\right)\right] = \frac{\tau}{1-\alpha}$$

$$\Theta_3 = \tau\left[1 + \underbrace{\frac{\alpha^2+2\alpha^3+3\alpha^4+\ldots}{\alpha+2\alpha^2+3\alpha^3+\ldots}}_{\alpha}\left[1+\beta\left[1+\beta \ \ldots\right]\right]\right] = \tau\left(1+\beta+\beta^2+\beta^3+\ldots\right)$$

$$= \frac{\tau}{1-\alpha}$$

Dann wird also wirklich $\Theta_1 = \Theta_3 = \dfrac{\tau}{1-\alpha} = \tau\,\dfrac{1}{1-e^{-k\tau}}$

$$1 - W_{(n)} = V_{(n)}$$

$$T_1 = \Theta_1\,\frac{W_{(n)}}{1-W_{(n)}} = \Theta_1\,\frac{1-V_{(n)}}{V_{(n)}} = \Theta_1\left[\frac{1}{V_{(n)}} - 1\right]$$

$$\Theta_1 = \tau\,\frac{V_{(n)}}{1-V_{(n)}}$$

$$\alpha+\alpha^2+\alpha^3 \ldots = \frac{1-\alpha}{\alpha}$$

$$\alpha+\alpha^2+\alpha^3 \ldots = \frac{\alpha}{1-\alpha} = \frac{1}{1-\alpha} - 1$$

$$\alpha + 2\alpha^2 + 3\alpha^3 \ldots = \frac{\alpha}{(1-\alpha)^2}$$

$$\Theta_1 = \frac{\tau}{1-M} \qquad \Theta_3 = \frac{\tau}{1-M}$$

$$P_n(0) = W_{(n)} \qquad \frac{M_1}{\Sigma(M+N)} = W_{(1)} \qquad \frac{M_2}{\Sigma(M+N)} = W_{(2)} = \left[W_{(1)}\right]^2 = \left(\frac{M_1}{\Sigma\, M+N}\right)^2 \qquad W_{(3)} = \left[W_{(1)}\right]^3$$

$$\frac{M_1 + 2M_2 + 3M_3}{M_1 + M_2 + M_3} =$$

$$M_2 = M_1^2 / (\Sigma\, M+N)$$

$$M_3 = M_1^3 / (\Sigma\, M+N)^2$$

$W(m\,n) = P_n(0)$

$W(n\,n\,n) \geq P_1(0)$ $\qquad T_3 > \tau\,[\underbrace{1+P(1+P(\cdots}_{\frac{\tau}{1-P}}\;)$

Wenn N Teilchen verteilt auf $-\infty$ bis $+\infty$ so ist deren mittlere Dichte in der Entfernung x

$$n_x\,dx = N\sqrt{\frac{\beta}{2\pi D}}\,e^{-\frac{\beta x^2}{2D}}\,dx$$

dieselben bewegen sich in jedem Moment ↑↓ mit mittl. Geschwind. $\frac{\Omega}{2}$

Also ist die Zahl der von unten aus die Ordinate x (pro Zeiteinheit) überschreitenden Teilchen

$\frac{n_x\,\Omega}{4}$ ebensoviel in umgekehrter Richtung

Also Wahrsch. dass ein (mit gleicher Wahrsch.) unten oder oben irgendwo befindliches Teilchen jene Ordinate in der Zeit dt überschreitet

$$= \frac{2C}{\sqrt{6\pi}}\sqrt{\frac{\beta}{2\pi D}}\,e^{-\frac{\beta x^2}{2D}}\,dt$$

Also mittlere Zeit zwischen zwei Überschreitungen:
2

$$T_1\,(= T_3\,?) = \frac{\sqrt{6\pi}}{2C}\sqrt{\frac{2\pi D}{\beta}}\,e^{\frac{\beta x^2}{2D}} = \frac{\sqrt{6\pi}}{2C}\sqrt{2\pi}\;\xi\,e^{\frac{x^2}{2\xi^2}} = \frac{\pi\sqrt{3}}{C}\,\xi\,e^{\frac{x^2}{2\xi^2}}$$

$\frac{\xi^2}{D}\;\frac{\xi}{x_0} : \frac{\xi}{C}$ [illegible]

Allgemein $T_1 = \frac{1}{[illegible]}\,[illegible]\,?$ $\qquad 1-P_n(0) = [illegible]$

$W(m\,n) = ?$ $\qquad 1-W(n\,n) = W(n\,m)$ $\qquad T_1 = \frac{\tau}{W(n\,m)}$

$W(n) = W(m\,n) + W(n\,n)$ $\qquad W(m\,n) + W(n\,m) = 1$

$W(n)\,P_n(0)$

$P_m(n) = ?$ $\qquad W(mm) = P_m(0)\, W(m)$

$P_m(0) + P_m(n) = 1$ $\qquad W(nn) = P_n(0)\, W(n)$

$P_n(0) + P_n(m) = 1$ $\qquad W(mn) = W(m)\, P_m(n) = W(n)\, P_n(m) = W(nm)$

$\cancel{W(nm) = W(n)\, P_n(m)}$

$\cancel{P_m(n) = 1 - P_n(0) = 1}$

$$P_m(n) = P_n(m)\,\frac{W(n)}{W(m)} = \left[1 - P_n(0)\right]\frac{W(n)}{W(m)} = \left[1 - P_n(0)\right]\frac{W(n)}{1 - W(n)}$$

$$\Theta_1 = \frac{\tau}{P_m(n)}$$

$$\frac{m(m-1)(m-2)\ldots\left(m - \frac{m-n}{2} + 1\right)\left(m - \frac{m-n}{2}\right)!}{1.\,2.\,3 \cdots \frac{m-n}{2} \quad \left(m - \frac{m-n}{2}\right)!} = \frac{m!}{\frac{m-n}{2}!\,\frac{m+n}{2}!}$$

$$\lim a_{nm} = \frac{n}{m\,\cancel{2^m}}\;\frac{\left(\frac{m}{\cancel{e}}\right)^m \cancel{\sqrt{2m\pi}}}{\left(\frac{m-n}{\cancel{2e}}\right)^{\frac{m-n}{2}}\left(\frac{m+n}{\cancel{2e}}\right)^{\frac{m+n}{2}}}\sqrt{\frac{m}{\frac{m-n}{2}\,\frac{m+n}{2}}}\;\frac{1}{\sqrt{2\pi}}$$

$$= \frac{n}{m}\sqrt{\left(\frac{m-n}{m}\right)^{m-n}\cdot\left(\frac{m+n}{m}\right)^{m+n}}\;\sqrt{\frac{2}{\pi m}}$$

$$= \cancel{\frac{n}{m}\sqrt{\left(1 - \frac{n}{m}\right)^m}} \qquad (m-n)\lg\left(1 - \frac{n}{m}\right) = (m-n)\left(\frac{n}{m} + \frac{n^2}{2m^2} + \frac{n^3}{3m^3}\right)$$
$$+ (m+n)\left(\frac{n}{m} - \frac{n^2}{2m^2} + \frac{n^3}{3m^3}\right)$$

$$= \cancel{n} - \frac{n^2}{2m} + \cancel{\frac{n^3}{3m^2}} \quad + \frac{n^2}{m} - \cancel{\frac{n^3}{2m^2}} + \frac{n^4}{3m^3} \;\Big\} = \frac{n^2}{m}$$
$$\cancel{-} \quad - \quad \cancel{-} \qquad + \quad \cancel{+} \quad +$$

$$= \frac{n}{m}\, e^{-\frac{n^2}{m}}\sqrt{\frac{2}{\pi m}} \qquad \cancel{\frac{n^2}{m} = x} \qquad \cancel{m = \frac{n^2}{x}} \qquad \sum_{n=0}^{\infty} a_{nm} = \int\sqrt{\frac{2}{\pi}}\, x^{-1/2} e^{-x}\, dx$$

$\cancel{dm = -\frac{n^2}{x^2}\,dx}$

$$= \sqrt{\frac{2}{\pi}}\;\underbrace{\Gamma\left(\tfrac{1}{2}\right)}_{=\sqrt{\pi}}$$

$$\cancel{\sqrt{\frac{2}{\pi}}\int_{\infty}^{\frac{n^2}{m}} \frac{n^2}{x^2}\,dx\,\frac{x}{n}\,e^{-x}\,\frac{\sqrt{x}}{n}}$$

$$= \sqrt{\frac{2}{\pi}}\int_{\frac{n^2}{m}}^{\infty}\frac{e^{-x}}{\sqrt{x}}\,dx = \sqrt{\frac{2}{\pi}}$$

0

1

1

1 1

2 1

2 3 1

5 4 1

5 9 5 1

14 14 6 1

14 28 20 7 1

1

1

2 1

3 3 1

5 ~~7~~ 6 4 1

10 10 5 1

20 15 6 1

35 21 7 1

$$\left(\frac{\alpha^2}{x}+x\right)^m = x^m + \binom{m}{1}\alpha^2 x^{m-2} + \binom{m}{2}\alpha^4 x^{m-4} + \dots + \alpha^{2m} x^{-m}$$

~~$(1+x)^4 = 24$~~

Durch Grenzfläche ~~#~~ von Anfang an durchgetreten

$$e^{-Dr^2t\,\frac{n}{h^2}}$$

seitliche Dimensionen beliebig gross

Wenn nun Dichte $\frac{n}{h}$ constant angenommen wird

aber lim $h=\infty$

so müsste dies die Ordnung ergeben, dass aus zwei mit gleichmässiger Dichte $\frac{n}{h}$ angefüllten Halbräumen bis zur Zeit t kein einziges Teilchen ausgetreten sei

lim $=1$ also Unsinn

Wahrsch. dass der positive Überschuss n bei keinem der ~~k~~ ersten Würfe aufgetreten sei = Wahrsch, dass er zum ersten Mal beim $(k+1)$ oder $(k+2)$ - - - Wurf auftreten wird (da er irgend einmal auftreten muss)

$\cancel{n=1}$ $\cancel{m=1}$ $\cancel{\frac{1}{2}}$

$\cancel{m=3}$ $\cancel{\frac{3}{8}}$

$\cancel{m=5}$ $\cancel{\frac{5}{16}}$ $\cancel{\frac{5}{16}\left(1-\frac{1}{2}\right)\left(1-\frac{3}{8}\right)=\frac{5}{32}\cdot\frac{5}{8}=\frac{25}{256}}$

$$W_{(n)k} = \sum_{m=k+1}^{\infty} a_{nm} = 1 - \sum_{m=n}^{m=k} a_{nm}$$

Wahrsch, dass keines der Teilchen (bis zur Zeit $m\tau$) sich an die Wand gesetzt habe:

$$= W_{(1)k} \cdot W_{(2)k-1}\, W_{(3)k}\, W_{(4)k-1} \cdots$$

$$W = \frac{8}{\pi^2}\left\{ \frac{1}{1} e^{-\frac{D\pi^2}{h^2}t} + \frac{1}{9} e^{-\frac{D\pi^2}{h^2}\cdot 9t} + \frac{1}{25} e^{-\frac{D\pi^2}{h^2}\cdot 25t} + \cdots \right\}$$

$$\alpha + \frac{\alpha^9}{9} + \frac{\alpha^{25}}{25} + \cdots$$

$\frac{h^2}{D\pi^2 n}$ ~~[illegible]~~ $D = v\lambda$ $\frac{Dn\pi^2}{h^2} = \frac{v\lambda n\pi^2}{h^2}$ für $\lambda = h$ identisch

$= \frac{v\,r}{h}$

Mittlere (ununterbrochene) Verweilungszeit in einem Volumen

falls mittlere Weglänge sehr gross im Vergleich zu den Dimensionen des Volumens

~~Mittl.~~ Durchschn. Erwartungszeit des Austrittes für ein irgendwo im Innern befindliches Molekül von der Grössenordnung $\frac{a}{v}$

---

falls mittlere Weglänge klein ist: (Da ist von Unabh. keine Rede, weil das ein Mol. austritt wird es sehr schnell wahrscheinl. Male nacheinander austreten)

wird sie dementsprechend grösser sein und zwar desto grösser je kleiner λ. lässt sich berechnen aus Diffusionstheorie, wird also nur abhängen von D und a, aber nicht von v!

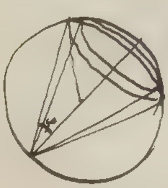
Anzahl der Durchtritte Ereignisse an der Oberfläche ist in (pro Zeiteinheit) beiden Fällen gleich!

falls Molekülzahl gleich und Geschwind. gleich [die Änderung in λ hervorgebracht durch Änderung der Moleküldimensionen]

$= 4a^2\pi n v \ldots$

Also durchschnittl. Dauer einer Anzahl = $\frac{1}{4a^2\pi n v}$

Stosszahl auf der (innern) Oberfläche eines kugelförmigen Gefässes:



Die Oberfläche sei diffus reflektierend, also jede ~~B~~ Bewegungsrichtung gleichberechtigt?

Wahrsch. einer Richtung φ … dφ

$\frac{2\pi \sin\varphi \, d\varphi}{2\pi}$

Anzahl der Stösse welche welche Mol. pro Sek. ausübt:

$\nu_\varphi = \frac{v}{2a\cos\varphi}$.

$$\cancel{\frac{\partial \rho_0}{\partial x} = -\alpha x} \qquad \cancel{\frac{\partial^2(\rho-\rho_0)}{\partial x^2} = D\frac{\partial(\rho-\rho_0)}{\partial t}}$$

$$\cancel{\rho_0 = -\frac{\alpha x^2}{2} + \beta}$$

$$\frac{\partial^2 \rho}{\partial x^2} + \alpha = D\frac{\partial \rho}{\partial t} \qquad ?$$

Also Gesamtzahl der Stöße pro Sek., welche durch ein im Innern befindliches Mol. erzeugt werden:

$$\nu = \frac{v}{2a}\int_0^{\frac{\pi}{2}} \frac{\sin\varphi\, d\varphi}{\cos\varphi} = \frac{v}{2a}\, \lg\cos\varphi \Big|_{\frac{\pi}{2}}^{0} = \infty \;!$$

Also ist eine solche Bewegung gar nicht möglich, es müssen die Normalrichtungen bevorzugt werden?

Dagegen ist bei kurzen mittleren Weglänge in dichten Gasen ohneweiters

$$\nu = 4a^2\pi\,\frac{nC}{\sqrt{6\pi}} = \frac{3N}{a}\,\frac{C}{\sqrt{6\pi}}$$

N = Gesamtzahl der Moleküle in der Kugel
also entfällt auf ein Molekül: $\frac{3}{a}\frac{C}{\sqrt{6\pi}}$ also durchschnittliche Dauer: $\frac{a}{C}\sqrt{\frac{6\pi}{3}}$

Der Fehler steckt darin: gleichförmige Richtungsverteilung gilt für Raumelemente, aber nicht für Stoßelemente.

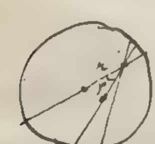
Grundlage: gleich wahrsch. ist der Aufenthalt in irgend einem Raumelement und dasselbe

Es muss hier ein Analogon zum Lambert'schen Cosinusgesetz construiert werden.

Auf die Richtung φ — $\varphi + d\varphi$ entfällt

$$2\pi \sin\varphi\cos\varphi\, d\varphi \quad dS \quad n$$

etwa was ganz das ähnliche zum $\frac{C}{\sqrt{6\pi}}$ führende Rechnung gibt.



Durchschn. Erwartungszeit (wobei alle Bewegungsrichtungen gleich wahrsch.)
für die aus einem Punkte in der Entfernung r ausgehenden Moleküle

$$\int_0^{\pi} \frac{2\pi\sin\varphi\, d\varphi}{4\pi}\cdot 2\,\frac{\sqrt{a^2 - r^2\sin^2\varphi}}{v} = -\frac{1}{v}\int \sqrt{a^2 - r^2 + r^2\cos^2\varphi}\; d(\cos\varphi)$$

Es wird jedenfalls einen Ausdruck von der Größenordnung $\frac{a}{C}$ geben, welcher aber mit durchschnittl. Dauer nicht identisch sein dürfte, denn es besteht da auch keine Unabhängigkeit d. Ereignisse: wenn ein Stoss auf die Wand erfolgt, muss sicher der nächste innerhalb der Zeit $\frac{2a}{v}$ erfolgen!

Kugel:

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial r^2}$$

$v = r\theta$

$\theta = 1$	$t = 0$
$\theta = 0$	$r = a$
$\frac{\partial\theta}{\partial r} = 0$	$r = 0$

$v = 0$

$\cancel{\frac{\partial v}{\partial r} = r\frac{\partial\theta}{\partial r}} \quad \cancel{\theta = \frac{v}{r}}$

$\frac{\partial^2 v}{\partial r^2} = 0$

$\theta =$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$$

für lim $r = 0$ muss also

$$\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial r^2} + \frac{2}{r}\frac{\partial\theta}{\partial r}$$

$$= \frac{1}{r}\frac{\partial^2 v}{\partial r^2}$$

$$v = \sum A_k e^{-\frac{Dk^2\pi^2 t}{a^2}} \sin\left(\frac{k\pi r}{a}\right)$$

$\sin k\pi = 0 \qquad k = 1, 2, 3, 4 \ldots$

$\sin 0 = 0$

$$\sum_{k=1}^{\infty} A_k \sin\frac{k\pi r}{a} = r$$

$$A_k = \frac{2}{a}\int_0^a r\sin\frac{k\pi r}{a}\,dr = \frac{2}{a}\left\{-\frac{a}{k\pi}\cdot r\cos\frac{k\pi r}{a}\Big|_0^a + \frac{a}{k\pi}\int_0^a \cancel{\underbrace{\cos\frac{k\pi r}{a}\,dr}_{\sin\frac{k\pi r}{a}}}\right\}$$

$$= -\frac{2}{k\pi}\left[a\cos k\pi\right]$$

$$A_k = -\frac{2a}{k\pi} \quad (k = 2m)$$

$$A_k = \frac{2a}{k\pi} \quad (k = 2m+1)$$

$$\theta = \frac{2a}{\pi r}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k} e^{-\frac{Dk^2\pi^2 t}{a^2}}\sin\frac{k\pi r}{a}$$

$\cancel{r = 0 \quad \theta = 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots\right)}$

~~[illegible]~~ mittlere Dichte: $\rho = \frac{1}{\frac{4}{3}a^3\pi}\int_0^a 4r^2\pi\,\theta\,dr = \frac{3}{a^3}\int_0^a r^2\theta\,dr$

Mittlere Dauer: $T = -\int_0^\infty t \frac{\partial \rho}{\partial t} dt = \cancel{-t\rho\Big|_0^\infty} + \int_0^\infty \rho\, dt$

$$\rho = \frac{3}{a^3} \frac{2a}{\pi} \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k} e^{-\frac{Dk^2\pi^2 t}{a^2}} \underbrace{\int_0^a r \sin\frac{k\pi r}{a} dr}_{(-1)^{k+1}\frac{a^2}{k\pi}} = \frac{6}{\pi^2} \sum_{k=1}^\infty \frac{1}{k^2} e^{-\frac{Dk^2\pi^2 t}{a^2}}$$

$$T = 6 \sum \int_0^\infty \frac{dt}{k^2\pi^2} e^{-\frac{Dk^2\pi^2 t}{a^2}} = \frac{6a^2}{D\pi^4} \sum_{k=1}^\infty \frac{1}{k^4} = \frac{6a^2}{D\pi^4}\left[1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots\right]$$

$$2\int_0^a \frac{2r\pi\, dr \cdot \sqrt{a^2-r^2}}{a^2\pi} = 4 \frac{\left(\sqrt{a^2-r^2}\right)^{3/2}}{3a^2}\Big|_0^a = \frac{4a}{3}$$

$$\tau = \frac{4a}{3v}$$

$$\bar{\tau} = \frac{\Sigma v\tau}{\Sigma v} = \frac{4a}{3} \frac{\Sigma}{\Sigma v} = \frac{4a}{3\Omega}$$

$$\overline{\xi^2} = \frac{C^2}{3}$$

$$\cancel{W(\xi)d\xi = \frac{2}{\alpha\sqrt{\pi}} e^{-\frac{\xi^2}{\alpha^2}} d\xi}$$

$$\cancel{\overline{\xi^2} = \alpha^2 \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} y^2 e^{-y^2} dy}$$

$$W(\xi)\, d\xi = e^{-\alpha\xi^2} d\xi$$

$$\overline{\xi^2} = \frac{\int_{-\infty}^{+\infty} \xi^2 e^{-\alpha\xi^2} d\xi}{\int_{-\infty}^{+\infty} e^{-\alpha\xi^2} d\xi}$$

$$\cancel{\bar{\xi} = \frac{\int_0^\infty \xi e^{-\alpha\xi^2} d\xi}{\int_0^\infty e^{-\alpha\xi^2} d\xi} = \frac{\frac{1}{\alpha}}{\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}} = \frac{2}{\sqrt{\pi}\sqrt{\alpha}} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{2}{3}}\frac{1}{C}}$$

$$\cancel{\bar{\xi} = \sqrt{\frac{8}{3\pi}}\frac{1}{C}}$$

$$= \frac{\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}}{\sqrt{\frac{\pi}{\alpha}}} = \frac{1}{2\alpha} = \frac{C^2}{3}$$

$$\alpha = \frac{3}{2}\frac{1}{C^2}$$

$$\bar{\tau} = \frac{4a}{3C}\sqrt{\frac{3\pi}{8}} = \frac{a}{C}\sqrt{\frac{2\pi}{3}}$$

$$\alpha v^2 = y$$

$$\int v^3 e^{-\alpha v^2} dv = \frac{1}{2\alpha^2}\int_0^\infty e^{-y} y\, dy$$

$$= \frac{1}{2\alpha^2}\, \cancel{y e^{-y}} - e^{-y}$$

$$\overline{v^2} = \frac{\int_{-\infty}^{+\infty} v^4 e^{-\alpha v^2} dv}{\int_{-\infty}^{+\infty} v^2 e^{-\alpha v^2} dv} = \frac{\frac{3}{4}\sqrt{\frac{\pi}{\alpha^5}}}{\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}} = \frac{3}{2}\frac{1}{\alpha}$$

$$\Omega = \frac{\int_0^\infty v^3 e^{-\alpha v^2} dv}{\int_0^\infty v^2 e^{-\alpha v^2} dv} = \frac{\frac{1}{2\alpha^2}}{\frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}} = \frac{2}{\sqrt{\alpha\pi}} = C\sqrt{\frac{8}{3\pi}}$$

$$\int dx\sqrt{1-x^2} = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsin x$$

$$4\pi\int_0^a \sqrt{a^2-x^2\sin^2\varphi}\cdot r^2\,dr \quad = 4\pi\frac{a^4}{\sin^3\varphi}\int\sqrt{1-\left(\frac{r\sin\varphi}{a}\right)^2}\ \left(\frac{r\sin\varphi}{a}\right)^2 d\left(\frac{r\sin\varphi}{a}\right)$$

$$= \frac{4\pi a^4}{\sin^3\varphi}\int_0^{\sin\varphi}\underbrace{\sqrt{1-x^2}\ \ x^2\,dx}$$

$$= \frac{x^3}{4}\sqrt{1-x^2} + \frac{1}{4}\int\underbrace{\frac{x^2\,dx}{\sqrt{1-x^2}}}_{-\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\arcsin x}$$

$$\frac{x^3\sqrt{1-x^2}}{4} - \frac{x\sqrt{1-x^2}}{8} + \frac{1}{8}\arcsin x$$

$$\frac{3x^2\sqrt{1-x^2}}{4} - \frac{x^4}{4\sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{8} + \frac{x^2}{8\sqrt{1-x^2}} + \frac{1}{8\sqrt{1-x^2}}$$

$$\frac{6x^2-6x^4-2x^4}{8\sqrt{1-x^2}} + \frac{x^2-1+x^2}{8\sqrt{1-x^2}} + \frac{\cancel{1}}{8\sqrt{1-x^2}} \quad = \quad \frac{-8x^4+8x^2}{8\sqrt{1-x^2}} = x^2\frac{1-x^2}{\sqrt{1-x^2}}$$

$$= \frac{\frac{4\pi a^4}{\sin^3\varphi}\left[\frac{\sin^3\varphi\cos\varphi}{4} - \frac{\sin\varphi\cos\varphi}{8} + \frac{1}{8}\varphi\right]}{\frac{4\pi}{3}a^3} =$$

$$\frac{3a}{4v}\int\underbrace{\left[\frac{\sin\varphi\cos\varphi}{\cancel{2}} - \frac{\cos\varphi}{\cancel{2}\sin\varphi} + \frac{1}{2}\frac{\varphi}{\sin\varphi}\right]}_{\frac{\varphi\sin^2\varphi\ -\cos^2\varphi}{2\sin\varphi}}d\varphi$$

$\cancel{\int \sqrt{1-a^2x^2}\, x^2\,dx}$ $\qquad \frac{\varepsilon r}{a} = x$

$$\int_0^a \sqrt{a^2-\varepsilon^2 r^2}\, r^2\,dr = a\int_0^a \sqrt{1-\left(\frac{\varepsilon r}{a}\right)^2}\, r^2\,dr = \frac{a^4}{\varepsilon^3}\int_0^\varepsilon \sqrt{1-x^2}\, x^2\,dx$$

$$= \frac{a^4}{\varepsilon^3}\left\{\frac{x^3\sqrt{1-x^2}}{4} - x\frac{\sqrt{1-x^2}}{8} + \frac{1}{8}\arcsin x\right\}$$

$x = \sin\varphi$

$$\int_0^\varphi \sin^2\varphi\cos^2\varphi\,d\varphi = \frac{1}{8}\left[\varphi - \frac{\sin 4\varphi}{4}\right] = \frac{1}{8}\left[\varphi - \frac{\sin 2\varphi\cos 2\varphi}{2}\right]$$

$$= \frac{1}{8}\left[\varphi - \sin\varphi\cos 2\varphi\cos\varphi\right] = \frac{1}{8}\left[\varphi - \sin\varphi\cos^3\varphi + \sin^3\varphi\cos\varphi\right]$$

$\cancel{= \frac{1}{8}\left[\varphi - 2\sin^2\varphi\cos^2\varphi\right]}$

$\cancel{\varphi - \frac{\varphi^3}{3!} - \varphi + \frac{16\varphi^3}{3!}}$ $\cancel{\frac{\varphi^3}{} = \frac{15}{3!}}$ $\cancel{\int \frac{\varphi}{\sin^2\varphi}\,d\varphi}$

$$= \frac{4\pi a^4}{8\,\frac{4\pi a^3}{3}}\int_0^\pi \frac{1}{\sin^2\varphi}\left[\varphi - \sin\varphi\cos^3\varphi + \sin^3\varphi\cos\varphi\right]d\varphi$$

$\cancel{\int \frac{\varphi^4}{\sin\varphi}}$ $\quad c = \sqrt{3R\theta\, m} = \sqrt{3H\theta\,\frac{m}{\mu}} = \sqrt{\frac{3H\theta}{N}}$ $\qquad C = \sqrt{\frac{3H\theta}{NM}}$

$$C = \sqrt{\frac{3H\theta}{6.10^{23}\cdot\frac{4}{3}a^3\pi\rho}} = \sqrt{\frac{3.293.\ 8{\cdot}32.10^7}{8.6.10^{23}\cdot\frac{4}{3}\pi.1{\cdot}1\ \ .(1{\cdot}9.10^{-6})^3}}$$

$\frac{n\Omega^2}{8} = \frac{C^2}{3}$

$\frac{9.\ 3\ \cancel{3}\ .\ \cancel{8{\cdot}16}\ 1{\cdot}03}{8.\ \cancel{13}.\ 2.\ \cancel{18}.\ 1{\cdot}9}\ \frac{10^9}{10^5} = \sqrt{\frac{9{\cdot}3}{7{\cdot}6}\ 10^4} = \sqrt{1{\cdot}22.10^3}$

$\frac{N\Omega}{4} = \frac{NC}{4}\sqrt{\frac{8}{3\pi}} = \frac{NC}{\sqrt{6\pi}}$ $\qquad$ 2135 854 $\qquad$ 1.71 .18 $\qquad = 3{\cdot}5\ .10^3$

$\frac{\cancel{\sqrt{6\pi}}\ \sqrt{188}}{2.\ 18{\cdot}5.\ 26}$ $\quad$ 112 $\quad$ 2 $\qquad = \sqrt{1{\cdot}22\ .10^2}$ $\qquad = 35\ \frac{m}{sek}$

$= \frac{1}{222} = 0{\cdot}45\ 10^{-2}$

$= \frac{1}{52\sqrt{19{\cdot}2}} = \frac{1}{52.\ 4{\cdot}27}$ $\quad \sqrt{\frac{1{\cdot}1}{19{\cdot}2}} = \sqrt{\frac{1{\cdot}14}{20}} = \frac{\sqrt{5{\cdot}7}}{\sqrt{100}} = \frac{2{\cdot}6}{10}$ $\qquad = 1{\cdot}1\ \frac{m}{sek}$

220 $\qquad$ 4.8 $\quad$ 576 $\qquad = 26\ \frac{cm}{sek}$

$$\frac{h^2}{D\,\Omega^2 (n+\nu)} = \frac{(2.10^{-4})^2}{9{\cdot}86 \cdot 1{\cdot}04 \cdot 10^{-7} (n+\nu)}$$

$9{\cdot}42$
$2{\cdot}12$
$\frac{1\,8\,5\,6}{9\,8\,6}$

$$= \frac{2 \cdot 10^{-2}}{18{\cdot}5} = 10^{-3}$$

$C = 26$

$D = 10^{-7} = \frac{C\Lambda}{3}$

$\Lambda = \frac{3 \cdot 10^{-7}}{26} = 10^{-8}$ cm

$9\,3$
$1{\cdot}1$
$9{\cdot}61 : 4 = 2{\cdot}402$

$\Delta^2 = 2\nu P = 1$

$\downarrow$

$\frac{2}{h}\sqrt{\frac{Dt}{\Omega}}$

$\sqrt{\frac{Dt}{\Omega}} = \left(\frac{h}{4\nu}\right)^2$

$t = \frac{\Omega}{D}\left(\frac{h}{4\nu}\right)^2$

$$\frac{\Omega\,(2.10^{-4})^2}{10^{-7} \cdot 16 \cdot (1{\cdot}55)^2} = \frac{12{\cdot}6 \cdot 10^{-1}}{16 \cdot \cancel{1{\cdot}55}} = \cancel{8} \cdot 0 = \frac{1}{30}$$

$2{\cdot}4$

$\cdot 72$

$= 0{\cdot}03$

$\frac{\sqrt{6\pi}}{4\nu\, C} = 0{\cdot}0045 \cdot \frac{18{\cdot}5}{1{\cdot}55}$ 12

$0{\cdot}054$

$2\,\mathcal{N}\, ds\, dt\, \Omega$
$\sqrt{6\pi}$

$\mathcal{N} = \frac{(n+\nu)}{\Omega h}$

$$= \frac{2\, ds\, dt\, (n+\nu)}{h\sqrt{6\pi}}$$

78
156
$\frac{9{\cdot}36}{}$ $\cdot 6{\cdot}6$
5616
$\frac{562}{0{\cdot}18}$

$$\frac{10^{7}}{\cancel{220}\ 11} \cdot \cancel{2} \cdot \cancel{10^{4}} \cdot \frac{\cancel{\ }}{39} \cdot \cancel{\ } \cdot \frac{1}{60}\,\frac{1}{24} = \frac{10^4}{11 \cdot 39 \cdot 6 \cdot 24} =$$

$$38 \quad = \frac{10^3}{6{\cdot}2} = 161^{\,d}$$

Wahrsch., dass kein Teilchen die Grenzfläche (bis t) überschritten habe:

$$W_k = \cancel{(1 - a_{1m})(1 -}$$

$$= \cancel{\left(1-\sum_{m=1}^{m=k} a_{1m}\right)\left(1-\sum_{m=2}^{m=k-1} a_{2m}\right)\left(1-\sum_{m=3} a_{3m}\right)} \dots$$

$$\left(1-\sum_{m=1}^{m=k} a_{1m}\right)\left(1-\sum_{m=2}^{m=k-1} a_{2m}\right)\left(1-\sum_{m=1}^{m=k-2} a_{3m}\right)\left(\dots\right.$$

$$\cancel{\sum_{m=k+1}^{\infty} a_{1m} \cdot \sum_{m=k}^{\infty} a_{2m} \cdot \sum_{m=k-1}^{\infty} a_{3m} \cdot \sum_{m=k-2}^{\infty} a_{4m} \cdot \sum_{m=k-4} a_{5m}}$$

$$\log W_k = \sum \log\left(1 - \sum_{m=k}^{m=k-n} a_{nm}\right)$$

$$\sum_{n=m}^{\infty} a_{nm}$$

$$\sum a_{33} + a_{53} + a_{73} + \quad = \sum_{n=m}^{\infty} \frac{n}{m} e^{-\frac{n^2}{m}} \sqrt{\frac{2}{\pi m}} = \frac{1}{2}\sqrt{\frac{1}{\pi k}} \sum_{n=2k}^{\infty} \frac{n}{k} e^{-\frac{n^2}{2k}}$$

$$\frac{n^2}{2k} = x \qquad \frac{1}{k} = \frac{2x}{n^2}$$

$$-\frac{n^2}{2k^2}\, dk = dx = -\frac{dk}{k}\cdot x$$

$$= \frac{1}{2\sqrt{\pi}} \int_{\frac{n^2}{2k}} \frac{dx}{x} \sqrt{\frac{x}{2}}\, e^{-x}$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{?}^{\infty} x^{-1/2} e^{-x}\, dx$$

$$\frac{dW_k}{dk} =$$

$$\sum_{m=k-n}^{\infty} a_{nm} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{n^2}{2k}}}^{\infty} e^{-z^2} dz = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{n^2}{m}}}^{\infty} e^{-z^2} dz$$

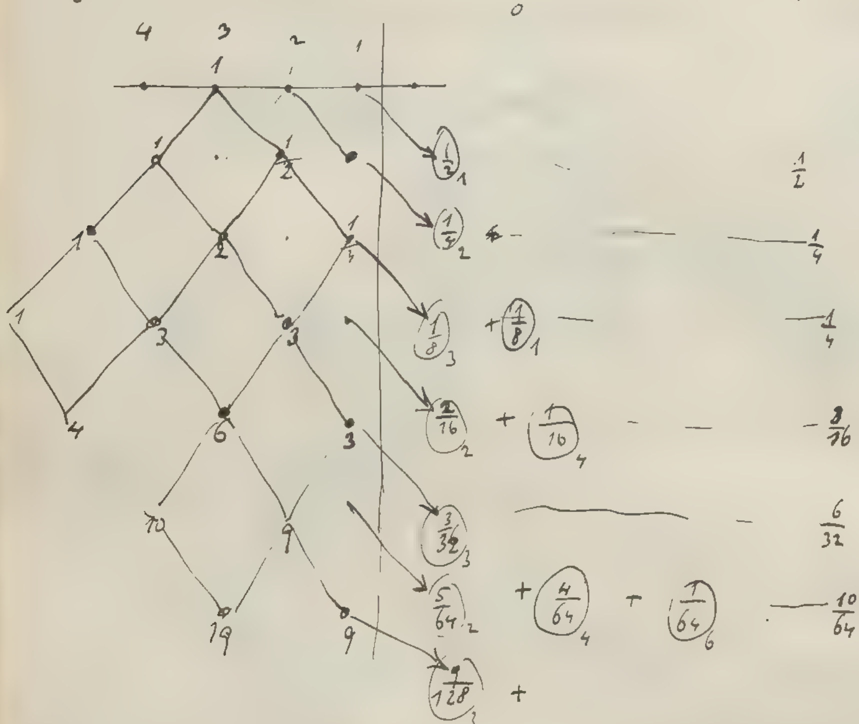
$$\cancel{= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{n^2}{k-n}}}^{\infty} e^{-z^2} dz \neq \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{n^2}{k-n}}}{2\sqrt{\frac{n}{k-n}}}}$$

$$1 - n < k$$

$$\cancel{\int_0^{k} \left[\log\frac{1}{\sqrt{2\pi}} - \frac{n^2}{k-n} - 2\log\frac{n}{\sqrt{k-n}}\right] dn}$$

$$\frac{n^2}{k-n} = z \qquad n^2 = kz \qquad n = -\frac{z}{2} \pm \sqrt{kz + \frac{z^2}{4}}$$

$$\int_0^k \frac{n^2}{k-n}\,dn = \cancel{-\int_k^0 \frac{(k-y)^2}{y}\,dy} = \int_0^k \frac{(k-y)^2}{y}\,dy \rightsquigarrow$$



$\frac{7}{8}\cdot\frac{3}{28} = \frac{3}{32}$

$\frac{7}{8}\cdot\left(1-\frac{3}{28}\right) = \frac{25}{32}$

$= 1 - \frac{1}{8} - \frac{3}{32}$

Zweierlei Arten Übergang:

1) Wahrsch. dass bis zum k ten Mal kein einziges Teilchen übergegangen sei = dass weder das erste bis zum kten Mal noch das zweite bis zum kten Mal, ... noch kte Teilchen bis zum kten Mal übergeht

$$\overbrace{1-\sum_{m=1}^{k} a_{1m}}\qquad \overbrace{1-\sum_{m=2}^{k} a_{2m}}\qquad \overbrace{1-\sum_{m=k}^{k} a_{km}}$$

$$\underbrace{= \sum_{m=k+1}^{\infty} a_{1m}}\qquad \underbrace{\sum_{m=k+1}^{\infty} a_{2m}}\qquad \underbrace{\sum_{m=k+1}^{\infty} a_{km}}$$

$k = \frac{t_1}{\tau}$

$$\frac{1}{\sqrt{2\pi}}\int_{\frac{1}{\sqrt{k+1}}}^{\infty} e^{-z^2}dz \cdot \frac{1}{\sqrt{2\pi}}\int_{\frac{2}{\sqrt{k+1}}}^{\infty} e^{-z^2}dz \cdots \frac{1}{\sqrt{2\pi}}\int_{\frac{k+1}{\sqrt{k+1}}}^{\infty} e^{-z^2}dz$$

$$W = \frac{1}{\sqrt{2\pi}} \int_{\frac{n}{\sqrt{k}}}^{\infty} e^{-z^2} dz = \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{n^2}{k}}}{2\frac{n}{\sqrt{k}}}$$

$$k! = \left(\frac{k}{e}\right)^k \sqrt{2\pi k}$$

$$\lg W = \lg \frac{\sqrt{k}}{2\sqrt{2\pi}} - \frac{n^2}{k} - \lg n$$

$$\sum_{n=1}^{k} \lg W = k \lg \frac{1}{2}\sqrt{\frac{k}{2\pi}} - \frac{k^2}{3} - \lg k!$$

$\bar{e}$

W. = Wahrsch. dass weder beim ersten Mal ein Teilchen übergegangen sei
noch beim zweiten Mal
noch beim dritten Mal ... usw.

$$W(0) + W(1) + W(2) + \dots = 1 \qquad W(2) = W(1)^2 \dots$$

$$W(1) + 2W(2) + 3W(3) \dots = A$$

$$W(0) + \alpha + \alpha^2 + \alpha^3 \dots = 1 = W(0) + \frac{\alpha}{1-\alpha} = W(0) + \frac{1-\beta}{\beta} = W(0) + \beta A$$

$$\alpha + 2\alpha^2 + 3\alpha^3 \dots = A = \frac{\alpha}{(1-\alpha)^2} = \frac{1-\beta}{\beta^2}$$

$$1 + \alpha + \alpha^2 \dots = \frac{1}{1-\alpha}$$

$$1 + 2\alpha + 3\alpha^2 \dots = \frac{1}{(1-\alpha)^2}$$

$$A\beta^2 = 1 - \beta$$

$$\beta^2 + \frac{\beta}{A} = \frac{1}{A}$$

$$\beta = -\frac{1}{2A} \pm \sqrt{\left(\frac{1}{2A}\right)^2 + \frac{1}{A}}$$

$$W(0) = 1 - \beta A = 1 + \frac{1}{2} \div \sqrt{\left(\frac{1}{2}\right)^2 + A}$$

$U_{\mathfrak{H}\tau}$ = Wahrsch. d. eins anfangs irgendwo innerhalb $\mathfrak{H}$ befindliches Teilchen bis zur Zeit t nicht kein einziges mal die Grenzfläche nicht überschritten habe

$U^N_{\mathfrak{H}\tau}$ = W. d. keines von N innerhalb $\mathfrak{H}$ bef. — — —

$$N = \frac{\nu}{h} H$$

Wenn es unabhängig von $\mathfrak{H}$ sein soll, so ist dies nur möglich falls $U_{\mathfrak{H}\tau} = V^{\frac{1}{H}}$ V unabhängig von H

$$\frac{1}{m}\left[m - \sqrt{\frac{2m}{\pi}}\right] = 1 - \sqrt{\frac{2}{m\pi}}$$

$$\left[1 - \sqrt{\frac{2}{m\pi}}\right]^{N} = e^{-\frac{\nu}{h} H \sqrt{\frac{2}{m\pi}}}$$

$$\left(1 - \sum_{m=1}^{k} a_{1m}\right) + \left(1 - \sum_{m=2}^{k} a_{2m}\right) + \left(1 - \sum_{m=3}^{k} a_{3m}\right) + \quad \underbrace{\left(1 - \sum_{m=n}^{k} a_{nm}\right)}_{\sum_{m=k+1}^{\infty} a_{nm}}$$

$$\frac{1}{\sqrt{2\pi}} \int_{\frac{1}{\sqrt{k}}}^{\infty} e^{-z^2} dz \quad - \quad - \quad - \quad - \quad \frac{1}{\sqrt{2\pi}} \int_{\frac{n}{\sqrt{k}}}^{\infty} e^{-z^2} dz$$

$$\frac{1}{n}\sqrt{\frac{k}{2\pi}} \int_1^n dz \int_z^\infty e^{-kx^2} dx = z \int_z^\infty - \int z\, e^{-kz^2} dz = \frac{e^{-kz^2}}{2k}\Bigg|_0^{\frac{n}{\sqrt{k}}} \frac{1}{n}\sqrt{\frac{k}{\pi}}$$

$$\frac{z}{\sqrt{k}} = x$$

$$\frac{1}{n}\sqrt{\frac{1}{2\pi}} \int_1^n dy \int_{\frac{y}{\sqrt{k}}}^\infty e^{-z^2} dz = y \int_{\frac{y}{\sqrt{k}}}^\infty e^{-z^2} dz - \frac{1}{\sqrt{k}} \int_{\frac{y}{\sqrt{k}}}^\infty y\, e^{-\frac{y^2}{k}} dy = \frac{1}{2n}\sqrt{\frac{k}{2\pi}}\, e^{-\frac{y^2}{k}}\Bigg|^n$$

$y = n$ $\quad y = 0$

$$\frac{1}{n\sqrt{2\pi}}\, n\, \frac{e^{-\frac{n^2}{k}}}{2\frac{n}{\sqrt{k}}}$$

$$= \frac{1}{2\sqrt{2\pi}} \frac{e^{-\frac{n^2}{k}} - 1}{\sqrt{\frac{n^2}{k}}} \neq \frac{1}{2\sqrt{2\pi}}\sqrt{\frac{n^2}{k}}$$ für kleine $\frac{n^2}{k}$

$$= \frac{1}{2\sqrt{2\pi}} \frac{\sqrt{k}}{n} \quad \text{für grosse } \frac{n^2}{k}$$

$$= \frac{n}{2\sqrt{2\pi}} \frac{1}{\sqrt{k}}$$

$$a_{nm} \neq \frac{n}{m}\sqrt{\frac{2}{\pi m}}\, e^{-\frac{n^2}{m}}$$

$$\frac{1}{9}\sqrt{\frac{2}{9\pi}}\; e^{-9} = \frac{1}{9\sqrt{14}} = \frac{1}{34} \qquad \frac{14}{512} = \frac{7}{256} = \frac{1}{36}$$

38

$n = m = 9$

$$\sqrt{\frac{2}{9\pi}}\; e^{-9} = \frac{1}{\sqrt{14}}\, e^{-9} = \frac{1}{3\cdot 7\cdot 28} = \frac{7}{}$$

$$\sum_{m=n}^{m=\infty} a_{nm} =$$

$$\frac{1}{2}\int \frac{\sqrt{2}}{\sqrt{\pi m}}\,\frac{n}{m}\, e^{-\frac{n^2}{m}}\, dm \qquad \frac{n^2}{m} = z = y^2 \qquad m = \frac{n^2}{z}$$

$$dm = -\frac{n^2}{z^2}\,dz$$

$$-2\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{z}\,\frac{z}{n^2}\, e^{-z}\,\frac{n^2}{z^2}\,dz = \frac{\sqrt{2}}{2\sqrt{\pi}}\int \frac{e^{-z}}{\sqrt{z}}\,dz = -\sqrt{\frac{2}{\pi}}\int_{\sqrt{n}}^{0} e^{-y^2}dy = \sqrt{\frac{2}{\pi}}\int_0^{\sqrt{n}} e^{-y^2}dy$$

$$\cancel{(1+x)^{\frac{m}{2}} = \sum \binom{\frac{m}{2}}{k} x^k} \qquad \cancel{\frac{\frac{m}{2}\left(\frac{m}{2}-1\right)\left(\frac{m}{2}-2\right)\cdots\left(\frac{m}{2}-k\right)}{1\cdot 2\cdots k} = \frac{m(m-2)(m-4)\cdots(m-2k)}{2\cdot 4\cdot 6\cdots 2k}}$$

$$\cancel{\left(\frac{1}{x}+x\right)^m = \binom{m}{0}x^{-m} + \binom{m}{1}x^{-m+2} + \binom{m}{2}x^{-m+4} \cdots \binom{m}{\frac{m}{2}}x^0 +}$$

(n gross)
$\frac{n^2}{m}$ klein

$$\frac{\nu}{h}H$$

$$\left(1-\frac{1}{2\sqrt{2\pi}}\sqrt{\frac{tD}{H}}\right)$$

$$\sum_{m=k}^{\infty} a_{nm} = \cancel{\sqrt{\frac{2}{\pi}}\int_0^{\sqrt{k}} e^{-y^2}dy} = \sqrt{\frac{2}{\pi}}\int_0^{\frac{n}{\sqrt{k}}} e^{-y^2}dy$$

$$\left[1-\frac{1}{2\sqrt{2\pi}}\frac{\sqrt{k}}{n}\right]^{N=\frac{\nu}{h}H} = e^{-\frac{1}{2\sqrt{2\pi}}\frac{\sqrt{k}}{n}N} \qquad k = \frac{t}{\tau}$$

$$\frac{\sqrt{k}}{n} = \frac{H}{\delta}$$

$$\frac{\delta}{\sqrt{2\tau}} = \sqrt{D}$$

$$e^{-\frac{1}{2\sqrt{2\pi}}\frac{\nu}{h}\frac{\delta\sqrt{t}}{\sqrt{\tau}}} = e^{-\frac{1}{2\sqrt{\pi}}\frac{\nu}{h}\sqrt{Dt}}$$

$$\cancel{-\frac{x e^{-\alpha x}}{\alpha} - \frac{e^{-\alpha x}}{\alpha^2}} \qquad \cancel{-\frac{e^{-\alpha x}}{\alpha} + x e^{-\alpha x} + \frac{e^{-\alpha x}}{\alpha}}$$

$$\int_0^{\infty} e^{-\alpha\sqrt{t}}\,dt = 2\int_0^{\infty} e^{-\alpha x}\, x\,dx = -x\cancel{\frac{e^{-\alpha x}}{\alpha}} + \frac{2}{\alpha^2} \qquad T = \frac{1}{\alpha^2} = \frac{h^2\, 4\pi}{\nu^2 D}$$

$$\cancel{-\alpha^2 t - 2\alpha x} \qquad e^{-\alpha^2 t - \alpha\sqrt{t}}$$

$$\alpha x + \tfrac{1}{2} = z$$

$$\int e^{-\alpha^2 t - \alpha\sqrt{t}}\,dt = \int_0^{\infty} e^{-\alpha^2 x^2 - \alpha x}\, x\,dx = \frac{e^{\frac{1}{4}}}{\alpha^2}\int_{\frac{1}{2}}^{\infty} e^{-z^2}\left(z-\tfrac{1}{2}\right)dz$$

$$\int_0^{\infty}\left(\alpha^2 + \frac{\alpha}{2\sqrt{t}}\right) e^{-\alpha^2 t - \alpha\sqrt{t}}\, t\,dt = \int \cancel{\alpha^2 + \alpha}$$

Nach Diffusionstheorie: übergegangene Menge

$$\int_0^t M\,dt = c\sqrt{\frac{D}{\pi}}\int \frac{dt}{\sqrt{t}} = 2c\sqrt{\frac{Dt}{\pi}}$$

Wahrsch., dass das eine Teilchen nicht übergegangen sei:

$$\frac{Hc - 2c\sqrt{\frac{Dt}{\pi}}}{Hc} = 1 - \frac{2}{H}\sqrt{\frac{Dt}{\pi}}$$

Falls nun $N = \frac{\nu}{h}H$ von einander unabhängige Teilchen vorhanden sind, Wahrsch. dass keines derselben übergegangen sei:

$$W_t = \left[1 - \frac{2}{H}\sqrt{\frac{Dt}{\pi}}\right]^{\frac{\nu}{h}H}$$

Nun lässt sich H beliebig gross wählen, so dass man immer setzen kann:

$$W_t = \lim \uparrow = e^{-\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}}$$

Für Schichten von endlichem H ist dies nicht anwendbar für Zeiten der Grössenordnung $t \sim \frac{H^2}{D}$

Darum müssen die inneren Teilchen anders berücksichtigt werden

Wahrsch. dass keine Zahlenänderung eingetreten sei:

$$W_t = e^{-\frac{D\pi^2 n t}{h^2} - \frac{4\nu}{h}\sqrt{\frac{Dt}{\pi}}} = e^{-\pi^2 n z - \frac{4\nu}{\sqrt{\pi}}\sqrt{z}}$$

$$\frac{Dt}{h^2} = z \qquad T = \int_0^\infty W_t\,dt \qquad \nu\pi^2 z = x^2$$

Wenn $n = \nu$:

$$T = \frac{h^2}{D}\int_0^\infty e^{-(\pi^2 z + 4\sqrt{\frac{z}{\pi}})\nu}\,dz = \frac{2h^2}{\pi^2 D\nu}\int_0^\infty e^{-(x^2 + \frac{4\sqrt{\nu}}{\sqrt{\pi^3}}x)}\,x\,dx$$

Falls nur die äusseren Teilchen berücksichtigt würden:

$$W = e^{-\frac{4\nu}{h}\sqrt{\frac{Dt}{\pi}}}$$

$$T = 2\int e^{-\frac{4\nu}{h}\sqrt{\frac{D}{\pi}}x}\,x\,dx \qquad \cancel{T = \frac{2h^2}{\pi^2 D\nu}\int_0^\infty e} = \frac{2}{\left(\frac{4\nu}{h}\sqrt{\frac{D}{\pi}}\right)^2} = \frac{\pi h^2}{8\nu^2 D}$$

dasselbe wie wenn $n=0$

$$J = \int_0^\infty e^{-x^2-\beta x}\, x\,dx = e^{\frac{\beta^2}{4}}\int_0^\infty e^{-\overbrace{(x+\frac{\beta}{2})}^{z}{}^2}\, x\,dx = e^{\frac{\beta^2}{4}}\int_{\frac{\beta}{2}}^\infty e^{-z^2}\left(z-\frac{\beta}{2}\right)dz$$

$$= e^{\frac{\beta^2}{4}}\left[\frac{e^{-z^2}}{2}\Bigg|_\infty^{\frac{\beta}{2}} - \frac{\beta}{2}\int_{\frac{\beta}{2}}^\infty e^{-z^2}dz\right] = \frac{1}{2} - \frac{\beta}{2}\, e^{\frac{\beta^2}{4}}\underbrace{\int_{\frac{\beta}{2}}^\infty e^{-z^2}dz}$$

für grosse $\frac{\beta}{2}$

$$= \frac{e^{-\frac{\beta^2}{4}}}{\beta}\left[1-\frac{2}{\beta^2}\right]$$

Jedenfalls muss J desto kleiner sein, je grösser β.

$\therefore \quad = \frac{1}{\beta^2} = \frac{\pi^3}{16\nu}$

also ist J enthalten zwischen den Grenzen: $\frac{1}{2} > J > 0$ $\left(\left(I = \frac{2h^2}{\pi^2 D\nu}\cdot\frac{\pi^3}{16\nu} = \frac{h^2\pi}{8D\nu^2}\right.\right.$

Sonst, wenn $n \geq \nu$

$$I = \frac{2h^2}{\pi^2 D n}\int_0^\infty e^{-\left[x^2+\frac{4\nu}{\sqrt{n\pi^3}}x\right]}\, x\,dx \qquad \beta = \frac{4\nu}{\sqrt{n\pi^3}} \qquad \Big| \; I =$$

Dagegen aus $\bar{\Delta}^2 = 2\nu P = 1 \quad = \frac{4\nu}{h}\sqrt{\frac{Dt}{\pi}}$

$P \downarrow \frac{2}{h}\sqrt{\frac{Dt}{\pi}}$

$$I = \frac{h^2\pi}{16\nu^2 D}$$

→ I muss desto kleiner sein, je grösser n; dabei kommt es auf das Verhältnis $\frac{n\sqrt{\pi^3}}{\nu\, 4}$ an; falls dieses gross gilt angenähert: $I = \frac{2h^2}{\pi^2 D n}$

Es ist doch unwahrscheinlich, dass im Falle grosser ν, n die I-Zahl unabhängig davon wird, ob in der Schichte selbst Teilchen sich befinden oder nicht!

$$\frac{8}{\pi^2}\left\{1-\frac{D\pi^2}{h^2}t+\left(\frac{D\pi^2}{h^2}t\right)^2\frac{1}{2!}-\right.$$

$$+\frac{1}{9}\left[1-9\frac{D\pi^2 t}{h^2}+\left(9\frac{D\pi^2}{h^2}t\right)^2\frac{1}{2!}-\cdots\right]$$

$$\left.+\frac{1}{25}\left[1-25\right.\right\}$$

$$\frac{dW}{dt}=\frac{8D}{h^2}\sum e^{-\frac{D k^2\pi^2}{h^2}t} \quad =\infty$$

$$t=0$$

$$\int_0^c \sin\frac{n\pi\alpha}{c}\,d\alpha=\frac{c}{n\pi}(\cos n\pi-1)=\cancel{\frac{c}{n\pi}(-2)}\ 0 \quad |\ n=2k$$

$$=-\frac{2c}{n\pi} \quad (n=2k+1)$$

$$u=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{c}\,e^{-D\left(\frac{n\pi}{c}\right)^2 t}$$

stimmt $\left\|\ \frac{\partial u}{\partial x}\right|_{x=0}=\frac{4}{c}\sum_{k=1}^{\infty}e^{-D\left(\frac{k\pi}{c}\right)^2 t}$

für $t=0$ divergente Reihe

das kann man nicht gliedweise ableiten?

Das Halbraum-Problem:

Wahrsch. dass gerade eines übergegangen sei:

$$N\frac{2}{H}\sqrt{\frac{Dt}{\pi}}\left[1-\frac{2}{H}\sqrt{\frac{Dt}{\pi}}\right]^{N-1}\neq\left(\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}\right)^{=\alpha}e^{-\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}}=\alpha e^{-\alpha}$$

Wahrsch. dass gerade zwei übergeg. sind:

$$\frac{N(N-1)}{1.2}\left(\frac{2}{H}\frac{Dt}{\pi}\right)^2\left[\quad\right]^{N-2}\neq\frac{\alpha^2}{2}e^{-\alpha}$$

etc.

Gesammte Wahrsch. $e^{-\alpha}\left[1+\alpha+\frac{\alpha^2}{2}+\frac{\alpha^3}{3!}+\cdots\right]=1$

~~[illegible]~~ Durchschn. übergehende Menge: $e^{-\alpha}\left[0.1+\alpha.1+2.\frac{\alpha^2}{2}+3\frac{\alpha^3}{3!}+\cdots\right]$

$$=e^{-\alpha}\left[\alpha+\frac{\alpha^2}{1}+\frac{\alpha^3}{2!}+\frac{\alpha^4}{3!}+\cdots\right]=\alpha=\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}$$

$$\frac{h^2\pi}{32\nu^2 D}\cdot \cancel{\pi}\sqrt{2\nu\pi}\; e^{\frac{\nu\delta^2}{2}}$$

$$= \frac{h^2\pi^{3/2}}{2\sqrt{2}D}\,\frac{1}{\nu^{3/2}}\, e^{\frac{\nu\delta^2}{2}} = \frac{h^2}{D}\left(\frac{\pi}{2}\right)^{3/2} e^{\frac{\delta^2}{2\xi^2}}\;\xi^3$$

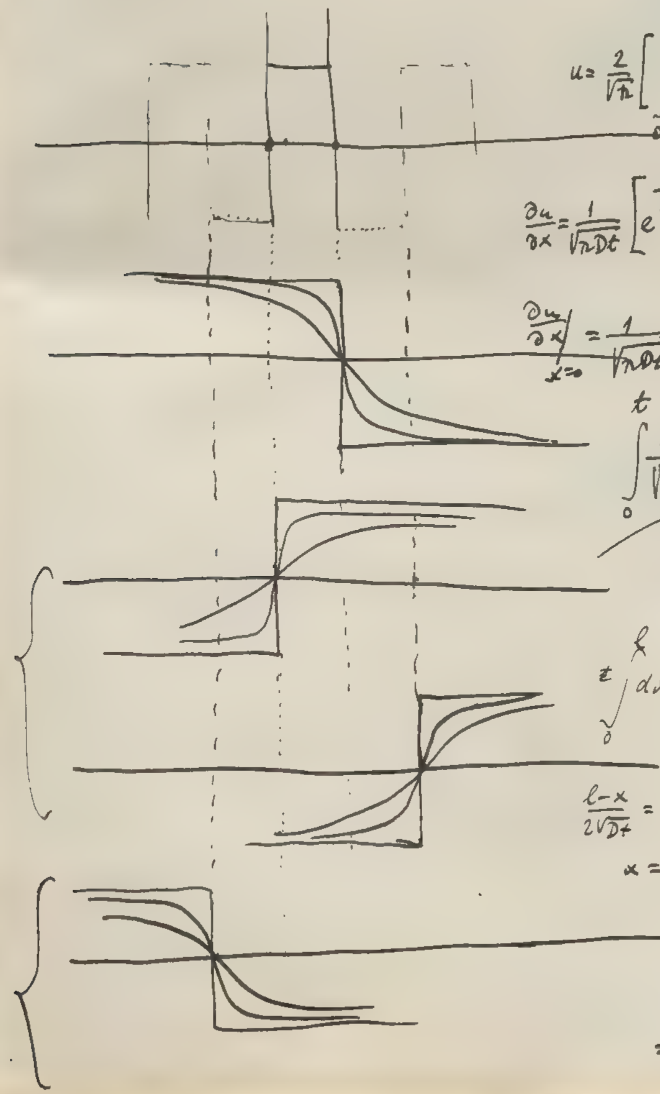
$$\lim W(n) = \lim \frac{e^{-\nu}\nu^n}{n!} = e^{n-\nu}\left(\frac{\nu}{n}\right)^n\frac{1}{\sqrt{2n\pi}} = e^{\nu\delta}\left(\frac{1}{1+\delta}\right)^{\nu(1+\delta)}\frac{1}{\sqrt{2\nu\pi}}$$

$$\lg W_{(n)} = \nu\delta - \nu(1+\delta)\left(\delta - \frac{\delta^2}{2}\right)\dots$$

$$= \nu\delta - \nu\delta + \nu\frac{\delta^2}{2} - \nu\delta^2$$

$$= -\nu\frac{\delta^2}{2}$$

$$= \frac{e^{-\frac{\nu\delta^2}{2}}}{\sqrt{2\nu\pi}}$$



$$u = \frac{2}{\sqrt{\pi}}\left[\int_0^{\frac{x}{2\sqrt{Dt}}} e^{-y^2}dy + \int_0^{\frac{l-x}{2\sqrt{Dt}}} - \int_0^{\frac{l+x}{\dots}} - \int_0^{\frac{2l-x}{\dots}} + \int_0^{\frac{2l+x}{}} + \int_0^{\frac{3l-x}{\dots}}\right.$$

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{\pi Dt}}\left[e^{-\frac{x^2}{4Dt}} - e^{-\frac{(l-x)^2}{4Dt}} - e^{-\frac{(l+x)^2}{4Dt}} + e^{-\frac{(2l-x)^2}{4Dt}} + e^{-\frac{(2l+x)^2}{4Dt}} - e^{-\frac{(3l-x)^2}{4Dt}}\right.$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \frac{1}{\sqrt{\pi Dt}}\left[1 - e^{-\frac{l^2}{4Dt}} - e^{-\frac{l^2}{4Dt}} + e^{-\frac{4l^2}{4Dt}} + e^{-\frac{4l^2}{4Dt}} - \dots - \frac{9l^2}{}\right.$$

$$\int_0^t \frac{1}{\sqrt{\pi Dt}}\, e^{-\frac{(nl)^2}{4Dt}}\,dt \qquad \frac{1}{\sqrt{Dt}} = z \qquad t = \frac{1}{z^2 D}$$

$$\frac{1}{\sqrt{\pi}D}\int_\infty^{\frac{1}{\sqrt{Dt}}} \frac{e^{-\frac{(nl)^2 z^2}{4}}}{z^2}\,dz = -\frac{1}{z}\,e^{\dots} - \int e^{\dots}\,dz$$

$$\int_0^l dx\int_0^{\frac{l-x}{2\sqrt{Dt}}} e^{-y^2}dy = x\left.\int_0^{\frac{l-x}{2\sqrt{Dt}}} e^{-y^2}dy\right|_0^l - \int_0^l x\, e^{-\left(\frac{l-x}{2\sqrt{Dt}}\right)^2}dx$$

$$\frac{l-x}{2\sqrt{Dt}} = z$$

$$x = l - 2z\sqrt{Dt}$$

$$= \int_0^{\frac{l}{2\sqrt{Dt}}}\left[2z\sqrt{Dt} - l\right]e^{-z^2}dz\,.\,2\sqrt{Dt}$$

$$= -2Dt\left.e^{-z^2}\right|_0^{\frac{l}{2\sqrt{Dt}}} - 2l\sqrt{Dt}\int_0^{\frac{l}{2\sqrt{Dt}}} e^{-z^2}dz$$

$$= 2Dt\left[1 - e^{-\frac{l^2}{4Dt}}\right] -$$

Jedenfalls gibt es einen Bereich genügend kurzer t wo angenähert gilt:

$$W = 1 - \frac{1}{h}\,\frac{4\sqrt{D}\sqrt{t}}{\sqrt{\pi}}$$ solange nämlich $\frac{4Dt}{h^2}$ sehr klein ist

Dagegen weite Art der Darstellung:

$$W = \frac{8}{\pi^2}\left[e^{-\frac{D\pi^2}{h^2}t} + \frac{1}{9}\, e^{-9\frac{D\pi^2}{h^2}t} + \frac{1}{25}\, e^{-25\frac{D\pi^2}{h^2}t} + \dots \right]$$

$$= 1 - \frac{8}{\pi^2}\sum_{k=1,3,5\dots}{}' \frac{1}{k^2}\left[1 - e^{-\frac{k^2 D\pi^2 t}{h^2}}\right]$$

W kann gleich $= \frac{8}{\pi^2}\, e^{-\frac{D\pi^2 t}{h^2}}$ gesetzt werden, sobald $\frac{D\pi^2 t}{h^2}$ gross ist

Für kürzere Zeiten dagegen erstere Darstellung besser

$$W^n = \left[1 - \frac{1}{h}\frac{4\sqrt{D}}{\sqrt{\pi}}\sqrt{t}\right]^n = e^{-\frac{4n\sqrt{D}}{h\sqrt{\pi}}\sqrt{t}}$$

Dann wird also im Ganzen:

$$T = \int e^{-\frac{4(n+\nu)}{h\sqrt{\pi D}}\sqrt{t}}\,dt = \frac{\pi h^2}{8(n+\nu)^2 D}$$ für $n = \nu$ $$T = \frac{\pi}{32}\,\frac{h^2}{D\nu^2}$$

$$\frac{h^2\pi}{32\nu^2 D}\sqrt{2\nu\pi}\; e^{\frac{\nu\delta^2}{2}}$$

$\nu = \frac{3.10^{19}}{4}$ ‖ 2.10

$\delta = \frac{1}{100}$

$e^{\frac{3}{8}\cdot 10^{15}}$

$\frac{3}{8}10^{15}.0{,}434$

$1{,}6.10^{14}$

$10^{17} - 30$

Kugelförmiger Raum

$$\frac{\partial(cr)}{\partial t} = \frac{\partial^2(cr)}{\partial r^2}$$

$$cr = \frac{2}{\sqrt{\pi}}\int_0^{\frac{a-r}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

$$\frac{\partial c}{\partial r} = \frac{1}{\sqrt{\pi D t}}$$

$$M = 4a^2\pi \cdot 2\sqrt{\frac{Dt}{\pi}}$$

$$W = 1 - \frac{4a^2\pi}{\frac{4}{3}a^3\pi}\,2\sqrt{\frac{Dt}{\pi}} = 1 - \frac{6}{a}\sqrt{\frac{Dt}{\pi}}$$

$$W^\nu = e^{-\frac{6\nu}{a}\sqrt{\frac{Dt}{\pi}}}$$ ebensoviel äussere

$$\int_0^\infty e^{-\alpha\sqrt{t}}\,dt = 2\int_0^\infty e^{-\alpha x}\,x\,dx = \frac{2}{\alpha^2}$$

$$I_3 = \frac{2}{\left[\frac{6(n+\nu)}{a}\sqrt{\frac{D}{\pi}}\right]^2} = \frac{a^2\pi}{18(n+\nu)^2 D} \quad \cdot \sqrt{2\nu\pi}\, e^{\frac{\nu\delta^2}{2}}$$

$a = 2.10^{-5}$; $\nu = \frac{\cancel{3}.10^{19}}{4} \cdot 10^{-15} \frac{4}{3}\pi$

$= 10^4.\pi.8$

$$\frac{a^2 \sqrt{2}}{18.4\,D}\left(\frac{\pi}{\nu}\right)^{3/2} \cdot e^{\frac{\nu\delta^2}{2}} =$$

$$\frac{4.10^{-10}\sqrt{2}}{72.D}\left(\frac{\cancel{\pi}}{8.10^4\cancel{\pi}}\right)^{3/2} e^{\frac{\pi}{2}.8}$$

34.4

$e^{4\pi} = e^{12.4}$

$12.4.\ 0.434$
496
372
49
5.38

18.16
108
288

$$= \frac{10^{-10}\sqrt{2}}{18.D.10^6.8\sqrt{8}} \cdot 3.5.10^5 = \frac{10^{-11}}{18.8.2.D} = \frac{10^{-13}}{2.88.D}$$

$$\frac{a^2\sqrt{2}}{72.D}\left(\frac{\pi}{\frac{4a^3\pi}{3}N}\right)^{3/2} e^{\frac{4a^3\pi}{3}N\frac{\delta^2}{2}} = 1. = \frac{3\sqrt{6}}{72.D.8.\,a^2\sqrt{a}\;N^{3/2}} \quad e^{\frac{2}{3}a^3\pi N\delta^2}$$

$a = 3.10^{-5}$ $\qquad \nu = \frac{3}{4}.10^{19}.\frac{4}{3}\pi\, 10^{-15}.\,27$

$= 27.\pi.10^4$ $\qquad e^{27\pi}$

$3.141.\ 27$
6283
2199
84.8

$0.434.\ 85$
217
3472
36.9

$\boxed{a = \frac{5}{2}.10^{-5}}$ $\qquad \nu = 10^4\pi.\frac{125}{8}$

$= 10^4\pi.15.6$

$15.6.\ 3.14$
468
156
62
49.0

$0.434.\ 49$
1736
3906
18.27

$$\frac{\frac{25}{4}.10^{-10}\sqrt{2}}{72.D} \cdot \frac{10^{18}}{(15.6.10^4)^{3/2}}.\,18$$

$$T = \frac{\sqrt{2}}{12}\,\frac{10^8.\,18}{10^6.\,15.4.D} = \frac{\sqrt{2}}{4.D}$$

Svedberg $k = 2.10^{-4}$

$D = 10^{-7}$

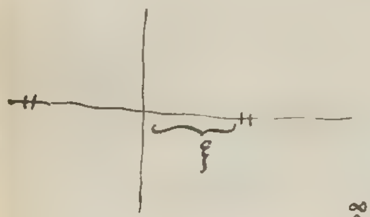
$v = 1'55$

$$T = \frac{\pi}{32} \frac{k^2}{D v^2} = \frac{\pi . 4.10^{-8}}{32 . 10^{-7} . (1'55)^2} = \frac{\pi}{8} \frac{10^{-1}}{2'4} = \frac{3'14}{19'2} 10^{-1} = \underline{1'6 . 10^{-2}}$$

$n = 17$

$$T = \frac{\pi}{8} \frac{4.10^{-8}}{10^{-7} (18'5)^2} = \frac{\pi . 10^{-1}}{2.18.19} = \frac{\pi . 10^{-1}}{684} = 3142 : 684 = 4'61 . 10^{-3}$$

Wenn ein Teilchen aus einer Lage ξ (fortg.) ausgeht, was ist die Wahrsch., dass es bis zur Zeit t (= kein einziges mal) nicht durch die Null-Ebene gegangen sei?



Verteilung

$$dx . u = \frac{1}{\sqrt{\pi D t}} \left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{(x+\xi)^2}{4Dt}} \right] dx \qquad \Big\| \frac{d\xi}{dx}$$

$$U = \int_0^\infty u \, dx = \frac{1}{\sqrt{\pi D t}} \int_0^\infty [\quad] dx$$

$$T = -\int_0^\infty t \frac{dU}{dt} dt = \int_0^\infty U \, dt$$ = durchschnittl. Wirkz., falls ein Teilchen ursprünglich in ξ

$$\overline{T} = \int_0^H T \frac{d\xi}{H} = \int_0^\infty dt \int_0^H U \frac{d\xi}{H}$$ = falls ein Teilchen im Bereich von Null bis H

[illegible]? von einem auf viele Teilchen

$$\overline{\overline{T}} = \lim_{\substack{H = \infty \\ \frac{\nu}{H} = \text{const}}} \int_0^\infty dt \left[\int_0^H U \frac{d\xi}{H} \right]^{\nu}$$

Ist die Mittelbildung überall richtig? Es sollten anfangs die verschiedenen Werte T gebildet werden, welche den verschiedenen örtlichen Teilchen-Anordnungen entsprechen. Jeder sollte mit der Wahrscheinlichkeit der betreffenden Teilchen-Anordnung multipliziert werden und daraus dann das Mittel zu nehmen.

$160 \cdot \frac{\cancel{4 \cdot 9 \cdot 10^{-7}} (10^{-4})}{\cancel{24 \cdot 10^{-7}}} = 80 \cdot 10^4 = \frac{8 \cdot 10^5}{360}$ d. $= 2200$ J.

$\sqrt{\frac{8\pi}{3}} \frac{a}{4\nu C} \sqrt{\nu\pi}\, e^{\frac{\nu\delta^2}{2}}$ 91

$= \sqrt{\frac{2\pi}{3}} \frac{a}{(n+\nu) C}$ $= \frac{a\pi}{\sqrt{3\nu}\, C} e^{\frac{\nu\delta^2}{2}}$

Zeiten Θ, T_1 für kugelförmigen Raum $O_2 - N_2$

$$T_1 = \frac{\sqrt{6\pi}}{\sqrt{\ldots}(n+\nu)\, C} \cdot \tfrac{4}{3} a^3 \pi \qquad \Theta_1 = \sqrt{2\nu\pi}\, e^{\frac{\nu\delta^2}{2}} \cdot T_1 = \frac{\sqrt{6\pi}\,\sqrt{2\nu\pi}}{8a^2\pi\nu C} e^{\frac{\nu\delta^2}{2}} \cdot \tfrac{4}{3} a^3 \pi$$

(wenn n & ν die im ganzen Raum befindliche Anzahl bedeuten) $= \frac{T_1}{W(n)}$

$$= \frac{\sqrt{3\nu}}{4a^2\nu C} e^{\frac{\nu\delta^2}{2}} = \frac{\sqrt{3}}{4a^2 C\sqrt{\nu}} e^{\frac{\nu\delta^2}{2}} \cdot \tfrac{4}{3} a^3 \pi$$

$$= \frac{\pi}{\sqrt{3}} \frac{a}{C\sqrt{\nu}} e^{\frac{\nu\delta^2}{2}}$$

Falls zwei Teilchen zwischen 0 und H:

Wahrsch., dass keines von beiden abgezogen sei, wenn das eine von ξ_1, das andere von ξ_2 ausgegangen ist:

$$= U_1 U_2$$

W. d. ...., wenn ξ_1 und ξ_2 beliebig innerhalb Null und H:

$$= \iint_0^H U_1 U_2 \frac{d\xi_1}{H} \frac{d\xi_2}{H} = \left[\frac{1}{H}\int_0^H U\, d\xi\right]^2$$

$$\overline{\overline{T}} = -\iint_0^H \frac{d\xi_1}{H} \frac{d\xi_2}{H} \int_0^\infty t \frac{d(U_1 U_2)}{dt} dt = \int_0^\infty dt \left[\frac{1}{H}\int_0^H U\, d\xi\right]^2$$

also ist meine Methode ganz richtig! Ebenso für n Teilchen!

$$\frac{e^{-\frac{\nu\delta^2}{2}}}{\sqrt{2\nu\pi}} = \frac{e^{-\frac{(n-\nu)^2}{2\nu}}}{\sqrt{2\nu\pi}} \qquad \sum_{n=-\infty}^{+\infty} e^{-\frac{(n-\nu)^2}{2\nu}} = 2\sum_{n=\nu}^{\infty} e^{-\frac{(n-\nu)^2}{2\nu}} = 2\sum_{n=0}^{\infty} e^{-\frac{n^2}{2\nu}} = ?$$

$\cancel{x^{-1} + x^{-4} + x^{-9} + x^{-16} \ldots}$

$$T = \frac{\sqrt{3}}{4}\,\frac{1}{a^2 C\sqrt{\nu}}\, e^{\frac{\nu\delta^2}{2}}$$

$C = 480\,\frac{m}{sk} = 4'8 \cdot 10^4$

$\nu = \frac{4}{3} a^3 \pi\, \frac{N}{4}$ $\qquad N = 3 \cdot 10^{19}$

$\delta = 10^{-2}$

$\underline{a = 1\ cm}$

$$\Theta_1 = \frac{T}{4t} = \frac{\sqrt{3}\ \cancel{\sqrt{x}}}{4 \cdot 4'8 \cdot 10^4 \sqrt{\pi}\,\sqrt{\cancel{x}\,10^{19}}}\, e^{+\frac{10^{19}\pi \cdot 10^{-4}}{2}}$$

$$= \sqrt{\frac{3}{10\pi}}\ \frac{10^{-13}}{4 \cdot 4'8 \cdot}\ e^{+\frac{10^{15}\pi}{2}}$$

$3'1415$

$0'4343 \cdot 1'57$
628
47
6
$0'681 \cdot 10^{14}$

$$\frac{4'8 \cdot 10^{10^{14}}}{10^{13}} = 4'8 \cdot 10^{10^{14}-13} \cdot 4$$

$$\neq 10^{10^{14}}$$

$a = 3\,10^{-5}$ $\qquad C = 4'8 \cdot 10^4$

$\nu = 27 \cdot 10^{-15} \cdot 10^{19}\pi = 2'7 \cdot 10^5 \cdot \pi = 8'5 \cdot 10^5$

$\delta = 10^{-2}$

49715
43136
9285

$8'48$

$$\Theta_1 = \frac{T}{4} = \frac{\sqrt{3}}{4}\ \frac{1}{9 \cdot 10^{-10} \cdot 4'8 \cdot 10^4 \sqrt{85 \cdot 10^4}}\, e^{42.}$$

$$= \frac{\cancel{\sqrt{3}} \cdot 10^4 \cdot 10^{18}\ \cancel{1'7} \cdot \cancel{\frac{4}{3}}\ \cancel{27} \cdot 10^{-15}\ \pi\ 546}{\cancel{4}\ \cancel{9}\ 4'8\ \cancel{10^4}\ \cancel{\sqrt{85}}\ 5'32} = \frac{10^7}{4'7} = 2 \cdot 10^6$$

$3'14$

$0'4343 \cdot 42$
17372
869
$18'24$

314
220
12
546

$a = 2 \cdot 10^{-5}$ $\qquad C = 4'8 \cdot 10^4$

$\nu = \cancel{3'1} \ 25 \cdot 10^4$

$5'429$

802
27
10

$$\Theta_1 = \frac{\sqrt{3}}{\cancel{4} \cdot \cancel{4} \cdot 10^{-10} \cdot 4'8 \cdot 10^4 \cdot \cancel{5} \cdot 10^2}\, e^{12'5} = \frac{\cancel{\sqrt{3}} \cdot 10^3\ \ 2'69 \cdot 10^5\ \cancel{4}\ \pi \cdot \cancel{8}\ 10^{-15}}{\cancel{38'48}\ \ \cancel{1'7 \cdot 1'73}\ \cancel{\sqrt{3}}} = \frac{8'44}{2'08} \cdot 10^{-7}$$

80 $\qquad 3'14$ $\qquad 2'08$

$= 4 \cdot 10^{-7}$

92

$$\cancel{\frac{T_1}{T_3}}\ \frac{\Theta_1}{\Theta_3} = \frac{\sqrt{6\pi}}{8a^2\pi \cancel{\nu} C}\ \frac{9\ \cancel{72 \cdot 4\nu^2} D}{a^2\pi} = \frac{9\sqrt{6}}{\pi\sqrt{\pi}}\ \frac{D\nu}{a^4 C} \qquad D = \frac{C\lambda}{3}$$

$$= \frac{9\sqrt{6}}{\pi\sqrt{\pi}}\ \frac{\nu\lambda}{3a^4} = \frac{3\sqrt{6}}{\pi\sqrt{\pi}}\ \frac{\lambda}{a^4}\ \cancel{\frac{3}{4}}\ \frac{N}{4}$$

$$= \frac{\sqrt{6}}{\pi}\lambda N \cdot \frac{4}{3}a^2\pi$$

$$= \frac{4}{3}\sqrt{6\pi}\cdot a^2\lambda N$$

$$4\cdot 10^{-10}\cdot 10^{-5}\cdot 3\cdot 10^{19}$$

$$T_1 = \frac{\sqrt{6\pi}}{8a^2\pi \frac{N}{4} C} \qquad \nu = \frac{\sqrt{6\pi}\cdot 3a}{2\cdot \nu C}$$

$$\cancel{\Theta = \sqrt{3}}$$

$$T_3 = \frac{a^2\pi}{72\cdot\nu^2 D} = \frac{a^2\pi}{72\cdot\left[\frac{4}{3}\pi a^3\frac{N}{4}\right]^2 D}$$

$$D = \frac{R\theta}{N}\ \frac{1}{3\pi a\mu} = \frac{\lambda}{3}\sqrt{\frac{2\theta}{}}$$

$$\frac{\nu}{h} =$$

---

für Schichten:

$$T_1 = \frac{\sqrt{6\pi}\,h}{4\nu C} \qquad \frac{T_1}{T_3} = \frac{\sqrt{6\pi}\,h}{4\nu C}\ \frac{32\nu^2\frac{C\lambda}{3}}{\pi h^2} = \frac{\sqrt{6\pi}\cdot 8}{3\pi}\ \frac{\nu\lambda}{h}$$

$$T_3 = \frac{\pi h^2}{32\nu^2 D}$$

Man kann doch ν beliebig gross machen, indem man (bei constantem h und const. Raumdichte) den Flächeninhalt der Schichte beliebig vermehrt. Warum soll da das Verhältnis $\frac{T_1}{T_3}$ beliebig wachsen?

$a = 1\cdot 10^{-5}$ $\qquad \nu = 3{,}14\cdot 10^4$ $\qquad e^{1{,}57}$ $\qquad \cancel{0{,}7729}$ $\qquad 0{,}6818$

$$\Theta_1 = \frac{\sqrt{3}\qquad \cancel{480}}{4\cdot 10^{-10}\cdot \cancel{48}\cdot 10^4\cdot\sqrt{3{,}14\cdot 10^2}} \neq \frac{10^4}{\cancel{4}}\cdot\frac{4}{3}\cdot 10^{-15}\cdot\pi = \underline{\underline{10^{-11}}}$$

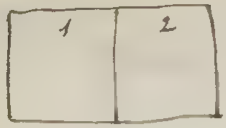
4343
2171
304
6818

$\underline{\underline{a = \frac{5}{2}\cdot 10^{-5}}}$ $\qquad \nu = \frac{125}{8}\cdot 3{,}14\cdot 10^4 = \frac{13}{8}\cdot 3\cdot 10^4 = 49\cdot 10^4$ $\qquad e^{2{,}45} =$

8686
1737
217
10,64

$$\Theta_1 = \frac{\cancel{\left(\frac{5}{2}\right)^{\frac{1}{2}}}}{\cancel{\left(\frac{5}{2}\right)^2}}\cdot\frac{4{,}37\cdot 10^{10}\cdot 10^{-11}}{\cancel{\sqrt{5}}\ 1{,}58} = \underline{\underline{0{,}3}}\ \text{sek}$$

$$\cancel{\frac{10^{-5}}{3\cdot 10^{2}\sqrt{10}\cdot 10^{2}}\ \frac{4\pi}{3} = 10^{-16}}\ ?$$



Wahrsch., dass eine Zahlyänderung um 1 eintritt

a) bei einer Halbfläche W

b) bei der ganzen: entweder Änderung in 1 und nicht in 2: $W(1-W)$

oder " 2 1: "

also Wahrsch. einer Änderung um 1 bei der ganzen Fläche $= 2W(1-W)$

Wenn W die ~~Dagegen~~ Wahrsch. einer Änderung überhaupt bedeutet, so ist

$$W_{ganz} = 2W_{halb}$$

Somit sollte man erwarten, dass die Wahrsch. einer Zahlenänderung bei konstanter Raumdichte proportional sein wird der Oberfläche der Schichte, also proportional v (resp. n)

$$\frac{D}{C} = \frac{\frac{1}{6\pi a\mu}\frac{R\theta}{N}}{\sqrt{\frac{3R\theta}{N\frac{4}{3}a^3\pi\rho}}} = \frac{\sqrt{\frac{R\theta}{N}}}{\sqrt{\frac{9\cdot 9\pi^2a^2\mu^2}{\frac{4}{\ }a^3\pi\rho}}} = \sqrt{\frac{R\theta}{N}}\cdot\frac{2}{9}\cdot\frac{1}{\mu}\sqrt{\frac{a\rho}{\pi}}$$

$$6\frac{D}{C^2} = \frac{2}{a\pi\mu}\frac{4}{9}a^3\pi\rho = \frac{8a^2\rho}{9\mu}$$

So kurze Zeiten dass man ~~das experimentelle~~

die Ungültigkeit der Diffusionstheorie nachweisen könnte sind wohl schwer experimentell herzustellen

Vielleicht bei Auflösungsgeschwindigkeit fester Körper?

Einfachstes Beispiel: Halbraum-Problem

Anzahl der Stösse pro Flächeneinheit : $2\frac{NC}{\sqrt{6\pi}}$

also ~~mittlere~~ durchschn. Dauer zwischen zwei Stössen auf Fläche F: $T_1 = \frac{\sqrt{6\pi}}{2FNC}$

Wahrsch., dass noch kein Teilchen übergegangen sei:

$$W = e^{-2NF\sqrt{\frac{Dt}{\pi}}}$$

Wahrsch. dass einer oder mehrere Überg. sei: $1-e^{-\frac{2NF}{3}\sqrt{\frac{Dt}{\pi}}}$; Wahrsch., dass gerade einer übergeg. sei: $\alpha e^{-\alpha}$

~~Mit~~ Durchschn. Zeit bis zum Übergange:

$$T_3 = -\int_0^\infty t\,\frac{dW}{dt}\,dt = \frac{2\pi}{4(NF)^2 D} = \frac{\pi}{2N^2 D F^2}$$

$$\int_0^\infty \frac{\alpha\sqrt{t^3}\,e^{-\alpha\sqrt{t}}\,dt}{\int \alpha\sqrt{t}\,e^{-\alpha\sqrt{t}}\,dt} = \frac{2\int_0^\infty \alpha x^4 e^{-\alpha x}\,dx}{2\int \alpha x^2 e^{-\alpha x}\,dx} = \frac{\frac{2}{\alpha^4}\int_0^\infty y^4 e^{-y}\,dy}{\frac{2}{\alpha^2}\int y^2 e^{-y}\,dy} = \frac{\cdots}{\alpha^2} \quad ?$$

$\sqrt{t} = x$

$t = x^2$

Aber Diffusionstheorie auch nur anwendbar für Zeiten welche eine gewisse untere Grenze überschreiten! Denn Übergangsgeschwindigkeit:

$$\frac{NC}{\sqrt{6\pi}} \geqq N\sqrt{\frac{D}{\pi t}}$$

$$t \geqq \frac{6D}{C^2} \qquad \text{d.h.} \quad T_3 = \frac{\pi h^2}{32\nu^2 D} \gg \frac{6D}{C^2}$$

$$\therefore \quad \frac{h}{\nu} \gtrapprox \sqrt{\frac{32\cdot 6}{\pi}}\cdot\frac{D}{C}$$

Dann ist für $=$: $T_1 = \frac{\sqrt{6\pi}}{4C}\sqrt{\frac{32\cdot 6}{\pi}}\,\frac{D}{C} = \frac{6\sqrt{32}}{4}\,\frac{D}{C^2}$ also von derselben Grössenordnung sonst wird aber T_1 kleiner sein

Wahrsch., dass bei $m=$ 8 Würfen der Maximalüberschuss ~~gerade~~ $n=$ 4 ~~beträgt~~ auftritt:

$$\frac{1}{16}+\frac{4}{64}+\frac{14}{256}-\frac{1}{32}-\frac{5}{128}=\frac{3}{32}+\frac{1}{64}=\frac{7}{64}$$

$$\binom{8}{2}\frac{1}{2^8}=\frac{8\cdot 7}{1\cdot 2}\,\frac{1}{2^8}=\frac{7}{2^6}=\frac{7}{64}$$

$m=8$. $n=3$

$$\frac{1}{8}+\frac{3}{32}+\frac{9}{128}-\frac{1}{16}-\frac{2}{32}-\frac{7}{128}=\frac{1}{16}+\frac{1}{32}+\frac{1}{64}=\frac{7}{64}$$

$m=8$ $n=2$

$$\frac{1}{4}+\frac{2}{16}+\frac{5}{64}+\frac{7}{128}-\frac{1}{8}-\frac{6}{64}-\frac{9}{128}=\frac{2}{8}-\frac{1}{32}=\frac{7}{32}$$

$$\binom{8}{3}\frac{1}{2^8}=\frac{\cancel{8}\cdot 7\cdot\cancel{6}}{1\cdot\cancel{2}\cdot\cancel{3}}\,\frac{1}{\cancel{8}\cdot 8\cdot 4}=\frac{7}{32}$$ stimmt

$$\cancel{\frac{N_1+N_2+N_3+\cdots}{N_1+2N_2+3N_3+\cdots}=W(\tau)}$$

~~Also~~ $\cancel{I_1=\frac{\tau}{W(\tau)}}$

$$W(k\tau)=\frac{N_k+N_{k+1}+\cdots}{N_1+2N_2+3N_3+\cdots}$$

$$I_3=\tau\sum_{k=1}^{\infty}{}' k\,W(k\tau)$$

$$W(k\tau)-W((k+1)\tau)=\frac{N_k}{N_1+2N_2+3N_3+\cdots}$$

$\|$ $1-W(\tau)=$ …

also muss sein:

$$\sum_{k=1}^{\infty}k\left[W(k\tau)-W((k+1)\tau)\right]=1$$

Wahrsch. dass auf n im nächsten Intervalle wieder n folge (und später ein beliebiger Wert)

$= P_n(0) = W(n,n)$

$$W(n,n) = \frac{N_2 + 2N_3 + 3N_4 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

Wahrsch. dass auf n im nächsten Intervalle wieder n folge (aber später ein „nicht-n")
dass also n nur durch zwei Intervalle ununterbrochen andauere

$$W(2\tau) = \frac{N_2 + N_3 + N_4 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

~~Wahrsch. dass auf n im nächsten Intervalle kein n folge, also dass nur durch 1 Intervalle ununterbrochen n folge~~ $= \frac{N_1 + N_2 + N_3 + \cdots}{}$

$W(1\tau) =$

Wahrsch. dass ein kfaches Intervall auftritt =

$$\frac{k N_k}{N_1 + 2N_2 + 3N_3 \cdots}$$

Wahrsch. dass der Zustand (durch k Intervalle noch weiter dauere

$$= \frac{N_k + N_{k+1} + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

$$T_3 = \tau\,\frac{N_1 + (1+2)N_2 + (1+2+3)N_3 + \cdots}{N_1 + 2N_2 + 3N_3 \cdots}$$

$$\cancel{= \tau\left[1 + \frac{N_2 + (1+2)N_3 + (1+2+3)N_4}{N_1 + 2N_2 + 3N_3 + \cdots}\right] = \tau\left[1 +\right.}$$

$$= \tau\left[\frac{N_1 + N_2 + N_3 \cdots}{N_1 + 2N_2 + 3N_3} + \frac{2N_2 + (2+3)N_3 + (2+3+4)N_4}{\cdots}\right]$$

$$= \tau\left[\frac{N_1 + N_2 + N_3}{\cdots} + \frac{2(N_2 + N_3 + N_4)}{\cdots} + \frac{3(N_3 + N_4 + N_5 + \cdots)}{\cdots}\right]$$

$$= \tau\left[W(1\tau) + 2W(2\tau) + 3W(3\tau) + \cdots\right] = \sum_{k=1}^{\infty} k\tau\, W(k\tau)$$

Nun ist offenbar: $W(1\tau) = 1 - W(n\,n) = 1 - P_n(0)$

Somit kann man setzen $T_1 = \frac{\tau}{W(1\tau)} = \frac{\tau}{1 - P_n(0)}$

$$\lim T_1 = \frac{1}{\lim\left(\frac{W(1\tau)}{\tau}\right)} = \frac{1}{\text{(Wahrsch., dass innerhalb } dt \text{ der n Zustand (zum ersten Male) in Nicht-n übergehe)} : dt}$$

$$\bar{W}(nnn) + \bar{W}(nnm) = \bar{W}_1(nn) \qquad \cancel{\bar{W}(n\,m\,m)}$$

$$(\bar{W}(nmn) + \bar{W}(nmm) = \bar{W}_1(nm) = \bar{W}(n) - \bar{W}_1(nn)$$

$$(\bar{W}(nnm) + \bar{W}(mnm) = \bar{W}_1(nm) =$$

$$(\bar{W}(nmn) + \bar{W}(mmm) = \bar{W}_1(mm) = 1 - 2\bar{W}(n) + \bar{W}_1(nn)$$

$$\bar{W}(nnn) + \bar{W}(nmn) = \bar{W}_2(nn)$$

$$\cancel{\bar{W}(nnm) + \bar{W}(nmm) = \bar{W}_2(nm)}$$

$$(\bar{W}(mnn) + \bar{W}(mmn) = \bar{W}_2(mm) = 1 - 2\bar{W}(n) + \bar{W}_2(nn)$$

$$\bar{W}(nmm) - \bar{W}(mnm) = \bar{W}_1(nn) - \bar{W}_2(nn)$$

$$\bar{W}(nnm) + \bar{W}(nmm) = \bar{W}(n) - \bar{W}_2(nn)$$

$$\bar{W}(nmn) - \bar{W}(nnm) = \bar{W}_2(nn) - \bar{W}_1(nn)$$

es fehlt eine Gleichung

Brillouin setzt, $\mathcal{N} = \frac{n\,\xi}{2} \qquad \xi = \sqrt{2Dt}$

$$= n\sqrt{\frac{Dt}{2}}$$

$$\frac{d\mathcal{N}}{dt} = \frac{n}{2}\sqrt{\frac{D}{2t}}$$

m. correkter scheint mir zu setzen: $\xi = \sqrt{\frac{2}{\pi}}\sqrt{2Dt}$

$$= 2\sqrt{\frac{Dt}{\pi}}$$

$$\mathcal{N} = n\sqrt{\frac{Dt}{\pi}}$$

$$\frac{d\mathcal{N}}{dt} = \frac{n}{2}\sqrt{\frac{D}{\pi t}}$$

$\frac{1}{2} \cdots\cdots \frac{4}{\pi}$

~~6.28~~ ~~3.14~~ 157

$\frac{276}{157} =$

119 (176

9

Nach Diffusionstheorie:

$$c = c_0 \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{Dt}}} e^{-y^2}\, dy \quad \ldots$$

für $t=0$: $c = c_0$

für $x = 0$: $c = 0$

Also gilt das bei Erhaltung der Null-Konzentration im O Punkt

$$\frac{\partial c}{\partial x} = \frac{2c_0}{\sqrt{\pi}} \frac{1}{2\sqrt{Dt}} \cancel{e^{-\frac{x^2}{4Dt}}} = \cancel{\frac{c_0}{\pi\sqrt{Dt}}} \quad \frac{c_0}{\sqrt{\pi D t}}$$

$$D\frac{\partial c}{\partial x} = c_0 \sqrt{\frac{D}{\pi t}}$$

Somit sollte sein: $\frac{dN}{dt} = n\sqrt{\frac{D}{\pi t}} =$

$N = 2n\sqrt{\frac{Dt}{\pi}} = A$

Somit: $\boxed{D = \frac{A^2\pi}{4n^2 t}}$ $= \frac{RT}{N}\frac{1}{6\pi\eta P}$

Dagegen würde bei Verteilung

$$c = \frac{c_0}{2}\left[1 + \frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{Dt}}} \cdots \right]$$

$t = 0$: $c = c_0$ $x > 0$; $\boxed{c = 0 \quad x < 0}$

$x = 0$: $c = \frac{c_0}{2}$

resultieren:

$$D\frac{\partial c}{\partial x} = \frac{c_0}{2}\sqrt{\frac{D}{\pi t}} \qquad N = n\sqrt{\frac{Dt}{\pi}}$$

$$\boxed{D = \frac{A^2\pi}{n^2 t} = \frac{RT}{N}\frac{1}{6\pi\eta P} \qquad N = RT\frac{n^2 t}{A^2\pi}\frac{1}{6\pi\eta P}}$$

Diese Verteilung resultiert in gewöhnlicher Weise durch Superposition der Ausg. aller c_0-Stellen). Daher ist es ganz natürlich, dass Westgrens korrigierte Berechnungsweise eben diesen Wert ergibt.

Das Setzen $\xi = \overline{\Delta x}$ ist offenbar falsch, da ja $\overline{\Delta x}$ das Mittel der weiter und weniger weit vorgerückten Verschiebungen bedeutet. ~~Er setzen wir~~ Falls die Wand ein durchlässiges Gitter wäre, welches den Teilchen wieder den Rückweg gestattet, wäre dies ganz richtig. Aber da ein an der Wand einmal angestossenes Teilchen daran kleben bleiben muss, wird so die betreffende Anzahl nicht unterschätzt.

Also ist wirklich: $N = RT\,\frac{4n^2 t}{A^2\pi}\,\frac{1}{6\pi\eta P}$ es wäre also: $N = 4.74 \cdot 10^{22}$! !

Diffusion (von Hg Dampf) lässt sich die Sache so führen, dass die Abhängigkeit von

Molekulardurchm. merkbar wird?

Es muss immer die diffundierende Menge ≦ Gesamtzahl der ~~stoss~~ wegfliegenden Moleküle sein

$$D\frac{\partial c}{\partial x} \leqq c\,\Omega$$

Nun kann D bei Gasen beliebig gross gemacht werden durch Abnahme des Druckes.

(Gesamt oder Partial-?)!

Also z.B. wenn $\Omega = 5.10^4$ $\quad \frac{1}{c}\frac{\partial c}{\partial x} = 1$

so würden bei $p = \frac{1}{100}$ mm Hg schon Divergenzen auftreten

---

Menge welche im Vacuum verdampfen würde:

$$c\,\Omega = \rho\,\Omega$$

z.B. $\vartheta = 18°$ $\quad p = 0.001$ mm Hg

$$\rho = \frac{10^{-3}}{760}\,\frac{200}{28}\,0.0013 = 1.2.10^{-8}$$

152
28
2128

$$\Omega = 48.10^4\sqrt{\frac{28}{200}} = 48.\sqrt{14}\,.10^3$$

$$= 1.4.10^4 \cdot \frac{1}{\sqrt{6\pi}}$$

$$= 3.10^3$$

$$\rho\,\Omega = 3.6.10^{-5}\,\frac{gr}{cm^2\,sek} = 0.12\,\frac{gr}{cm^2.h}$$

Ebenso ist auch

$$\underset{c_v c \rho \lambda}{k}\frac{\partial\vartheta}{\partial x} \leqq c_v.\rho.\Omega.\vartheta$$

$$\frac{\lambda}{\partial x} \leqq 1$$

Könnte vielleicht in sehr dünnen Schichten (Oberfläche von Flammen, gelegentlich erwärmter Draht in strömenden Gasen) merklich werden

Dabei wird aber Temperatursprung stören

Fokker Ann. 43 p. 812 — Anwendung auf mein Beispiel (Seite) 36

$$W(x)\, f(x)\, \tau - W(x)\, \overline{R} + \frac{1}{2} \frac{\partial}{\partial x}\left\{ W(x)\, \overline{R^2} \right\} = 0$$

R = Änderung von x infolge unregelm. äusseren Einwirkung

$f(x)$ = Geschw. mit welcher x abnehmen würde, wenn sich selbst überlassen

$$f(x) = +\alpha\mu x = \beta x$$

$$\overline{R} = 0$$

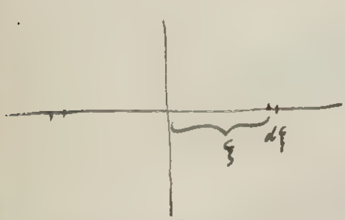
$$\overline{R^2} = 2Dt$$

$$\beta x\, W(x) + D \frac{\partial}{\partial x} W(x) = 0$$

$$\frac{dW}{W} = -\frac{\beta}{D} x\, dx$$

$$W(x) = e^{-\frac{\beta}{2D}x^2} \quad \text{stimmt}$$

Wenn ein Teilchen aus einer Lage ξ – $\xi + d\xi$ ausgeht so ist die Wahrsch. dass es bis zur Zeit t kein einziges mal durch die Null-Ebene gegangen sei:



$$U = \int_0^\infty \cdot\, dx = \frac{1}{2\sqrt{\pi D t}} \int_0^\infty \left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{(x+\xi)^2}{4Dt}} \right] dx$$

$$= \frac{1}{\sqrt{\pi}} \left[\int_{-\frac{\xi}{2\sqrt{Dt}}}^{\infty} e^{-y^2} - \int_{\frac{\xi}{2\sqrt{Dt}}}^{\infty} e^{-y^2}\, dy \right] = \frac{1}{\sqrt{\pi}} \int_{-\frac{\xi}{2\sqrt{Dt}}}^{+\frac{\xi}{2\sqrt{Dt}}} e^{-y^2}\, dy$$

und die Wahrsch., dass es innerhalb der Zeit t – $t + dt$ zum ersten Male durch jene Ebene treten ist:

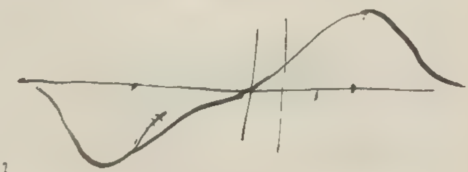
$$-\frac{dU}{dt}\, dt = \frac{2}{\sqrt{\pi}} \frac{\xi}{4\sqrt{Dt^3}} e^{-\frac{\xi^2}{4Dt}}\, dt$$

Maximum: $-\frac{3}{2\sqrt{t^5}} + \frac{\xi^2}{4Dt^2} \frac{1}{\sqrt{t^3}} = 0 \quad \therefore \quad \frac{\xi^2}{4Dt} = \frac{3}{2}$

Wie ist die Verteilung (in einer Säule) in welcher zur Zeit $t=0$ im Punkte $x=0 \dots x+dx$ ~~$\xi \dots \xi+d\xi$~~ die Konzentration 1 herrscht und in der Ebene $\xi = a - ct$ die Konzentration Null?

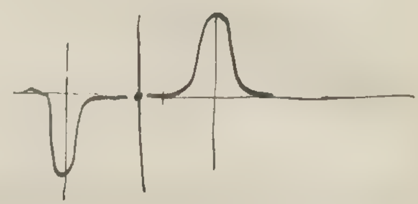
Also die Null-Konzentrations-Ebene soll sich mit konstanter Geschwindigkeit nähern!

Mit der Zeit veränderliche Grenzbedingung!



Darf man setzen: ?

$$\cancel{u = \frac{1}{\sqrt{\pi D t}}\left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{[x+\xi+2(a-ct)]^2}{4Dt}}\right]} \; ?$$



$$u = \frac{1}{\sqrt{\pi D t}}\left[e^{-\frac{x^2}{4Dt}} - e^{-\frac{[2(a-ct)-x]^2}{4Dt}}\right]$$

Dann wäre allerdings $u=0$ für

$$[2(a-ct)]^2 - 2x[2(a-ct)] = 0$$

$$x = a - ct$$

aber die Differentialgleichung $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$

wäre wohl nicht erfüllt! Also ist jene Annahme unbrauchbar!

Es ist das dieselbe Aufgabe wie:

Diffusion verbunden mit konvektiver Strömung (von Geschwindigkeit c)

$$x = \xi - ct$$

$$du = \frac{\partial u}{\partial x}\underbrace{dx}_{= d\xi - c\,dt} + \left(\frac{\partial u}{\partial t}\right)_{x=\text{const}} dt = \frac{\partial u}{\partial \xi} d\xi + \left(\frac{\partial u}{\partial t}\right)_{\xi=\text{const}} dt$$

$$\left(\frac{\partial u}{\partial t}\right)_{\xi=\text{const}} = \left(\frac{\partial u}{\partial t}\right)_{x=\text{const}} - c\frac{\partial u}{\partial x}$$

$$\left(\frac{\partial u}{\partial x}\right) = \frac{\partial u}{\partial \xi}$$

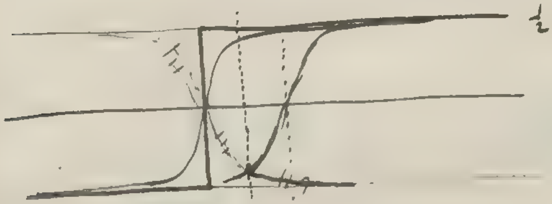
Also Transformation auf neue Koordinaten:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial \xi^2} - c\frac{\partial u}{\partial \xi}$$ (Bewegung der Masse in Richtung positiver $\xi \rightarrow$)

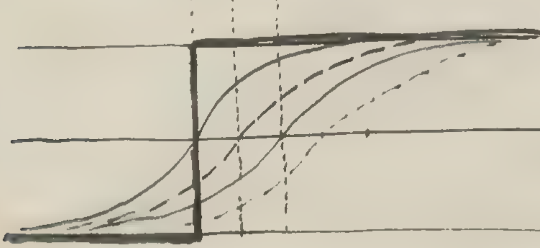
mit Grenzbedingung $u=0$ für $\xi=0$

Partikuläre Lösung: $u = A(ct - \xi)$

$f_0 = f(x) =$



$$f_1 = \frac{1}{2}\left[f(x) + f(x-2c\tau)\right]$$



$$f_2 = \frac{1}{4}\left[f(x) + 2f(x-2c\tau) + f(x-4c\tau)\right]$$

$$f_3 = \frac{1}{2}\left[f_2 + f_2(-2c\tau)\right]$$

$$= \frac{1}{8}\left[f(x) + 3f(x-2c\tau) + 3f(x-4c\tau) + f(x-6c\tau)\right]$$

~~f(x, t+2τ)~~

$$f(x,(n+1)\tau) = \frac{1}{2}\left[f(x, n\tau) + f(x-2c\tau, n\tau)\right]_{t=(n+1)\tau}$$

$$\cancel{f(x,(n+1)\tau) = f(x,n\tau) - c\tau\left.\frac{\partial f}{\partial x}\right|_{n\tau} + c^2\tau^2\left.\frac{\partial^2 f}{\partial x^2}\right|_{n\tau}}$$

$$\cancel{\frac{\partial f}{\partial t} = -c\frac{\partial f}{\partial x} + c^2\tau\frac{\partial^2 f}{\partial x^2}}$$

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} \qquad \xleftarrow{c}$$

$u = 0$

$x = 0$

$$u = e^{-\gamma t} f(x)$$

$$D\frac{\partial^2 f}{\partial x^2} + c\frac{\partial f}{\partial x} + \gamma f = 0$$

$$f = e^{\alpha x}$$

$$D\alpha^2 + c\alpha + \gamma = 0$$

$$\alpha = -\frac{c}{2D} \pm \sqrt{\frac{c^2}{4D^2} - \frac{\gamma}{D}}$$

Supp. $\frac{\gamma}{D} > \frac{c^2}{4D^2}$

sonst ist die Grenzbedingung unerfüllbar

$\cancel{u = e^{-\gamma t} \ldots}$

$$\alpha = -\frac{c}{2D} \pm i\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}$$

$$u = e^{-\gamma t - \frac{c}{2D}x} \sin\left(x\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right)$$

$$u = e^{-\gamma t - \frac{cx}{2D}} \sin\left[x\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right] \qquad \beta^2 = \frac{\gamma}{D} - \frac{c^2}{4D^2}$$

$$\gamma = \beta^2 D + \frac{c^2}{4D}$$

~~Lässt sich auch so darstellen~~

$\cancel{\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial u}{\partial x} + c u\right)}$

$$U = \frac{e^{-\frac{(x-\xi)^2}{4Dt}}}{2\sqrt{D t \pi}}$$

$$u = \frac{e^{-\frac{cx}{2D} - \frac{c^2 t}{4D} - \frac{(x-\xi)^2}{4Dt}}}{2\sqrt{\pi D t}}$$ ist ein partikuläres Integral

zur Zeit $t = 0$ und $x = -\infty$

$u = 0$ überall mit Ausnahme $x = \xi$

zur Zeit $t = \infty$ $u = 0$

Damit $\int u\,dx = 1$ wird, muss man setzen

$$u = \frac{e^{\frac{c(\xi - x)}{2D} - \frac{c^2 t}{4D} - \frac{(x-\xi)^2}{4Dt}}}{2\sqrt{t}\sqrt{\pi D}}$$

$$\int_{-\infty}^{+\infty} u\,dx = \cancel{e^{-\frac{c^2 t}{4D}}} = e^{-\frac{c\xi}{2D}}$$

$= \text{constans}$

$$= \frac{e^{-\frac{(x-\xi+ct)^2}{4Dt}}}{2\sqrt{\pi D t}}$$ das nichts Neues!

Versuch: $u = e^{-\alpha t} v$

$$\frac{\partial u}{\partial t} = -\alpha u + e^{-\alpha t}\frac{\partial v}{\partial t}$$

$$\frac{\partial u}{\partial x} = e^{-\alpha t}\frac{\partial v}{\partial x}$$

$$-\alpha e^{-\alpha t}.v + e^{-\alpha t}\frac{\partial v}{\partial t} = D e^{-\alpha t}\frac{\partial^2 v}{\partial x^2} + c e^{-\alpha t}\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial x^2} + c\frac{\partial v}{\partial x} + \alpha v$$

$$c\frac{\partial v}{\partial x} + \alpha v = 0$$

$$v = f_1(t)\, e^{-\frac{\alpha}{c}x}$$

$$\frac{df}{dt} = D\frac{\alpha^2}{c^2} f$$

Versuch: $u = e^{-\alpha x} v$ $\qquad u = v. e^{-\frac{cx}{2D}}$

$$\frac{\partial u}{\partial x} = -\alpha e^{-\alpha x}.v + e^{-\alpha x}\frac{\partial v}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \alpha^2 e^{-\alpha x}.v - 2\alpha e^{-\alpha x}.\frac{\partial v}{\partial x} + e^{-\alpha x}\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = D\left[\frac{\partial^2 v}{\partial x^2} - 2\alpha\frac{\partial v}{\partial x} + \alpha^2 v\right] + c\left[\frac{\partial v}{\partial x} - \alpha v\right]$$

$$= D\frac{\partial^2 v}{\partial x^2} + \underbrace{(c - 2\alpha D)}_{=0}\frac{\partial v}{\partial x} + \underbrace{(\alpha^2 D - c\alpha)}_{\frac{c^2}{4D} - \frac{c^2}{2D} = -\frac{c^2}{4D}} v$$

$$\alpha = \frac{c}{2D}$$

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial x^2} - \frac{c^2}{4D}v \qquad u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}}.\mathcal{U}$$

$$v = e^{-\frac{c^2}{4D}t}.\mathcal{U} \qquad \frac{\partial \mathcal{U}}{\partial t} = D\frac{\partial^2 \mathcal{U}}{\partial x^2} \qquad = e^{-\frac{c}{4D}(2x+ct)}.\mathcal{U}$$

$$u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \cdot U$$

wenn nun ~~...~~ $U = \frac{1}{\sqrt{\pi}} \int^{\frac{x}{2\sqrt{Dt}}} e^{-y^2} dy$

angenommen wird

so ist für: $t=0, x>0$: $u = \frac{1}{2} e^{-\frac{cx}{2D}}$ | für $x=0$, t beliebig $u=0$

Andrer Versuch:

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + c \frac{\partial u}{\partial x}$$

$$u = \varphi[\overbrace{(ct+x)}^{z}, t]$$

$$\cancel{\frac{\partial u}{\partial t} = \frac{\partial \varphi}{\partial z} c}$$

$$\cancel{\frac{\partial u}{\partial x} = -\frac{\partial \varphi}{\partial z} + \frac{\partial \varphi}{\partial x}}$$

$$\cancel{\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 \varphi}{\partial x^2} - 2\frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \varphi}{\partial z^2}}$$

$$\frac{\partial u}{\partial t} = c \frac{\partial \varphi}{\partial z} + \frac{\partial \varphi}{\partial t}$$

$$\frac{\partial u}{\partial x} = + \frac{\partial \varphi}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 \varphi}{\partial z^2}$$

$$\cancel{c \frac{\partial \varphi}{\partial z}} + \frac{\partial \varphi}{\partial t} = D \frac{\partial^2 \varphi}{\partial z^2} + \cancel{c \frac{\partial \varphi}{\partial z}}$$

Also ist jede Lösung von: $\frac{\partial \varphi}{\partial t} = D \frac{\partial^2 \varphi}{\partial z^2}$

anwendbar, wenn statt z eingesetzt wird; $z = ct + x$

$$\frac{1}{2a\sqrt{t}} \int_{-\infty}^{0} \Phi(z)\, e^{-\frac{(z-ct)^2}{4a^2 t}} dz = -\int_0^{\infty} \frac{e^{-\frac{(z-ct)^2}{4a^2 t}}}{2a\sqrt{t}} dz = -\int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2} d\beta$$

$$u_x = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(x + 2a\beta\sqrt{t})\, e^{-\beta^2} d\beta \qquad \int_{-\infty}^{+\infty}$$

für ~~——~~ $x > \cancel{\ }0$ $\quad u = \text{const} = 1$ <u>für $t = 0$</u>

$x = ct \qquad u = 0 \qquad\qquad x = 0 \quad u = 0$

$$0 = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(\underbrace{ct + 2a\beta\sqrt{t}}_{z})\, e^{-\beta^2} d\beta \qquad \int_{-\infty}^{+\infty} \Phi(\underbrace{2a\beta\sqrt{t}}_{})\, e^{-\beta^2} d\beta = 0$$

$$\Phi(+x) \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\beta^2} d\beta = 1 \qquad \therefore \Phi(+x) = 1$$

$$ct + 2a\beta\sqrt{t} = z$$

$$\beta = \frac{z - ct}{2a\sqrt{t}}$$

$$0 = \frac{1}{\sqrt{\pi}} \int_{\beta=-\infty}^{\beta = -\frac{c}{2a}\sqrt{t}} \Phi(-z)\, e^{-\beta^2} d\beta + \frac{1}{\sqrt{\pi}} \int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2} d\beta$$

$$\int_{\beta=-\infty}^{\beta=-\frac{c}{2a}\sqrt{t}} \Phi(\underbrace{ct + 2a\beta\sqrt{t}}_{<0})\, e^{-\beta^2} d\beta = -\int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2} d\beta \qquad \int_{z=-\infty}^{z=0} \Phi(z)\, e^{-\left(\frac{z-ct}{2a\sqrt{t}}\right)^2} \frac{dz}{2a\sqrt{t}} = -\int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2} d\beta$$

$$u_x = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\beta = -\frac{x}{2a\sqrt{t}}} \Phi_{(z)}^{<0}\, e^{-\beta^2} d\beta + \int_{z=0}^{\infty} e^{-\beta^2} d\beta \qquad = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{0} \Phi(z)\, e^{-\left(\frac{z-x}{2a\sqrt{t}}\right)^2} \frac{dz}{2a\sqrt{t}} + \int_{-\frac{x}{2a\sqrt{t}}}^{\infty} e^{-\beta^2} d\beta$$

$z = 0 \;\downarrow\; \beta = -\dfrac{x}{2a\sqrt{t}}$

$$u_x = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(x + 2a\beta\sqrt{t})\, e^{-\beta^2} d\beta = \int_{-\infty}^{-\frac{x}{2a\sqrt{t}}} + \int_{-\frac{x}{2a\sqrt{t}}}^{\infty}$$

Annahme:

$x > 0 \qquad t = 0 \qquad u = 1$

$x = 0 \qquad t = t \qquad u = 0$

$$\int_{-\infty}^{+\infty} \Phi(2a\beta\sqrt{t})\, e^{-\beta^2} d\beta = 0 = \int_{-\infty}^{0} + \int_{0}^{\infty} = \frac{\sqrt{\pi}}{2} + \int_{-\infty}^{0} = \frac{\sqrt{\pi}}{2} + \int_{0}^{\infty} \Phi(-2a\beta\sqrt{t})\, e^{-\beta^2} d\beta$$

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(+x)\, e^{-\beta^2} d\beta = 1 = \Phi(+x) \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\beta^2} d\beta = \Phi(x) = 1$$

$$\frac{1}{2a\sqrt{t}} \int_0^\infty \Phi(-z)\, e^{-\frac{z^2}{4a^2t}} dz = +\frac{\sqrt{\pi}}{2} \longrightarrow$$

Czy z tego z konieczności wynika że $\Phi(-z) = 1$?

$$u_x = \frac{1}{\sqrt{\pi}} \int_0^\infty \Phi(-z)\, e^{-\frac{(z-x)^2}{4a^2t}} \frac{dz}{\cdots} - \frac{1}{\sqrt{\pi}} \int_0^\infty \Phi(z)\, e^{-\frac{(z+x)^2}{4a^2t}} \frac{dz}{\cdots}$$

$$\cancel{\frac{1}{2a\sqrt{\pi t}} \int_0^\infty \left[e^{-\frac{(\alpha-x)^2}{4a^2t}} - e^{-\frac{(\alpha+x)^2}{4a^2t}} \right] d\alpha = \frac{1}{2a\sqrt{\pi t}} \int_{-\frac{x}{2a\sqrt{t}}}^{\frac{x}{2a\sqrt{t}}} e^{-\beta^2} d\beta}$$

$$\alpha \int_0^\infty \Phi(-z)\, e^{-\alpha^2 z^2} dz = \frac{\sqrt{\pi}}{2}$$

$$\int_0^\infty \Phi(\gamma z)\, e^{-z^2} dz = \frac{\sqrt{\pi}}{2}$$ dla jakiegokolwiek γ

czy to koniecznie pociąga za sobą $\Phi = 1$?

→ ~~Tak tu prosze w kartce równaniu ani rodzaj ze stosujemy się~~

$$\int_0^\infty \Phi^{(n)}_{y(x)}\, z^n e^{-z^2} dz \neq 0 \qquad \text{przy czym } \Phi^{(n)}_{y} = \int e^{i\alpha(x-\xi)} d\xi = \frac{e^{i\alpha(x-\xi)}}{i\alpha}\Big|_0^\infty = \frac{e^{i\alpha x}}{i\alpha}$$

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x}$$

$$u = e^{i\alpha x - \gamma t} \qquad u = e^{-i\alpha x - \gamma t}$$

$$-D\alpha^2 + c\alpha i + \gamma = 0$$

$$\gamma = D\alpha^2 - c\alpha i \qquad \gamma = D\alpha^2 + c\alpha i$$

$$u = \frac{e^{i\alpha x + c\alpha i t - D\alpha^2 t} - e^{-i\alpha x - c\alpha i t - D\alpha^2 t}}{2i}$$

$$= e^{-D\alpha^2 t}\, \cancel{\cos(\alpha x + c\alpha t) + i}\, \sin(\alpha x + c\alpha t) +$$

$$= e^{-D\alpha^2 t} \sin_{\cos} \alpha(x + ct)$$

Ale w takim razie nie było by $u_{x=0} = 0$?

W każdym razie warunki: $= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\alpha \int_{-\infty}^{+\infty}$

$$u_{t=0} = \int_{-\infty}^{+\infty}\left[f(\alpha)\, e^{i\alpha x} d\alpha + g(\alpha)\, e^{-i\alpha x} d\alpha\right] = 1 \quad \big(\big(x > 0$$

$$u_{x=0} = \int_{-\infty}^{+\infty} e^{-D\alpha^2 t}\left[f(\alpha)\, e^{ic\alpha t} d\alpha + g(\alpha)\, e^{-ic\alpha t} d\alpha\right] = 0$$

z tego:

$$u_{xt} = \int_{-\infty}^{+\infty}\left[f(\alpha)\, e^{i\alpha(x+ct)} + g(\alpha)\, e^{-i\alpha(x+ct)}\right] e^{-D\alpha^2 t} d\alpha$$

Trzeba współczynniki z równań pierwszego wyznaczyć

Wege ~~[illegible]~~

$$\cancel{\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2}}$$

$$u = \int_{\gamma = -\infty = \frac{c^2}{4D}}^{+\infty} e^{-\gamma t - \frac{cx}{2D}} \sin\left[x \sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right] f(\gamma)\, d\gamma$$

erfüllt die Bedingung $x=0$ $u=0$ für beliebige t

für $t=0$ soll sein:

$$u = \underbrace{\int_{-\infty}^{+\infty} e^{-\frac{cx}{2D}} \sin\left(x \sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right) f(\gamma)\, d\gamma}_{} = 1 \qquad \text{für } x > 0$$

$$= e^{-\frac{cx}{2D}} \int_{-\infty}^{+\infty} f(\gamma) \sin\left(x \overbrace{\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}}^{\beta}\right) d\gamma = 1$$

$$f(x) = \frac{1}{\pi} \int_0^\infty d\alpha \int_{-\infty}^{+\infty} f(\gamma) \cos\alpha(\gamma - x)\, d\gamma$$

Umgekehrt könnte man auch setzen:

$$\beta^2 = \frac{\gamma}{D} - \frac{c^2}{4D^2}$$

$$\gamma = \beta^2 D + \frac{c^2}{4D}$$

$$u = \int_{\beta=0}^{\infty} e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t - \frac{cx}{2D}} \sin\beta x \cdot f(\beta)\, d\beta$$

$$u_0 = 1 = e^{-\frac{cx}{2D}} \int_0^\infty f(\beta) \sin\beta x\, d\beta = \Phi(x)$$

$$f(y) = \frac{2}{\pi}\int_0^\infty \sin xy \, dx \int_0^\infty f(\beta) \sin \beta x \, d\beta$$

$$f(y) = e^{-ny} \qquad \int_0^\infty e^{-n\beta} \sin\beta x \, d\beta = \frac{x}{x^2+n^2}$$

Gdyby było c ze znakiem przeciwnym

$$1 = e^{\frac{cx}{2D}} \int_0^\infty f(\beta) \sin\beta x \, d\beta$$

można by przyjąć także $f(\beta) = \dfrac{\beta}{\beta^2 + \left(\frac{c}{2D}\right)^2}$

Więc byłoby:

$$u = \int_{\beta=0}^\infty e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t + \frac{cx}{2D}} \frac{\beta}{\beta^2 + \frac{c^2}{4D^2}} \cdot \sin\beta x \cdot d\beta$$

Spełnia równanie

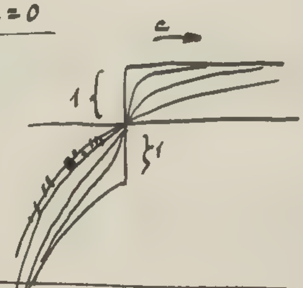
$$\frac{\partial u}{\partial t} - D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} = 0$$

oraz warunki $\quad t=0 \quad x>0 \quad u=1$

$x=0 \quad u=0$

Dla $x<0$ będzie $u = -e^{\frac{cx}{D}}$

dla $t=0$



Czy jest ciągłość dla $x=0$ między jednym a drugim rozwiązaniem? Trzeba by stwierdzić $\left(\frac{\partial u}{\partial x}\right)_- = \left(\frac{\partial u}{\partial x}\right)_+ \Big|_{x=0}$?

$$f(x) = \frac{1}{\pi}\int_0^\infty d\alpha \int_{-\infty}^{+\infty} f(y)\left[\cos\alpha y \cos\alpha x + \sin\alpha y \sin\alpha x\right] dy$$

jeśli $f(-y) = -f(y)$

$$f(x) = \frac{2}{\pi}\int_0^\infty d\alpha \int_0^{+\infty} f(y)\sin\alpha y \sin\alpha x \, dy \qquad = \sqrt{\varepsilon}\cdot\frac{e^{-\frac{(x-x_0)^2}{\varepsilon}}}{\sqrt{\pi}}\cdot e^{-\frac{cx}{2D}} = \Phi(x)$$

$$\int_0^{+\infty} e^{-\frac{(y-x_0)^2}{\varepsilon}+\frac{cy}{2D}} \sin\alpha y \, dy =$$

$$f(\beta) = \frac{2}{\pi}\int_0^\infty e^{-\frac{(y-x_0)^2}{\varepsilon}+\frac{cy}{2D}} \sin\beta y \, dy$$

$$u = \frac{2}{\pi}\sqrt{\frac{\varepsilon}{\pi}}\int_0^\infty\int_0^\infty e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t - \frac{cx}{2D} - \frac{(y-x_0)^2}{\varepsilon} + \frac{cy}{2D}} \cdot \sin\beta x \sin\beta y \, dy \, d\beta$$

~~dla~~ $t=0$: $u = \frac{2}{\pi}\sqrt{\frac{\varepsilon}{\pi}}\iint e^{-\frac{cx}{2D}}\, e^{-\frac{(y-x_0)^2}{\varepsilon}+\frac{cy}{2D}} \sin\beta y \, dy \cdot \sin\beta x \, d\beta$

$$= \cancel{e^{-\frac{cx}{2D}}\sqrt{\frac{\varepsilon}{\pi}}\, e^{\frac{cx}{2D}}\, e^{-\frac{(x-x_0)^2}{\varepsilon}}}$$

$\cancel{\int e^{-Ay^2}}$ $$\int_0^\infty e^{-A(y-b)^2}\sin ay \, dy = ?$$

$$\int_{-b}^\infty e^{-Az^2}\sin a(z+b)\, dz = \cos ab \int_{-b}^\infty e^{-Az^2}\sin az \, dz$$

$$+ \sin ab \int_{-b}^\infty e^{-Az^2}\cos az \, dz$$

W każdym razie

$$\lim_{t=0} \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-\frac{(y-x_0)^2}{2} + \frac{cy}{2D}} \sin\beta y \, dy = \sin\beta x_0$$

$$u = \frac{2}{\pi} \int_0^\infty e^{-\left(\beta^2 D + \frac{c^2}{4D}\right) t - \frac{cx}{2D}} \sin\beta x_0 \sin\beta x \, d\beta$$

$t=0$:

$$u = e^{-\frac{cx}{2D}} \frac{2}{\pi} \int_0^\infty \sin\beta x \sin\beta x_0 \, d\beta$$

$x \lessgtr x_0 \quad = \,?$

$x = x_0 \quad \int_0^\infty (\sin\beta x_0)^2 d\beta$

$$\omega = \int_0^\infty e^{-\alpha^2} \sin\alpha x \, d\alpha$$

$$\frac{\partial\omega}{\partial x} = \int_0^\infty \alpha e^{-\alpha^2} \cos\alpha x \, d\alpha$$

$$\left(e^{-\alpha^2} \cos\alpha x\right)\Big|_0^\infty = -2\int_0^\infty \alpha e^{-\alpha^2} \cos\alpha x \, d\alpha - x\int_0^\infty e^{-\alpha^2} \sin\alpha x \, d\alpha$$

$$-1 = -2\frac{d\omega}{dx} - x\omega$$

$$2\frac{d\omega}{dx} + \omega x = 1 \quad \Big| \; y$$

$$2\frac{dy}{dx} + yx = 0 \quad \Big| \; \omega$$

$$2\frac{\left(y\frac{d\omega}{dx} - \omega\frac{dy}{dx}\right)}{y^2} = \frac{1}{y}$$

$$2\frac{d}{dx}\left(\frac{\omega}{y}\right) = \frac{1}{y}$$

$$\omega = \frac{y}{2}\int \frac{1}{y} dx = \frac{1}{2} e^{-\frac{x^2}{4}} \left[\int^x e^{\frac{x^2}{4}} dx + C\right]$$

$$\int_0^\infty e^{-\alpha^2} \sin\alpha x \, d\alpha = \frac{e^{-\frac{x^2}{4}}}{2} \int_0^x e^{\frac{y^2}{4}} dy$$

$$-\frac{x}{2} e^{-\frac{x^2}{4}} \int$$

Versuch

$$u = A\frac{2}{\pi} \int e^{-\frac{c^2}{4D}t - \frac{cx}{2D}} \int_0^\infty e^{-\beta^2 D t} \underbrace{\sin\beta x \sin\beta x_0}_{\left[-\cos\beta(x+x_0) + \cos\beta(x-x_0)\right]} \, d\beta$$

$$= A\frac{2}{\pi} \underbrace{e^{-\frac{c^2 t}{4D} - \frac{cx}{2D}}}_{U} \cdot \underbrace{\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \left\{ e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}} \right\}}_{V}$$

$$\frac{\partial u}{\partial t} - D\frac{\partial^2 u}{\partial x^2} - c\frac{\partial u}{\partial x} = U\Big(\underbrace{\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}}_{=0} - c\frac{\partial}{\partial x}\Big)V + V\Big(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2} - c\frac{\partial}{\partial x}\Big)U - 2D\frac{\partial U}{\partial x}\frac{\partial V}{\partial x}$$

$$= +c\,\frac{2(x-x_0)}{4Dt} V_1 - c\,\frac{2(x+x_0)}{4Dt} V_2 + \cancel{(V_1 - V_2)\left(-\frac{c^2}{4D} - \frac{c^2}{4D} + \frac{c^2}{2D}\right)} + 2\cancel{D}\,\frac{c}{2\cancel{D}}\left[-\frac{(x-x_0)}{2Dt}V_1 + \frac{(x+x_0)}{2Dt}V_2\right]$$

$$= 0$$

Also Differentialgleichung ist erfüllt

Anfangsbedingungen:

für $x = 0$ $\quad u = 0$ bei beliebigem t

für $t = 0$ $\quad u = 0$ für alle x, mit Ausnahme $x = x_0$

daselbst ist $\int_0^\infty u\,dx = 2\, e^{-\frac{cx_0}{2D}} \cdot A = 1$

$$A = \frac{1}{2} e^{\frac{cx_0}{2D}}$$

Somit ist:

$$\lim_{x_0=\infty} u = e^{-\frac{x^2}{4Dt}}\left[e^{\frac{xx_0}{2Dt}} - e^{-\frac{xx_0}{2Dt}}\right] =$$

$$u = \frac{e^{-\frac{c^2t}{4D} - \frac{c(x-x_0)}{2D}}}{2\sqrt{\pi D t}} \left\{ e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}} \right\}$$

unter Bedingung: $= e^{-\frac{[ct+x-x_0]^2}{4Dt}}$

$x+x_0 \gg x-x_0$

also wenn x und x_0 [illegible]

$$= \frac{1}{2\sqrt{\pi D t}} \left\{ e^{-\frac{(x-x_0+ct)^2}{4Dt}} - e^{-\frac{(x+x_0)^2 + 2ct(x-x_0) + c^2t^2}{4Dt}} \right\}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \frac{e^{-\frac{c^2t}{4D} + \frac{cx_0}{2D}}}{2\sqrt{\pi D t}} \underbrace{\left[-\frac{2(x-x_0)}{4Dt} + \frac{2(x+x_0)}{4Dt} \right]_{x=0}}_{\frac{x_0}{Dt}} e^{-\frac{x_0^2}{4Dt}} = \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi (Dt)^3}} \cdot x_0$$

Wahrsch. dass ein Teilchen (von x_0 ausgehend) „zum ersten Mal" zwischen t und $t+dt$ durch die Null Ebene durchtritt:

Vielleicht aber $D \left.\frac{\partial u}{\partial x}\right|_0 + c\, u|_0 =$ ist identisch damit da $u_0 = 0$

$$D\left(\frac{\partial u}{\partial x}\right)_{x=0} dt = x_0 \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi D t^3}}\, dt$$

Dagegen Fletcher: $b = x_0$ $\quad h = \frac{1}{4D}$

$$\frac{1}{2}\sqrt{\frac{1}{4\pi D}}\,\frac{1}{\sqrt{t^3}}(x_0+ct)\, e^{-\frac{(x_0-ct)^2}{4Dt}}\, dt = \frac{(x_0+ct)\, e^{-\frac{(x_0-ct)^2}{4Dt}}}{4\sqrt{\pi D t^3}}$$

Also ist bei raschem Fall, ~~wo~~ wo annähernd $x_0 = ct$, das Resultat dasselbe, dagegen im Grenzfalle verschwindend langsamen Fallens (im Vergl. zur Br. B.) Resultat $\frac{1}{2}$ mal.

Durchschnittliche Fallzeit:

$$\frac{x_0}{2\sqrt{D}\,c} - \frac{t}{2\sqrt{D}} = \frac{z\sqrt{t}}{c}$$

$$T = x_0 \int_0^{\infty} \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\, dt$$

$$\frac{x_0 - ct}{2\sqrt{Dt}} = z \qquad t + \frac{2z}{c}\sqrt{Dt} = \frac{x_0}{c}$$

$$\frac{x_0}{2c\sqrt{D}} = \frac{t}{2\sqrt{D}} + \frac{z}{c}\sqrt{t}$$

$$\cancel{\sqrt{t} = -\frac{z}{2c} \pm \sqrt{\frac{x_0}{2c\sqrt{D}} + \frac{z^2}{4c^2}}}$$

$$\sqrt{t} = -\frac{z\sqrt{D}}{c} \pm \sqrt{\frac{x_0}{c} + \frac{z^2 D}{c^2}}$$

$$\frac{1}{2}\frac{dt}{\sqrt{t}} = dz\left\{-\frac{1}{2c} \pm \frac{z}{4c^2}\,\frac{1}{\sqrt{\frac{z^2}{4c^2} + \frac{x_0}{2c\sqrt{D}}}}\right\}$$

$$T = \frac{-x_0}{2c\sqrt{\pi D}}\left\{\int_{z=+\infty}^{z=0}\left[1 + \frac{z}{2c}\,\frac{1}{\sqrt{\frac{z^2}{4c^2}+\frac{x_0}{2c\sqrt{D}}}}\right]e^{-z^2}dz + \int_{z=0}^{z=-\infty}\left[1 - \frac{z}{2c}\,\frac{1}{\sqrt{\frac{z^2}{4c^2}+\frac{x_0}{2c\sqrt{D}}}}\right]e^{-z^2}dz\right\}$$

$$= \frac{x_0}{2c\sqrt{\pi D}}\left\{\underbrace{\int_{-\infty}^{+\infty} e^{-z^2}dz}_{\sqrt{\pi}} \underbrace{- \int_{-\infty}^{0} + \int_0^{+\infty} \frac{z\, e^{-z^2}}{2c\sqrt{\frac{z^2}{4c^2}+\frac{x_0}{2c\sqrt{D}}}}\,dz}_{= 2\int_0^{+\infty} \frac{z\,e^{-z^2}}{\sqrt{z^2 + \frac{2cx_0}{\sqrt{D}}}}\,dz}\right\}$$

$$= \int_0^{\infty} \frac{e^{-y}\,dy}{\sqrt{y + \frac{2cx_0}{\sqrt{D}}}} = e^{\frac{2cx_0}{\sqrt{D}}}\int_{\frac{2cx_0}{\sqrt{D}}}^{\infty} \frac{e^{-z}dz}{\sqrt{z}} = 2e^{\frac{2cx_0}{\sqrt{D}}}\int_{x=\sqrt{\frac{2cx_0}{\sqrt{D}}}}^{\infty} e^{-x^2}dx$$

$$\frac{1}{2}\frac{dt}{\sqrt{t}} = \left\{ -\frac{\sqrt{D}}{c} \pm \frac{\frac{z D}{c^2}}{\sqrt{\frac{x_0}{c} + \frac{z^2 D}{c^2}}} \right\} dz$$

?

$$T = \frac{+x_0}{c\sqrt{\pi}} \left\{ \int_{-\infty}^{+\infty} e^{-z^2} dz + 2\int_0^\infty \frac{z\, e^{-z^2} dz}{\sqrt{z^2 + \frac{x_0 c}{D}}} \right\}$$

$$T = \frac{x_0}{c}\left\{ 1 + \frac{1}{\sqrt{\pi}}\, e^{\frac{c x_0}{D}} \underbrace{\int_{x=\sqrt{\frac{c x_0}{D}}}^{\infty} e^{-x^2} dx}_{} \right\} \quad ?$$

$$= e^{-x^2}\left[\frac{1}{2x} - \frac{2}{(2x)^3} \cdots\right] = \frac{e^{-\frac{c x_0}{D}}}{2\sqrt{\frac{c x_0}{D}}}\left[1 - \frac{D}{2 c x_0}\right]$$

?

$$= \frac{x_0}{c}\left\{ 1 + \cancel{\frac{1}{2\sqrt{\pi}}\sqrt{\frac{D}{c x_0}}\left[1 - \cdots\right]} \right\}$$

~~Also für kleines~~ $\frac{c x_0}{D}$: $T = \frac{3 x_0}{2c}$

~~großes~~ : $T = \frac{x_0}{c}$

für kleines $\frac{c x_0}{D}$: $T = \infty$!

großes $T = \frac{x_0}{c}$

Dagegen bei Fletcher: $T = \frac{x_0}{c} + \frac{D}{c^2} = \frac{x_0}{c}\left\{1 + \frac{D}{c x_0}\right\}$

$$\frac{T D}{x_0^2} = \varepsilon + \frac{1}{2\sqrt{\pi}}\sqrt{\varepsilon}^3$$

Angenähert für kleine ε:

$$\varepsilon = \frac{T D}{x_0^2}\left[1 - \frac{1}{2\sqrt{\pi}}\sqrt{\frac{T D}{x_0^2}}\right] \quad ?$$

Somit: $c = \frac{D}{\varepsilon x_0} = \frac{x_0}{T}\cdot\frac{1}{1 - \frac{1}{2 x_0}\sqrt{\frac{T D}{\pi}}}$

$$\cancel{c = \frac{x_0}{T}\left(1 + \frac{1}{2 x_0}\sqrt{\frac{T D}{\pi}}\right) = \frac{x_0}{T} + \frac{1}{2}\sqrt{\frac{D}{T\pi}}}$$

während Fletcher: $c = \frac{x_0}{T}\left(1 + \frac{T D}{x_0^2}\right)$

Im extremen Fall $c=0$:

$$I = \frac{\xi}{\sqrt{\pi}} \int_{t=0}^{\infty} \frac{e^{-\frac{\xi^2}{4Dt}}}{2\sqrt{Dt}}\, dt$$

$$\frac{\xi^2}{4Dt} = y^2$$

$$\sqrt{\frac{t}{D}} = \frac{\xi}{2D y}$$

$$\frac{dt}{2\sqrt{Dt}} = \frac{dy}{2D} \cdot \frac{\xi}{y^2}$$

$$= -\frac{\xi^2}{\sqrt{\pi}} \frac{1}{2D} \int_{\infty}^{0} \frac{e^{-y^2}\, dy}{y^2} = \infty$$

Wien und Ehrenhaft setzen:

$$\overline{\lambda^2} = \frac{\overline{\Lambda^2}}{t} = \frac{1}{n} \sum \frac{\Lambda^2}{t} = \frac{1}{n} \sum \frac{(l - v_m t)^2}{t} = \frac{l^2}{t_m^2} \left[\overline{\frac{(t_m - t)^2}{t}}\right] =$$

$$= \frac{l^2}{n} \sum \frac{\left(1 - \frac{t}{t_m}\right)^2}{t} = l^2 \left[\overline{\left(\frac{1}{t}\right)} - \frac{2}{t^m} + \frac{\bar{t}}{(t^m)^2}\right]$$

$$= l^2 \left[\overline{\left(\frac{1}{t}\right)} - \frac{1}{t_m}\right] \qquad (\bar{t} = t_m$$

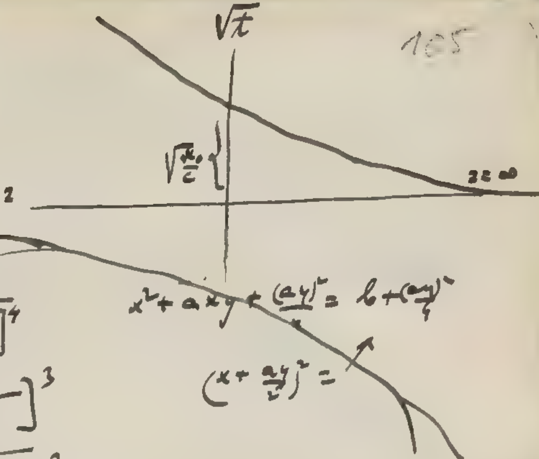
$$\overline{\left(\frac{1}{t}\right)} = x_0 \int_0^{\infty} \frac{1}{t^2} \frac{e^{-\frac{(x_0 - ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\, dt$$

$$\sqrt{t} = -\frac{z\sqrt{D}}{c} \pm \sqrt{\frac{x_0}{c} + \frac{z^2 D}{c^2}}$$

$$= \frac{\sqrt{D}}{c}\left[-z \pm \sqrt{z^2 + \frac{c x_0}{D}}\right]$$

$$\frac{1}{2} \frac{dt}{\sqrt{t}} = \frac{\sqrt{D}}{c}\left[-1 \pm \frac{z}{\sqrt{}}\right] dz = \frac{\mp\sqrt{D}}{c} \left(\frac{-z \pm \sqrt{}}{\sqrt{}}\right) dz$$

$$\overline{\left(\frac{1}{t}\right)} = \frac{x_0}{\sqrt{\pi D}} \left(\int_{\infty}^{0} + \int_{0}^{-\infty} \right)$$



$$x^2 + axy + \frac{(ay)^2}{4} = b + \frac{(ay)^2}{4}$$

$$\left(x + \frac{ay}{2}\right)^2 =$$

$$= \frac{x_0}{\sqrt{\pi D}} \frac{\sqrt{D}}{c} \int_{-\infty}^{+\infty} \frac{z - \sqrt{z^2 + \frac{cx_0}{D}} \quad e^{-z^2} dz}{\sqrt{z^2 + \frac{cx_0}{D}} \left(\frac{\sqrt{D}}{c}\right)^4 \left[z - \sqrt{\quad}\right]^4}$$

$$= -\frac{x_0 c^3}{\cancel{2}\sqrt{\pi} D^2} \int_{-\infty}^{+\infty} \frac{e^{-z^2} dz}{\sqrt{z^2 + \frac{cx_0}{D}} \left[z - \sqrt{z^2 + \frac{cx_0}{D}}\right]^3} \frac{\left[z + \sqrt{\quad}\right]^3}{\left[z + \sqrt{\quad}\right]^3}$$

$$= \frac{x_0 c^3}{D^2 \sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{e^{-z^2} dz \left[\cancel{z^3} + 3z^2 \sqrt{\quad} + \cancel{3z\left(z^2 + \frac{cx_0}{D}\right)} + \sqrt{\quad}^3\right]}{\left(\frac{cx_0}{D}\right)^3 \sqrt{z^2 + \frac{cx_0}{D}}}$$

$$= \frac{x_0 c^3}{D^2 \sqrt{\pi}} \int \frac{e^{-z^2} dz}{\left(\frac{cx_0}{D}\right)^3} \left(3z^2 + z^2 + \frac{cx_0}{D}\right) = \frac{D}{x_0^2 \sqrt{\pi}} \int_{-\infty}^{+\infty} \left[4z^2 + \frac{cx_0}{D}\right] e^{-z^2} dz$$

$$= \frac{c}{x_0} + \frac{4D}{x_0^2 \sqrt{\pi}} \int_{-\infty}^{+\infty} \underbrace{z^2 e^{-z^2} dz}_{\frac{\sqrt{\pi}}{2}}$$

$$= \frac{c}{x_0} + \frac{2D}{x_0^2}$$

$$\overline{\left(\frac{1}{t}\right)} - \frac{1}{t_m} = \frac{2D}{x_0^2}$$

$\overline{\lambda^2} = \cancel{t}\, 2D$! Also wäre Weiss' (Berechnungs-)Methode ganz richtig!

$$u = \int e^{-\frac{cx}{2D}} \begin{Bmatrix} \sin\beta x \\ \cos\beta x \end{Bmatrix} \cdot e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t} f(\beta)\, d\beta$$

$$\bar{t}^n = -\frac{b}{c\sqrt{\pi}} \int \frac{e^{-z^2} dz}{\sqrt{z^2 + \frac{cb}{D}}} \left[-z + \sqrt{z^2 + \frac{cb}{D}}\right]^{-1+2n} \cdot \frac{D^{n-1}}{c^{2n-2}}$$

$$= +\frac{b D^{n-1}}{c^{2n-1}\sqrt{\pi}} \int \left(e^{-z^2} dz\right) \sqrt{z^2 + \frac{cb}{D}}^{\,2n-2}$$

Warunek dla $x=0$:

$$D\frac{\partial u}{\partial x} + cu = 0$$

$$u = e^{-\frac{cx}{2D}} \sin(\beta x + \varepsilon)$$

$$\frac{\partial u}{\partial x} = e^{-\frac{cx}{2D}} \beta \cos(\beta x + \varepsilon) - \frac{c}{2D} u$$

$$D\beta \cos\varepsilon + \frac{c}{2} \sin\varepsilon = 0$$

$$\operatorname{tg}\varepsilon = -\frac{2D\beta}{c}$$

$$u = e^{-\frac{cx}{2D}} \int \frac{\sin\beta x - \cos\beta x \cdot \frac{2D\beta}{c}}{\sqrt{1 + \frac{4D^2\beta^2}{c^2}}} e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t} f(\beta)\, d\beta$$

$$= e^{-\frac{cx}{2D} - \frac{c^2}{4D}t} \int_0^\infty e^{-\beta^2 Dt} \left[\sin\beta x - \frac{2D\beta}{c} \cos\beta x\right] f(\beta)\, d\beta$$

$$\cancel{\frac{d}{dz}\left[\frac{e^{-z^2}}{\sqrt{z^2+\alpha}}\right] = -2\frac{z e^{-z^2}}{\sqrt{z^2+\alpha}} - \frac{z e^{-z^2}}{\sqrt{z^2+\alpha}^{\,3}}}$$

$$e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t} \sin\left(\beta x - \operatorname{arctg}\frac{2D\beta}{c}\right) f(\beta)\, d\beta$$

$$\cancel{\int_\alpha^\infty \frac{e^{-x} dx}{\sqrt{x+\alpha}} = -\frac{e^{-x}}{\sqrt{x+\alpha}} + \frac{1}{2}\int \frac{e^{-x}}{\sqrt{x+\alpha}^{\,3}}}$$

$$\cancel{= \frac{x e^{-x}}{\sqrt{x+\alpha}} + \int x \frac{e^{-x}}{\sqrt{x+\alpha}} dx - \frac{1}{2}\int \frac{x e^{-x}}{\sqrt{\;}^{\,3}} dx}$$

$$y_2^2 = \left[\frac{x_0 - ct_2}{2\sqrt{Dt_2}}\right]^2 + \frac{cx_0}{D}$$

$$\frac{dt}{\sqrt{t}} = 2\frac{\sqrt{D}}{c}\left[-1 + \frac{z}{\sqrt{z^2+\frac{cx_0}{D}}}\right]dz \qquad \bar{t} = x_0\int_0^\infty \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\,dt$$

$$\sqrt{t} = \frac{\sqrt{D}}{c}\left[-z + \sqrt{z^2+\frac{cx_0}{D}}\right]$$

$$\bar{t} = \frac{x_0}{c\sqrt{\pi}}\int_{\infty}^{-\infty} e^{-z^2}\left[-1+\frac{z}{\sqrt{z^2+\frac{cx_0}{D}}}\right]dz = \frac{x_0}{c\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-z^2}dz = \frac{x_0}{c}$$

Probe ob $W = 1$:

$$\bar{W} = \frac{x_0 c}{D\sqrt{\pi}}\int_{\infty}^{-\infty} e^{-z^2}\,\frac{-1+\frac{z}{\sqrt{z^2+\frac{cx_0}{D}}}}{\left[-z+\sqrt{\quad}\right]^2}\,dz = \frac{x_0 c}{D\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{e^{-z^2}}{\sqrt{z^2+\frac{cx_0}{D}}}\;\frac{z+\sqrt{\quad}}{\frac{cx_0}{D}}\,dz$$

$$= \frac{1}{\sqrt{\pi}}\left\{\int_{-\infty}^{+\infty} e^{-z^2} + \cancel{\int_{-\infty}^{+\infty}\frac{z\,e^{-z^2}}{\sqrt{z^2+\dots}}}\,dz\right\} = 1 \quad \text{stimmt}$$

$$W\Big|_{t_1}^{t_2} = \frac{1}{\sqrt{\pi}}\int_{\frac{x_0-ct_2}{2\sqrt{Dt_2}}}^{\frac{x_0-ct_1}{2\sqrt{Dt_1}}}\frac{e^{-z^2}}{\sqrt{z^2+\frac{cx_0}{D}}}\left[z+\sqrt{\quad}\right]dz$$

$$= \frac{1}{\sqrt{\pi}}\int_{z_2}^{z_1} e^{-z^2}dz + \frac{1}{\sqrt{\pi}}\int\frac{z\,e^{-z^2}}{\sqrt{z^2+\frac{cx_0}{D}}}\,dz$$

$$\text{Fletcher: } \frac{1}{2\sqrt{\pi}}\int\frac{e^{-z^2}}{\sqrt{\quad}}\left[z+\sqrt{\quad}\right]\left\{1+\frac{\cancel{c}}{x_0}\frac{D}{\cancel{c}}\left[-z+\sqrt{\quad}\right]^2\right\}$$

$$= \frac{1}{2\sqrt{\pi}}\int e^{-z^2}\left[\frac{z}{\sqrt{\quad}}+1\right] +$$

$$+ \frac{1}{2\sqrt{\pi}}\,\cancel{\frac{\sqrt{\quad}}{cx_0}}\int\frac{e^{-z^2}}{\sqrt{\quad}}\,\cancel{\frac{cx_0}{D}}\left[-z+\sqrt{\quad}\right] = \frac{1}{\sqrt{\pi}}\int e^{-z^2}dz$$

wie Fletcher angibt (4.91)

$$\frac{cx_0}{D} = 4\,b\,V\,h \qquad y^2 = z^2+\frac{cx_0}{D}$$

$$\frac{1}{\sqrt{\pi}}\int_{z_2}^{z_1}\frac{z\,e^{-z^2}dz}{\underbrace{\sqrt{z^2+\frac{cx_0}{D}}}_{y^2}} = \frac{1}{\sqrt{\pi}}\int_{y_2}^{y_1}\frac{\cancel{y}\,e^{-y^2}dy}{\cancel{y}}\cdot e^{+\frac{cx_0}{D}} = \frac{e^{\frac{cx_0}{D}}}{\sqrt{\pi}}\int_{\sqrt{z_2^2+\frac{cx_0}{D}}}^{\sqrt{z_1^2+\frac{cx_0}{D}}} e^{-y^2}dy$$

$$\bar{W}\Big|_{t=0}^{t_1} = \frac{1}{\sqrt{\pi}}\int$$

Für $0 < t - t_g$ | $t_g < t < \infty$

$z = \infty \quad z = 0$ | $z = 0 \quad z = \infty$

$$\overline{W} = \frac{1}{2} \pm \frac{e^{\frac{cx_0}{D}}}{\sqrt{\pi}} \int_{y^2 = \frac{cx_0}{D}}^{\infty} e^{-y^2}\, dy \doteq \frac{1}{2} \pm \frac{1}{\sqrt{\pi}} \quad ?$$

$$\bar{t}_a = x_0 \int_0^{t_g} \frac{e^{-\frac{(x_0 - ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\, dt = \frac{x_0}{2\sqrt{\pi D}} \int_\infty^0 e^{-z^2}\, \frac{4\sqrt{D}}{c} \left[-1 + \frac{z}{\sqrt{z^2 + \frac{cx_0}{D}}}\right] dz$$

$$= \frac{x_0}{c} \left[\frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{2 e^{-z^2}\, dz}{\sqrt{z^2 + \frac{cx_0}{D}}}\right] \qquad t_a^+ = x_0 \int_{t_g}^{\infty} \cdots = \frac{x_0}{c}\left[\frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^\infty \sim \right]$$

$$\overline{W}_a = \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{2 e^{-z^2}\, dz}{\sqrt{\cdots}}$$

$$\left(\frac{t_a}{W_a}\right)^- \doteq \frac{x_0}{c} \left[1 - \frac{4}{\sqrt{\pi}} \int_0^\infty \frac{2 e^{-z^2}\, dz}{\sqrt{z^2 + \frac{cx_0}{D}}}\right] \qquad \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{2 e^{-z^2}\, dz}{\sqrt{z^2 + \frac{cx_0}{D}}} = \frac{e^{\frac{cx_0}{D}}}{\sqrt{\pi}} \int_{y^2 = \frac{cx_0}{D}}^{\infty} e^{-y^2}\, dy$$

Zum.: $c = 0{\cdot}00211$

$x_0 = 0{\cdot}0373$

$$\frac{cx_0}{D} = 0{\cdot}0021 \,.\, 0{\cdot}037 \,.\, 142 \,.\, 10^4 = 0{\cdot}21 \,.\, 3{\cdot}7 \,.\, 142 = 100$$

$$\frac{1}{D} = 41 = 142 \,.\, 10^4$$

$$\int_\beta^\infty e^{-y^2}\, dy \doteq \frac{e^{-\beta^2}}{2\beta} = \frac{e^{-\frac{cx_0}{D}}}{2\sqrt{\frac{cx_0}{D}}}$$

$$t^- = \frac{x_0}{c} \left[1 - \frac{2}{\sqrt{\pi}} \sqrt{\frac{D}{cx_0}}\right]$$

Fletcher $\frac{t^+ - t^-}{2} = t_g \frac{2}{\sqrt{\pi}} \sqrt{\frac{x_0 c}{D}} =$

$$\frac{t^+ - t^-}{2} = \frac{x_0}{c} \frac{2}{\sqrt{\pi}} \sqrt{\frac{D}{cx_0}} = \frac{2}{\sqrt{\pi}} \frac{1}{c} \sqrt{\frac{Dx_0}{c}}$$

$$= \frac{2}{\sqrt{\pi}}$$

also scheinbar dasselbe Resultat
aber Unterschied darin, dass hier t_g dem wirklichen Durchschnitt
entspricht, bei Fletcher doch noch eine (falsche) Korrektur
dazukommt

$$\tau = \frac{t^+ - t^-}{2} = \frac{2}{\sqrt{\pi}}\,\frac{1}{c}\sqrt{D t_g} = \frac{2}{\sqrt{\pi}}$$

$$\frac{x_0 + ct}{2\sqrt{\pi D t}} = $$

$$\frac{\tau^2 \pi c^2}{4 t_g} = D$$

$$\Delta = e^{\frac{c x_0}{D}} \int_{\sqrt{z_1^2 + \frac{c x_0}{D}}}^{\sqrt{z_2^2 + \frac{c x_0}{D}}} e^{-z^2}\,dz = \cancel{e^{\frac{c x_0}{D}}} \cdot \left(\frac{e^{-z_2^2 - \cancel{\frac{c x_0}{D}}}}{2\sqrt{z_2^2 + \frac{c x_0}{D}}} - \frac{e^{-z^2 - \cancel{\frac{c x_0}{D}}}}{2\sqrt{\ldots}} \right)$$

Maximalwert für $z_2 = 0$, $z_1 = \infty$: $\Delta = \frac{1}{2\sqrt{\frac{c x_0}{D}}} = \frac{1}{2\sqrt{43 \cdot 6}} \neq \frac{1}{13}$

$$\cancel{\int \frac{(x_0 + ct)\, e^{-\frac{(x_0 - ct)^2}{4 D t}}}{\sqrt{\pi D t}}\,dt = \frac{1}{2\sqrt{\pi}} \int_0 \left[x + \frac{c\sqrt{b}}{\sqrt{D}} \right] e^{-z^2}\,dt = \frac{1}{2\sqrt{\pi}} \int_\infty \left[\sqrt{z^2 + \frac{c x_0}{D}} \right] e^{-z^2}\,dt}$$

$$\cancel{\bar{t} = \frac{-1}{2\sqrt{\pi}} \cdot \frac{2D}{c^2} \int_\infty \left[z - \sqrt{z^2 + \frac{c x_0}{D}} \right]^2 e^{-z^2}\,dz} = \frac{D}{c^2\sqrt{\pi}} \int_{-\infty}^{0} \left[z - \sqrt{z^2 + \frac{c x_0}{D}} \right]^2 e^{-z^2}\,dz$$

$$t^+ = \frac{D}{c^2\sqrt{\pi}} \int \left[z - \sqrt{\quad} \right]^2 e^{-z^2}\,dz$$

$$\cancel{= \frac{D}{c^2\sqrt{\pi}} \int_{-\infty}^{0} \left[z - \sqrt{\quad} \right]^2 e^{-z^2}\,dz}$$

$$\frac{t^+ + t^-}{2} = \frac{D}{c^2\sqrt{\pi}} \int_0^\infty 2z\sqrt{z^2 + \frac{c x_0}{D}}\; e^{-z^2}\,dz$$

$$\bar{w}^+ = \frac{1}{\sqrt{\pi}} \int e^{-z^2} \left[1 + \frac{z}{\sqrt{z^2 + \frac{c x_0}{D}}} \right] dz \qquad \tau = \frac{2b}{c}\,\frac{J}{1 - J^2} \qquad J = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{z\, e^{-z^2}}{\sqrt{z^2 + \frac{cb}{D}}}\,dz$$

$$= \frac{2 e^{\frac{cb}{D}}}{\sqrt{\pi}} \underbrace{\int_{\sqrt{\frac{cb}{D}}}^{\infty} e^{-y^2}\,dy}_{\frac{e^{-\frac{cb}{D}}}{2\sqrt{\frac{cb}{D}}} \left[1 - \frac{D}{2cb} \right]}$$

$$\tau = \frac{2b}{c}\;\frac{\sqrt{\frac{D}{cb\pi}} \left[1 - \frac{D}{2cb} \right]}{\quad} \left[1 + \frac{D}{cb\pi} \right]$$

$$\neq \frac{2}{c}\sqrt{\frac{Db}{c\pi}} = 2 t_g \sqrt{\frac{D}{cb\pi}}$$

$$\bar{t}_{\frac{1}{2} + \delta} = \bar{t}_{\frac{1}{2}} \left(1 + \frac{2\delta}{\sqrt{\pi}} \right) \qquad \frac{\delta t_{1/2}}{t_{1/2}} = \frac{D}{cb}$$

$$\delta z = \frac{c\,\delta t}{2\sqrt{Dt}} = \frac{D}{2b}\,\frac{t}{\sqrt{Dt}} = \frac{\sqrt{Db}}{2b\sqrt{c}} = \frac{1}{2}\sqrt{\frac{D}{bc}}$$

$$\Delta\tau = \bar{t}_{1/2}\,\frac{2\delta}{\sqrt{\pi}} = \frac{t_g}{\sqrt{\pi}}\sqrt{\frac{D}{bc}}$$

$$W(n) = \frac{e^{-\nu}\nu^n}{n!}$$

$$\overline{\sum n \lg n}$$

$$= \frac{N}{\nu} e^{-\nu}\left[1\frac{\nu^1}{1!}\lg(1) + 2\frac{\nu^2}{2!}\lg 2 + 3\frac{\nu^3}{3!}\lg 3 + \cdots \quad \cdots\right]$$

$$H \qquad = N e^{-\nu}\left[\frac{\nu}{1}\lg 2 + \frac{\nu^2}{2!}\lg 3 + \frac{\nu^3}{3!}\lg 4 + \cdots \quad \right]$$

$$\lim_{\nu=0} H = 0$$

$$\Delta\{\cancel{\underset{\lg W}{n \lg n} + (N-n)\lg(N-n)}\}$$

$$W = e^{\cancel{n \lg n + (N-n)\lg(N-n)}} = n^n (N-n)^{N-n}$$

$$\lg W = -\sum n \lg n = \cancel{\ldots} - \sum \nu(1+\delta)\left[\lg \nu + \lg(1+\delta)\right] = \overbrace{\sum \lg \nu}^{W_0} \cdot \# + \cancel{\nu \lg \nu \sum \delta}^{=0}$$

$$+ \underbrace{\nu \sum (1+\delta)\lg(1+\delta)}$$

$$= \nu \quad \delta - \frac{\delta^2}{2} + \frac{\delta^3}{3}$$

$$+ \delta^2 - \frac{\delta^3}{2}$$

$$= \lg W_0 - \nu \sum \frac{\delta^2}{2} \ldots$$

$$W = A e^{-\nu \sum \frac{\delta^2}{2}}$$

$$\cancel{\lg W = \sum n \lg n = n_1 \lg n_1 + \sum_{n_1}^{\infty} n \lg n \quad \lg W_1 \quad \sum_2^{\infty} n \lg n = \int^{\infty} n \lg n \, dn}$$

$$\Delta H = \lg \frac{W_2}{W_1}$$

$W(v)\,dv = c\,v^2 e^{-\frac{v^2}{\alpha^2}}\,dv$

$\frac{\partial W}{\partial v^2} = 0 \qquad e^{-\frac{v^2}{\alpha^2}}\left[1-\frac{v^2}{\alpha^2}\right] = 0$

$v^2 = \alpha^2$

$W(\xi)\,d\xi = c\,e^{-\frac{\xi^2}{\alpha^2}}\,d\xi$

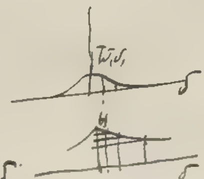
$W(\xi^2)\,d(\xi^2) = 2\xi\,c\,e^{-\frac{\xi^2}{\alpha^2}}\,d\xi$

$\frac{\partial}{\partial \xi^2}\left(e^{-\frac{\xi^2}{\alpha^2}}\right) =$

$H = \sum n \lg n$

$n = \nu(1+\delta)$

$W(\delta)\,d\delta = \sqrt{\frac{2}{\nu\pi}}\; e^{-\frac{\nu\delta^2}{2}}\,d\delta$



$W(H)\,dH = ?$

Max: $\frac{\partial W(H)}{\partial H} = 0$

$\frac{dW}{d\delta}\frac{d\delta}{dH}$ .. $dH = (1+\lg n)\,dn$

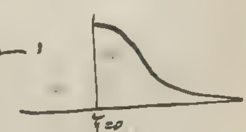
$= -\nu\delta\, e^{-\frac{\nu\delta^2}{2}} \cdot \underbrace{\frac{d\delta}{dn}}_{\frac{1}{\nu}}\, \frac{dn}{dH} = \frac{-\delta\, e^{-\frac{\nu\delta^2}{2}}}{1+\lg n} = 0$

$\delta = 0$

Ebenso wenn es sich um den Absolutwert der $|\xi|$ handelt, so ist der $\underbrace{\text{wahrscheinlichste Wert}}$ $= 0$

aber der ~~wahrsch~~ durchschnittliche ist endlich

denn Diagramm:



p. 119 $dN \;\; f_0 C_0 \int c\,d\sigma \cdot \varphi' - \varphi$
$\;\; f_0\, b \int f_0 C_0 \qquad v_3 = \ldots$ $\qquad \int c\,d\varepsilon\; \Sigma_\omega\, d\sigma\, \varphi$

$$\Delta \sum_{\omega\, d\omega} \varphi = B_5(\varphi) = \int_0^\infty\!\!\int\!\!\int_0^{2\pi}\!\!\int (\varphi' - \varphi)\, f F_1\, g\, b\, d\omega\, d\omega_1\, db\, d\varepsilon$$

ungenau, denn es ist zwar $f F_1\, g\, b\, d\omega\, d\omega_1\, db\, d\varepsilon$ — die durchschnittliche Anzahl von $f_0 C_0$

welche die f .. erleiden, aber

Wirkliche Dichte der Elemente $do\, d\omega$ ...... φ dann ist allerdings $\quad f = \overline{\varphi}$

und $\sum f\, do\, d\omega = \sum \varphi\, do\, d\omega = \sum f(1+\delta)\, do\, d\omega$

Es ist aber das wirkliche $H = \sum \varphi \lg \varphi\, do\, d\omega < \sum f \lg f\, do\, d\omega$

Wenn man Molekülraum in genügend kleine Teile teilt, so dass immer nur 0, manchmal 1 Molek vorkommen, wie soll Stosszahl berechnet werden?

$$\sum n^2 = e^{-\nu}\left[0 + \frac{\nu}{1}\cdot 0 + \frac{\nu^2}{2}\,2 + \frac{\nu^3}{3!}\,3.2 + \frac{\nu^4}{4!}\,4.3 + \frac{\nu^5}{5!}\,5.4 + \cdots \right]$$

$$\frac{d}{dx}\left(\frac{e^x}{x}\right) = \frac{d}{dx}\sum' \frac{x^{n-1}}{n!} = n-1\,\frac{x^{n-2}}{n!} = e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)$$

$$\frac{1}{x} + x + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!}$$

$$\frac{d^2}{dx^2}\left(\frac{e^x}{x}\right) = \sum (n-1)(n-2)\frac{x^{n-2}}{n!} = e^x\left(\frac{1}{x} - \frac{2}{x^2} + \frac{2}{x^3}\right)$$

$$-\frac{1}{x^2} + \frac{1}{2!} + \frac{2x}{3!} + \frac{3x^2}{4!}$$

$$+\frac{2}{x^3} + \frac{1.2}{3!} + \frac{2.3\,x}{4!} + \cdots$$

$$2 + \frac{1.2}{3!}x^3 + \frac{2.3}{4!}x^4 + \frac{3.4}{5!}x^5 + \cdots = e^x(2 - 2x + x^2)$$

$$\sum n^2 = e^{-\nu}\left[e^{\nu}(2 - 2\nu + \nu^2) - 2\right] + \frac{\nu^2}{2}e^{-\nu} = 2 - 2\nu + \nu^2 - 2e^{-\nu} + \frac{\nu^2}{2}e^{-\nu}$$

$$= e^{-\nu}\left[\frac{\nu^2}{1!} + \frac{\nu^3}{1} + \frac{\nu^4}{2!} + \frac{\nu^5}{3!} + \cdots\right] = \underline{\underline{\nu^2}}$$

$$\bar{y} = \frac{\int_0^\infty y\, e^{-\alpha y}\,dy}{\int_0^\infty e^{-\alpha y}\,dy} = \frac{-\frac{e^{-\alpha y}}{\alpha}\cdot y + \frac{1}{\alpha}\int e^{-\alpha y}\,dy}{1} = \frac{1}{\alpha}$$

$$\alpha = \frac{g}{R\theta} = \frac{\mu g}{H\theta} = \frac{4}{3}\frac{a^3\pi(\rho-\rho')g\,N}{H\theta}$$

$$\frac{\bar{y}\,F}{T} = \bar{y}\cdot\frac{4}{3}\pi a^3(\rho-\rho')g =$$

$$D\frac{\partial u}{\partial x} + cu = \frac{cu}{2} + \frac{x_0}{2t}(A+B)\,e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}}\;\frac{1}{2}\sqrt{\frac{\pi}{Dt}}$$

$$= \frac{1}{2}\sqrt{\frac{\pi}{Dt}}\,e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}}\underbrace{\left[\frac{c}{2}(A-B) + \frac{x_0}{2t}(A+B)\right]}_{\text{falls } = 0} = 0 \text{ für alle } t$$

$$\boxed{u = e^{-\frac{cx}{2D}+\frac{c^2}{4D}t}\int_{-\infty}^{\infty} e^{-\beta D t}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\,d\beta}$$

$t=0$:

$$u = \cancel{e^{\frac{cx}{2D}}}\int_{-\infty}^{\infty}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\,d\beta = \sqrt{\frac{2}{\pi}}\, e^{-\frac{(x-x_0)^2}{2}\cancel{\frac{x}{2D}}}$$

$$= \frac{1}{\pi}\int_0^{\infty} d\alpha \int_{-\infty}^{+\infty}\varphi(y)\left[\cos\alpha y\cos\alpha x + \sin\alpha y\sin\alpha x\right] dy$$

$$= \frac{1}{\pi}\iint \sin\beta x\cdot\varphi(y)\sin\beta y\, d\beta\, dy$$

$$+ \iint \cos\beta x\,\varphi(y)\cos\beta y\, d\beta\, dy$$

$$f(\beta) = \int\varphi(y)\sin\beta y\, dy$$

$$-\frac{2D\beta}{c} f(\beta) = \int\varphi(y)\cos\beta y\, dy$$

$$\cancel{f(\beta) = \sqrt{\tfrac{2}{\pi}}\, e^{-\frac{(x-y)^2}{2}}\cos\beta y\, dy}$$

$$\cancel{\frac{\partial u}{\partial x} = -\frac{c}{2D}u - \frac{(x-x_0)}{2Dt}\mathcal{U}V_1 + \frac{(x+x_0)}{2Dt}\mathcal{U}V_2}$$

Versuch:

$$u = \underbrace{e^{-\frac{c^2t}{4D}-\frac{cx}{2D}}}_{\mathcal{U}}\ \frac{1}{2}\sqrt{\frac{\pi}{Dt}}\Big\{\underbrace{A\, e^{-\frac{(x-x_0)^2}{4Dt}}}_{V_1} + \underbrace{B\, e^{-\frac{(x+x_0)^2}{4Dt}}}_{V_2}\Big\} = \mathcal{U}\,(V_1 - V_2)$$

$$\frac{\partial u}{\partial t} - D\frac{\partial^2 u}{\partial x^2} - c\frac{\partial u}{\partial x} = -c\,\mathcal{U}\frac{\partial(V_1-V_2)}{\partial x} - 2D\frac{\partial\mathcal{U}}{\partial x}\frac{\partial(V_1-V_2)}{\partial x} = -\underbrace{\left[c\,\mathcal{U} + 2D\frac{\partial\mathcal{U}}{\partial x}\right]}_{\left[\cancel{c - 2D\frac{c}{2D}}\right]\mathcal{U}}\frac{\partial(V_1-V_2)}{\partial x} = 0$$

also Diff-Gleichung ist erfüllt

Anfangsbedingungen: für $t=0$

$u = 0$ für alle x mit Ausnahme $x = x_0$

für $x=0$:

$$\begin{cases} u = e^{-\frac{c^2t}{4D}}\,\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\,(A-B)\, e^{-\frac{x_0^2}{4Dt}} \\[2ex] \dfrac{\partial u}{\partial x} = -\dfrac{c}{2D}u + \dfrac{x_0}{2Dt}\, e^{-\frac{c^2t}{4D}}(A+B)\, e^{-\frac{x_0^2}{4Dt}}\,\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \end{cases}$$

$$\varphi(y) = \sqrt{\frac{\varepsilon}{\pi}}\, e^{-\frac{(x_0-y)^2}{\varepsilon}}$$

$$f(\beta) = \int_{-\infty}^{+\infty} \varphi(y) \sin y\beta \, dy \qquad \Big| \qquad -\frac{2D\beta}{c} f(\beta) = \int_{-\infty}^{+\infty} \varphi(y) \cos y\beta \, dy$$

$$u_0 = \int \left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\, d\beta$$

$$u = e^{-\frac{cx}{2D} \mp \frac{c^2t}{4D}} \int_{-\infty}^{\infty} e^{-D\beta^2 t}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\, d\beta \rightarrow \boxed{\text{erfüllt Diff. Gleichung (allgemein) und Bedingung } D\frac{\partial u}{\partial x} + cu = 0 \Big|_{x=0}}$$

$$= e^{-\frac{cx}{2D} \mp \frac{c^2t}{4D}} \int_{-\infty}^{+\infty}\left[e^{-D\beta^2 t} \sin\beta x \, d\beta \int_{-\infty}^{\infty} \varphi(y) \sin y\beta \, dy + e^{-D\beta^2 t} \cos\beta x \, d\beta \int_{-\infty}^{\infty} \varphi(y)\cos y\beta \, dy\right]$$

$$= e^{-\frac{cx}{2D} \mp \frac{c^2t}{4D}} \int_0^{\infty}\int_{-\infty} e^{-D\beta^2 t} \cos\beta(y-x) \, \varphi(y) \, dy$$

$$= e^{-\frac{cx}{2D} \mp \frac{c^2t}{4D}} \int_0^{\infty} d\beta \int_{-\infty}^{\infty} dy \; e^{-D\beta^2 t}\, e^{-\frac{(x_0-y)^2}{\varepsilon}} \sqrt{\frac{\varepsilon}{\pi}} \cos\beta(y-x)$$

$$\lim_{\varepsilon=0} \sqrt{\frac{\varepsilon}{\pi}} \int_{-\infty}^{+\infty} e^{-\frac{(x_0-y)^2}{\varepsilon}} \cos\beta(y-x)\, dy = \cos\beta(x_0-x)$$

$$u = e^{-\frac{cx}{2D} \mp \frac{c^2t}{4D}} \int_0^{\infty} \underbrace{e^{-D\beta^2 t} \cos\beta(x_0-x)\, d\beta} \qquad \alpha = \beta\sqrt{Dt}$$

$$= \frac{1}{\sqrt{Dt}} \int_0^{\infty} e^{-\alpha^2} \cos\frac{\alpha(x_0-x)}{\sqrt{Dt}}\, d\alpha$$

$$\int_0^\infty e^{-\alpha^2}\cos\alpha x\,d\alpha = \frac{\sqrt{\pi}}{2}\,e^{-\frac{x^2}{4}}$$

$$\int_0^\infty e^{-\alpha^2}\sin\alpha x\,d\alpha = \frac{1}{2}\,e^{-\frac{x^2}{4}}\int_0^x e^{\frac{y^2}{4}}dy$$

$$\cancel{u = e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\frac{-(x_0-x)^2}{4Dt} \ldots \frac{}{2\sqrt{Dt}}}$$

$$u = e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\int_{-\infty}^{+\infty} e^{-D\beta^2 t}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right]d\beta \underbrace{\int_{-\infty}^{+\infty}\sqrt{\frac{\varepsilon}{\pi}}\,e^{-\frac{(x_0-y)^2}{\varepsilon}}\sin y\beta\,dy}$$

Vrench

$\downarrow$

lim = $\sin x_0\beta$

$f(\beta) = \sin\beta x_0$

$$= 2e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\int_0^{+\infty} e^{-D\beta^2 t}\left[\sin\beta x\sin\beta x_0 - \frac{2D\beta}{c}\cos\beta x\sin\beta x_0\right]d\beta$$

$$\cancel{\int_0^\infty e^{-D\beta^2 t}\sin\beta x\,d\beta = \frac{e^{-D\beta^2 t}}{2Dt}\sin\beta x + \int\frac{e^{-D\beta^2 t}}{2Dt}\cos\beta x\,d\beta}$$

$$\int_0^\infty e^{-\alpha^2}\cos\alpha x\,dx = \frac{\sqrt{\pi}}{2}\,e^{-\frac{x^2}{4}}$$

$2\sin\beta x_0\cos\beta x = \sin\beta(x_0-x) + \sin\beta(x_0+x)$

$$\int_0^\infty \alpha\,e^{-\alpha^2}\sin\alpha x\,dx = \frac{\sqrt{\pi}}{4}\,x\,e^{-\frac{x^2}{4}}$$

$$\parallel \frac{2\alpha\sqrt{D}}{c\sqrt{t}}$$

$$u = \frac{e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}}{\sqrt{Dt}}\int e^{-\alpha^2}\left[\cos\alpha\frac{(x-x_0)}{\sqrt{Dt}} - \cos\left(\alpha\frac{(x+x_0)}{\sqrt{t}}\right) - \frac{2D}{c}\frac{\alpha}{\sqrt{Dt}}\left[\sin\left(\alpha\frac{(x_0-x)}{\sqrt{Dt}}\right) + \sin\left(\alpha\frac{(x_0+x)}{\sqrt{Dt}}\right)\right]\right]$$

$$\cancel{= e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\left\{e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}}\right\} - \frac{2\sqrt{\pi}}{c\,t}\left\{e^{-\frac{(x-x_0)^2}{4Dt}}\int_0^{\frac{x-x_0}{2\sqrt{Dt}}} e^{z^2}dz + e^{-\frac{(x+x_0)^2}{4Dt}}\int_0^{\frac{x+x_0}{2\sqrt{Dt}}} e^{+z^2}dz\right\}\right]}$$

$$\cancel{u_{1\,t=0} = e^{-\frac{c^2t}{4D}}\left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\ldots - \frac{2\sqrt{\pi}}{ct}\,2\left\{e^{-\frac{x_0^2}{4Dt}}\int_0^{\frac{x_0}{2\sqrt{Dt}}} e^{z^2}dz\right\}\right]}$$

$$u = e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\left\{e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}}\right\} - \frac{\sqrt{\pi}}{2ct}\left\{\frac{x_0-x}{\sqrt{Dt}}e^{-\frac{(x_0-x)^2}{4Dt}} + \frac{x_0+x}{\sqrt{Dt}}e^{-\frac{(x_0+x)^2}{4Dt}}\right\}\right]$$

$$u\Big|_{x=0} = e^{-\frac{c^2t}{4D}}\left[\cancel{\frac{1}{2}\sqrt{\frac{\pi}{Dt}}} - \frac{\sqrt{\pi}}{ct}\frac{x_0}{\sqrt{Dt}}e^{-\frac{x_0^2}{4Dt}}\right]$$

$$\frac{\partial u}{\partial x}\Big|_0 = -\frac{c}{2D}u + e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\left[-\frac{(x-x_0)}{2Dt}e^{\cdots} + \frac{(x+x_0)}{2Dt}e^{\cdots}\right] - \frac{\sqrt{\pi}}{2ct\sqrt{Dt}}\left[\cancel{e} + \frac{(x_0-x)^2}{2Dt} - \frac{(x_0+x)^2}{2Dt}\right]e^{-\frac{x_0^2}{4Dt}}\right]$$

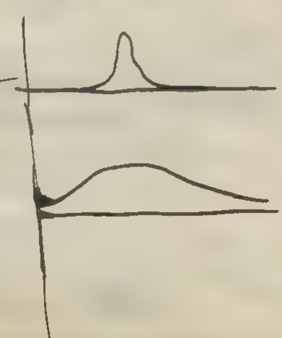
$$= -\frac{c}{2D}u + e^{-\frac{c^2t}{4D}}\left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\frac{x_0}{Dt}e^{-\frac{x_0^2}{4Dt}}\right] = \frac{\sqrt{\pi}\,x_0}{Dt\sqrt{Dt}}e^{-\frac{c^2t}{4D}-\frac{x_0^2}{4Dt}}$$

$$u = \boxed{u} - \frac{2}{ct}e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\int_0^\infty e^{-\alpha^2}\cdot\alpha\left[\sin\left(\alpha\frac{x_0-x}{\sqrt{Dt}}\right) + \sin\left(\alpha\frac{x_0+x}{\sqrt{Dt}}\right)\right]d\alpha$$

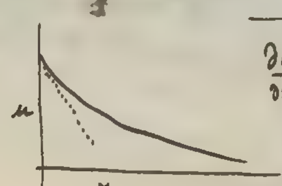
$$u\Big|_{x=0} = \cancel{\boxed{u}} - \frac{4}{ct}e^{-\frac{c^2t}{4D}}\underbrace{\int_0^\infty e^{-\alpha^2}\cdot\alpha\,\sin\left(\frac{\alpha x_0}{\sqrt{Dt}}\right)d\alpha}_{\frac{\sqrt{\pi}}{4}\frac{x_0}{\sqrt{Dt}}e^{-\frac{x_0^2}{4Dt}}} = -\frac{\sqrt{\pi}\,x_0}{ct\sqrt{Dt}}e^{-\frac{c^2t}{4D}-\frac{x_0^2}{4Dt}}$$

$$\frac{\partial u}{\partial x}\Big|_0 = \frac{\partial\boxed{u}}{\partial x}\Big|_0 + \frac{1}{2D}\frac{\sqrt{\pi}\,x_0}{t\sqrt{Dt}}e^{-\frac{c^2t}{4D}-\frac{x_0^2}{4Dt}} - \frac{2}{ct}e^{-\frac{c^2t}{4D}}\int_0^\infty e^{-\alpha^2}\frac{\alpha^2}{\sqrt{Dt}}\cancel{\left[-\cos\frac{\alpha x_0}{\sqrt{Dt}} + \cos\frac{\alpha x_0}{\sqrt{Dt}}\right]}$$

$$-\frac{c}{2D}\cancel{u} + \frac{1}{2}\sqrt{\frac{\pi}{Dt}}e^{-\frac{c^2t}{4D}}\frac{x_0}{Dt}e^{-\frac{x_0^2}{4Dt}} = \sqrt{\frac{\pi}{Dt}}\frac{x_0}{Dt}e^{-\frac{c^2t}{4D}-\frac{x_0^2}{4Dt}}$$



$cu + D\frac{\partial u}{\partial x} = 0\Big|_{x=0}$



$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x}$$

$\lim \frac{\partial u}{\partial t} = 0$

umm: $u = u_0 e^{-\frac{c}{D}x}$

$$u = A e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \cdot \frac{1}{2}\sqrt{\frac{\pi}{Dt}} \left\{ e^{-\frac{(x-x_0)^2}{4Dt}}\left[1-\frac{x_0-x}{ct}\right] - e^{-\frac{(x+x_0)^2}{4Dt}}\left[1+\frac{x_0+x}{ct}\right]\right\}$$

$\lim\limits_{t=0} u = 0$ mit Ausnahme von $x = x_0$

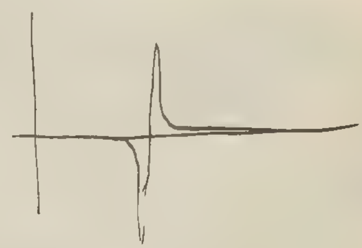
$$\int_0^\infty \lim_{t=0} u \, dx = A e^{-\frac{cx_0}{2D}} \left\{ \underbrace{\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \int_{-\infty}^{+\infty} e^{-\frac{x^2}{4Dt}} dx}_{=1} \right\} - \cancel{\frac{\sqrt{\pi}}{ct} \int_{-\infty}^{+\infty} \frac{x}{2\sqrt{Dt}} e^{-\frac{x^2}{4Dt}} dx}$$

$$A = e^{\frac{cx_0}{2D}}$$

$$u = \frac{1}{2}\sqrt{\frac{\pi}{Dt}}\, e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} \left[e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}} - \frac{x_0-x}{ct} e^{-\frac{(x_0-x)^2}{4Dt}} - \frac{x_0+x}{ct} e^{-\frac{(x_0+x)^2}{4Dt}} \right]$$

$$u = \frac{1}{2}\sqrt{\frac{\pi}{Dt}}\, e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} \left[e^{-\frac{(x-x_0)^2}{4Dt}}\left(1-\frac{x_0-x}{ct}\right) - e^{-\frac{(x+x_0)^2}{4Dt}}\left(1+\frac{x_0+x}{ct}\right) \right]$$

Entspricht einem andern Anfangszustand, nämlich:



sodass $\int_0^\infty u \, dx = 0$

daher auch $\lim u = 0$

$$-\frac{2D\beta}{c} f(\beta) = \int_{-\infty}^{+\infty} \varphi(y) \cos\beta y \, dy \qquad f(\beta) = -\frac{c}{2D\beta} \int_{-\infty}^{\infty} \varphi(y) \cos\beta y \, dy$$

$$\lim \int \varphi(y) \cos\beta y \, dy = \lim_{\varepsilon \to 0} \sqrt{\frac{\varepsilon}{\pi}} \int e^{-\frac{(x_0-y)^2}{\varepsilon}} \cos\beta y \, dy = \cos\beta x_0$$

Versuch

$$\boxed{f(\beta) = \frac{c}{2D} \frac{\cos\beta x_0}{\beta}}$$

$$u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \int_{-\infty}^{\infty} e^{-D\beta^2 t} \cos\beta x_0 \left[\cos\beta x - \frac{c}{2D\beta} \sin\beta x\right] d\beta$$

$$D\beta^2 t = \alpha^2$$

$$\frac{1}{\beta} = \frac{\sqrt{Dt}}{\alpha}$$

$$J(x) = \int_0^\infty \frac{e^{-\alpha^2} \sin\alpha x}{\alpha} \, d\alpha = ? \qquad \frac{\partial J}{\partial x} = \int_0^\infty e^{-\alpha^2} \cos\alpha x \, d\alpha = \frac{\sqrt{\pi}}{2} e^{-\frac{x^2}{4}}$$

$$J = \frac{\sqrt{\pi}}{2} \int_0^x e^{-\frac{x^2}{4}} dx = \sqrt{\pi} \int_0^{\frac{x}{2}} e^{-z^2} dz$$

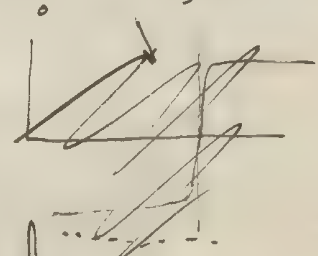
$$\int_0^\infty e^{-px^2} \cos qx \, dx = \frac{1}{2} e^{-\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}$$

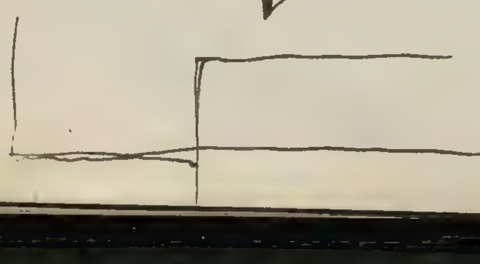
$$\int_0^\infty e^{-px^2} \sin qx \, dx = \sum_0^\infty (-1)^n \frac{1}{(n+2)^{n+1}} \frac{q^{2n+1}}{p^{2n+1}}$$

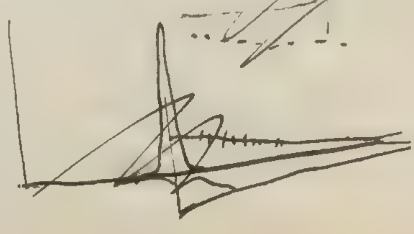
$$u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \left\{ \int_0^\infty \frac{e^{-\alpha^2}}{\sqrt{Dt}} \left[\cos\frac{\alpha(x-x_0)}{\sqrt{Dt}} + \cos\frac{\alpha(x+x_0)}{\sqrt{Dt}}\right] d\alpha - \frac{c}{2D} \int_0^\infty \frac{e^{-\alpha^2}}{\alpha} \left[\sin\frac{\alpha(x-x_0)}{\sqrt{Dt}} + \sin\frac{\alpha(x+x_0)}{\sqrt{Dt}}\right] d\alpha \right\}$$

$$= e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \left\{ \frac{1}{2}\sqrt{\frac{\pi}{Dt}} \left[e^{-\frac{(x-x_0)^2}{4Dt}} + e^{-\frac{(x+x_0)^2}{4Dt}}\right] - \frac{c\sqrt{\pi}}{2D} \left[\int_0^{\frac{x-x_0}{2\sqrt{Dt}}} e^{-z^2} dz + \int_0^{\frac{x+x_0}{2\sqrt{Dt}}} e^{-z^2} dz\right] \right\}$$

$t = 0$







$$u_0 = \int_{-\infty}^{+\infty}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\,d\beta = \varphi(x) \qquad f(\beta) = ?$$

$$= \frac{1}{\pi}\int_0^\infty d\alpha \int_{-\infty}^{+\infty} \varphi(y)\left[\cos\alpha y\cos\alpha x + \sin\alpha y\sin\alpha x\right] dy$$

$$= \frac{1}{\pi}\int_0^\infty d\alpha \left[\cos\alpha x \int_{-\infty}^{+\infty}\varphi(y)\cos\alpha y\,dy + \sin\alpha x \int_{-\infty}^{+\infty}\varphi(y)\sin\alpha y\,dy\right]$$

Solche Funktion dass $\varphi(x) = 0$ für alle x mit Ausnahme $x = x_0$ und

$$\int_0^\infty \varphi(x)\,dx = 1 = \int_{-\infty}^{+\infty}\left\{-\frac{\cos\beta x}{\beta}\Big|_0^\infty - \frac{2D}{c}\sin\beta x\Big|_0^\infty\right\} f(\beta)\,d\beta = \int_{-\infty}^{+\infty}\frac{f(\beta)}{\beta}\,d\beta = 1$$

Verschiedenes Resultat je nach Reihenfolge der Limes!

Jedenfalls[2] muss die richtige Lösung die Form besitzen:

$$u = e^{-\frac{cx}{2D} - \frac{c^2}{4D}t}\int_{-\infty}^{\infty} e^{-\beta^2 Dt}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\,d\beta$$

$$\int_0^\infty e^{-mx}\sin\beta x\,dx = \frac{\beta}{\beta^2+m^2}$$

$$\int_0^\infty e^{-mx}\cos\beta x\,dx = \frac{m}{\beta^2+m^2}$$

$$\int_0^\infty u\,dx = e^{-\frac{c^2}{4D}t}\int_{-\infty}^{+\infty} e^{-\beta^2 Dt}\left[\frac{\beta}{\beta^2+\frac{c^2}{4D^2}} - \frac{2D\beta}{c}\,\frac{\frac{c}{2D}}{\beta^2+\frac{c^2}{4D^2}}\right] f(\beta)\,d\beta = \underline{\underline{0!}}$$

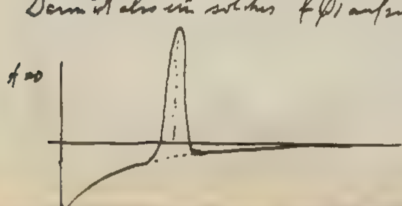
Also ist diese Form ~~[illegible]~~ nicht geeignet (nicht allgemein genug)

Vielleicht so, dass man die Lösung

$$u = e^{-\frac{cx}{D}}$$

hinzuaddiert, welche sowohl Dgl. wie Grenzbedingung erfüllt und $\int_0^\infty u\,dx = \text{const}$ macht

Dann ist also ein solches $f(\beta)$ aufzusuchen, welches für $t = 0$ ~~[illegible]~~ eine u-Verteilung ergibt.



Wenn:

$$\int_{-\infty}^{\infty}\left[\sin\beta x - \frac{2D}{c}\beta\cos\beta x\right] f(\beta)\,d\beta = \varphi(x)$$

$$= \frac{1}{2\pi}\left\{\int_{-\infty}^{+\infty} d\beta\,\cos\beta x\int_{-\infty}^{+\infty}\varphi(y)\cos\beta y\,dy + \int_{-\infty}^{+\infty} d\beta\,\sin\beta x\int_{-\infty}^{+\infty}\varphi(y)\sin\beta y\,dy\right\}$$

Kann man daraus schliessen, dass:

$$f(\beta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\varphi(y)\sin\beta y\,dy \qquad -\frac{2D}{c}\beta\, f(\beta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\varphi(y)\cos\beta y\,dy$$

sein muss? (Riem. Weber II p. 91)

~~[illegible]~~ Dann wäre:

$$u = \frac{e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}}{2\pi}\int_{-\infty}^{+\infty} e^{-\beta^2 Dt}\left[\sin\beta x\int_{-\infty}^{+\infty}\varphi(y)\sin\beta y\,dy + \cos\beta x\int\varphi(y)\cos\beta y\,dy\right]d\beta$$

Erfüllt dies die Bedingung $D\frac{\partial u}{\partial x} + cu\Big|_{x=0} = 0$? welche durch obige Wahl erfüllt wird?

$$u_0 = \frac{e^{-\frac{c^2t}{4D}}}{2\pi}\int e^{-\beta^2 Dt}\,d\beta\int\varphi(y)\cos\beta y\,dy$$

$$\frac{\partial u}{\partial x}\Big|_0 = -\frac{c}{2D}u_0 + \frac{e^{-\frac{c^2t}{4D}}}{2\pi}\int e^{-\beta^2 Dt}\,\beta\,d\beta\int\varphi(y)\sin\beta y\,dy$$

Dann müsste sein:

$$\int_{-\infty}^{+\infty} e^{-\beta^2 Dt}\,d\beta\left\{\beta\int_{-\infty}^{\infty}\varphi(y)\sin\beta y\,dy + \frac{c}{2D}\int_{-\infty}^{\infty}\varphi(y)\cos\beta y\,dy\right\} = 0$$

Blödsinn!

(Burkhardt) Encykl. II A 12 (1043)

$$e^{-x}\int_0^{\infty}\frac{\xi^{a-1}[\cos x\xi - \xi\sin x\xi]}{1+\xi^2}\,d\xi = \Gamma(a)\cos\frac{a\pi}{2}\int_x^{\infty} x^{-a}e^{-x}\,dx$$

$$\int_0^{\infty}\frac{e^{-\xi^2}}{\lambda^2+\xi^2}\begin{Bmatrix}\lambda\cos x\xi\\ \xi\sin x\xi\end{Bmatrix}d\xi = \tfrac{1}{2}\sqrt{\pi}\,e^{\lambda^2}\left[e^{-\lambda x}\int_{-\infty}^{\frac{x}{2}-\lambda} e^{-z^2}dz \pm e^{\lambda x}\int_{\frac{x}{2}+\lambda}^{\infty} e^{-z^2}dz\right]$$

(847) (848)

$$\int_0^{\infty}\frac{\cos x\xi}{\lambda^2+\xi^2}\,d\xi = \frac{\pi}{2\lambda}e^{-\lambda x}$$

$$\int_0^{\infty}\frac{\xi\sin x\xi\,d\xi}{\lambda^2+\xi^2} = \frac{\pi}{2}e^{-\lambda x}$$

$$\int_0^{\infty}\frac{\cos x\xi + \frac{\xi}{\lambda}\sin x\xi}{\lambda^2+\xi^2}\,d\xi = \frac{\pi}{\lambda}e^{-\lambda x} \quad (x>0)$$

Jedenfalls muss die Voraussetzung genügen auch wie $\frac{\partial\varphi}{\partial x} + \frac{c}{D}\varphi = 0 \,\Big|_{x=0}$

$$\therefore \int_{-\infty}^{+\infty} \left[\beta\cos\beta x - \frac{2D}{c}\beta^2\sin\beta x + \frac{c}{D}\sin\beta x - 2\beta\cos\beta x\right] f(\beta)\, d\beta \Big|_{x=0} = 0 \qquad = -\int_{-\infty}^{+\infty}\beta f(\beta)\, d\beta$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\beta \left\{ \left[\beta\cos\beta x + \frac{c}{D}\sin\beta x\right]\int\varphi(\mu)\sin\beta\mu\, d\mu + \left[-\beta\sin\beta x + \frac{c}{D}\cos\beta x\right]\int\varphi(\mu)\cos\beta\mu\, d\mu \right\}\Big|_{x=0}$$

$$= \int\beta\, d\beta\int\varphi(\mu)\sin\beta\mu\, d\mu + \int\frac{c}{D}d\beta\int\varphi(\mu)\cos\beta\mu\, d\mu = 0$$

Also wird jene ~~B~~ Bedingung tatsächlich für $t=0$ erfüllt sein, ob aber auch für später?

~~Hilfsatz~~

$$u = \frac{e^{-\frac{cx}{2D} - \frac{c^2t}{4D}}}{2\pi}\int e^{-\beta^2 Dt}\, d\beta\int\varphi(\mu)\cos\beta(x-\mu)\, d\mu = \frac{e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}}{2\pi}\int\varphi(\mu)\cdot\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\, e^{-\frac{(x-\mu)^2}{4Dt}}\, d\mu$$

$$\frac{\partial u}{\partial x} = -\frac{c}{2D}u - \frac{e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}}{2\pi}\int\varphi(\mu)\,\tfrac{1}{2}\sqrt{\pi}\cdot\frac{x-\mu}{2Dt}\, e^{-\frac{(x-\mu)^2}{4Dt}}\, d\mu$$

$$\int\left[\frac{\mu}{2Dt} + \frac{c}{2D}\right]\varphi(\mu)\, e^{-\frac{\mu^2}{4Dt}}\, d\mu = 0\ ?$$

Vielleicht liegt die Schwierigkeit wirklich darin dass schon die Anfangsbedingung $u = \varphi(x)\,\Big|_{t=0}$ mit der ~~[illegible]~~ Gr. Bed. verträglich sein muss und dabei ~~[illegible]~~ $\int_0^\infty\varphi\, dx = 0$

$$\sin\beta x - \frac{2D}{c}\beta\cos\beta x \qquad \Big|\ \cancel{a_0} + b_0 x$$

$$\frac{\partial\varphi}{\partial\beta} = \cancel{x}\cos\beta x - \frac{2D}{c}\cos\beta x + \frac{2Dx}{c}\beta\sin\beta x \qquad \Big|\ a_1 + b_1 x$$

$$\frac{\partial^2}{\partial\beta^2} = -x^2\sin\beta x + \frac{4Dx}{c}\sin\beta x + \cancel{\frac{2Dx}{c}\sin\beta x} + \frac{2Dx^2}{c}\beta\cos\beta x \qquad \Big|\ a_2$$

$$\cancel{b_2 = \frac{2D}{c}\beta}\ \Big\|\ \cancel{\left(\frac{2D}{c}\right)^2 y\cos y x}$$

$$a_0 = 0 \qquad b_0 + \frac{2D}{c}\beta a_1 + \frac{b_1 D}{c}a_2 = 0 \ \Big\|\ \frac{2D}{c}\beta b_0 + a_1 = 0$$

$b_1 = 0$

$$(1033)\quad \int_0^\infty \left\{\begin{matrix}\cos x\xi\\ \sin x\xi\end{matrix}\right\}\frac{\xi^{a-1}\, d\xi}{1+\xi^2} = \frac{\pi}{2}\left\{\begin{matrix}\frac{\cos x}{\sin\frac{a\pi}{2}}\\ \frac{\sin x}{\cos\frac{a\pi}{2}}\end{matrix}\right\} + \frac{1}{2}\Gamma(a)\left\{\begin{matrix}\cos\frac{a\pi}{2}\\ \sin\frac{a\pi}{2}\end{matrix}\right\}\left[e^{-x}\int_0^x x^{-m}e^{x}\, dx + e^{x}\int_0^x x^{-m}\, e^{-x}\, dx\right]$$

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x}$$

$u = e^{-\gamma t} f(x)$ $\qquad f = e^{\alpha x}$

$D\alpha^2 + c\alpha + \gamma = 0$ $\qquad \alpha = \alpha_1 + i\alpha_2$, $\gamma = \gamma_1 + i\gamma_2$ $\quad\Big|\quad \alpha_1 - i\alpha_2$, $\gamma_1 - i\gamma_2$

$$\left.\begin{aligned} D(\alpha_1^2 - \alpha_2^2) + c\alpha_1 + \gamma_1 &= 0 \\ 2D\alpha_1\alpha_2 + c\alpha_2 + \gamma_2 &= 0 \end{aligned}\right\}$$

$$u = e^{-\gamma_1 t + \alpha_1 x}\underbrace{\left[e^{i(\gamma_2 t + \alpha_2 x)} + e^{-i(\gamma_2 t + \alpha_2 x)}\right]}_{\substack{\cos \\ \sin}(\gamma_2 t + \alpha_2 x)}$$

$$u = e^{\alpha_1 x + t\{c\alpha_1 + D(\alpha_1^2 - \alpha_2^2)\}}\ \substack{\cos \\ \sin}\left[t(2D\alpha_1\alpha_2 + c\alpha_2) + \alpha_2 x\right]$$

1). Spezialfall: $\alpha_1 = -\frac{c}{2D}$ $\rightarrow$: $\gamma_2 = 0$

$\alpha_2 = \beta$

$$e^{-\frac{cx}{2D} - \frac{c^2}{4D}t - Dt\beta^2}$$

$t = 0$ $\qquad u = e^{\alpha_1 x}\ \substack{\cos \\ \sin}(\alpha_2 x)$

2). Spezialfall

$\alpha_1 = 0$ gibt zeitlich Wärmeleitung für $x = ct$ fort...

$$\bar{u} = \frac{D}{c} + \frac{1}{\sqrt{\pi}}\left[\sqrt{2Dt}\int_{-\frac{x_0+ct}{2\sqrt{Dt}}}^{\infty} u e^{-u^2}du + (x_0 - ct)\int_{-\frac{x_0+ct}{2\sqrt{Dt}}}^{\infty} e^{-u^2}du\right] - \frac{D}{c\sqrt{\pi}}\int_{-\infty}^{\frac{x_0+ct}{2\sqrt{Dt}}} e^{-u^2}du - \frac{D}{c\sqrt{\pi}}e^{\frac{cx_0}{D}}\int_{\frac{x_0+ct}{2\sqrt{Dt}}}^{\infty} e^{-u^2}du$$

und die durchschn. Entropie!

Also gilt auch für den ~~...~~ durchschnittl. Wert nicht das zeitl. Folgende

$$\int_0^\infty u\,dx = e^{+t\left[c\alpha_1 + D(\alpha_1^2-\alpha_2^2)\right]} \frac{\cos}{\sin} t\,(2D\alpha\alpha_2 + c\alpha_2) \int_0^\infty e^{\alpha_1 x} \frac{\cos}{\sin} \alpha_2 x\,dx$$

Aber alle diese Lösungen müssen $\lim_{t\to\infty} \int_0 u\,dx = 0$ geben, falls Exponent (bei t) negativ

Somit wenn Bedingung $D\frac{\partial u}{\partial x} + cu = 0$

auch erfüllt sein soll (d.h. dass nichts durch die Anfangsebene hindurchtritt) so muss schon von allem Anfang an $\int_0^\infty u\,dx = 0$ sein und demgemäss muss φ gewählt werden

Falls die wirkliche Verteilung dieser Bedingung nicht Genüge leistet, muss Lösung zusammengesetzt werden aus $u = Ae^{-\frac{cx}{D}}$ und der übrigen welche $\int_0^\infty u\,dx = 0$ erfüllt.

Für $c = 0$ reduziert sich auf die Lösung welche ich früher angegeben habe

<u>Richtige Lösung</u>:

$$u = \frac{c}{D} e^{-\frac{cx}{D}} + \cancel{D} \frac{1}{2\sqrt{\pi D t}} \left[e^{-\frac{(x-x_0)^2}{4Dt}} + e^{-\frac{(x+x_0)^2}{4Dt}} \right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} - \frac{c}{\sqrt{\pi}\,D} e^{-\frac{cx}{D}} \int_{-\infty}^{\frac{x+x_0-ct}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

stimmt | Dgl. u. Bed.

Das entspricht auch dem Falle dass ein Niederschlag auf einem Filter durch einen Wasserstrom gewaschen wird

So kann man durch entsprechende Wahl der Durchflussgeschwindigkeit alle möglichen Fälle realisieren welche sich bei Schwere nicht gut verwirklichen lassen ~~F~~ (vorausgesetzt dass der Niederschlag nicht zusammenklebt!)

$$u = \cancel{e^{-\frac{cx}{D}} \left[1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x+x_0-ct}{2\sqrt{Dt}}} e^{-z^2}\,dz \right]} + \frac{D}{C} \frac{1}{2\sqrt{\pi D t}} \cancel{\left[e^{-\frac{(x-x_0+ct)^2}{4Dt}} + e^{-\frac{(x+x_0+ct)^2}{4Dt} + \frac{cx_0}{D}} \right]}$$

Grenzfälle für $t = 0$, $t = \infty$ stimmen

Nun sieht man wenn $\frac{\partial u}{\partial x} = 0$ gesetzt wird (wahrscheinlichster Wert) so resultiert dafür nicht

~~$x = x_0 - ct$~~ $x = x_0 - ct$ also gibt das gewöhnliche Fallgesetz nicht den wahrscheinlichsten Fall an.

$$\bar{u} = \int_0^\infty x e^{-\frac{cx}{D}}\,dx + \frac{D}{C} \frac{1}{2\sqrt{\pi D t}} \left[\int_0^\infty x e^{-\frac{(x-x_0+ct)^2}{4Dt}}\,dx + e^{+\frac{cx_0}{D}} \cancel{\int_0^\infty x e^{-\frac{x^2+2x(x_0+ct)+(x_0+ct)^2}{4Dt}}\,dx} \right] - \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty x e^{-\frac{cx}{D}}\,dx \int^{\frac{x+x_0-ct}{\sqrt{\ }}} e^{-z^2}\,dz$$

$$= -\frac{1}{\sqrt{\pi}} \left(\frac{D}{C}\right)^2 \int_{-\infty}^{\frac{x_0-ct}{2\sqrt{Dt}}} e^{-z^2}\,dz - \frac{D}{2c\sqrt{D\pi t}} e^{\frac{cx_0}{D}} \int \left(x + \frac{D}{c}\right) e^{-\frac{(x+x_0+ct)^2}{4Dt}}\,dx$$

$$\lim a_{mn} = \frac{n}{m\,2^m}\;\frac{m(m-1)(m-2)\cdots\overbrace{\left(m-\frac{m-n}{2}+1\right)}^{\frac{m+n}{2}+1}}{1.2.3\cdots\frac{m-n}{2}}\;\frac{\frac{m+n}{2}!}{\frac{m+n}{2}!} = \frac{n}{m\,2^m}\;\frac{m!}{\frac{m-n}{2}!\,\frac{m+n}{2}!}$$

$$\log\frac{m!}{\frac{m-n}{2}!\,\frac{m+n}{2}!} = \frac{\left(\frac{m}{e}\right)^m \quad 2\sqrt{m}}{\left(\frac{m-n}{2e}\right)^{\frac{m-n}{2}}\left(\frac{m+n}{2e}\right)^{\frac{m+n}{2}}\sqrt{m^2-n^2}\,\sqrt{2\pi}}$$

$$= m\log m - \frac{m(1-\delta)}{2}\log\frac{m}{2}(1-\delta) - \frac{m}{2}(1+\delta)\log\frac{m}{2}(1+\delta) + \cdots$$

$$= \cancel{m\log m - \frac{m}{2}(1-\delta)\log m} + \frac{m}{2}(1-\delta)\log 2 + \frac{m}{2}(1-\delta)\left(\delta+\frac{\delta^2}{2}\right)$$

$$\cancel{-\frac{m}{2}(1+\delta)\log m} + \frac{m}{2}(1+\delta)\log 2 - \frac{m}{2}(1+\delta)\left(\delta-\frac{\delta^2}{2}\right)$$

$$= m\log 2 + \left[\begin{array}{l}\delta-\delta^2+\frac{\delta^2}{2}\\ -\delta-\delta^2+\frac{\delta^2}{2}\end{array}\right] =$$

$$\lim a_{mn} = \frac{n}{m}\,e^{-\frac{\delta^2 m}{2}}\sqrt{\frac{2}{\pi}}\;\frac{1}{\sqrt{m}}$$

$$\frac{m+n}{2} \quad \frac{m-n}{2}$$

$$+ \quad -$$

$$= \frac{n}{m}\sqrt{\frac{2}{m\pi}}\,e^{-\frac{1}{2}\left(\frac{n^2}{m}\right)}$$

$$m = \frac{t}{\tau} \qquad D = \frac{\delta^2}{2\tau}$$

$$e^{-\frac{x^2\tau}{2\delta^2 t}} = e^{-\frac{x^2}{4Dt}}$$

$$n\delta = x$$

$$\lim\frac{a_{mn}}{2\tau} = \frac{x}{2t\delta}\sqrt{\frac{2\tau}{\pi t}}\,e^{-\frac{x^2}{2Dt}} = \frac{x}{2t\sqrt{\pi D t}}\,e^{-\frac{x^2}{4Dt}}$$

$$z = \frac{x}{2\sqrt{Dt}}$$

$$dz = -\frac{dt}{4\sqrt{Dt^3}}\cdot x$$

$$\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-\frac{x^2}{4Dt}}\frac{x}{t\,2\sqrt{Dt}}\,dt = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-z^2}dz = 1$$

$$\int_0^\infty \frac{b}{2\sqrt{\pi D t^3}}\,e^{-\frac{b^2}{4Dt}}\,db = \frac{\sqrt{D}}{\sqrt{\pi t}}\int_0^\infty e^{-x}\,dx = \sqrt{\frac{D}{\pi t}}$$

$$\int \frac{e^{-z^2}dz}{\sqrt{z^2+\frac{cb}{D}}}\left[\sqrt{z^2+\frac{cb}{D}}-z\right]^{2n-1} = \int e^{-z^2}dz\left\{\sqrt{z^2+\alpha}^{\,2n-2}-\binom{2n-1}{1}z\sqrt{z^2+\alpha}^{\,2n-3}+\binom{2n-1}{2}z^2\sqrt{z^2+\alpha}^{\,2n-4}+\cdots\right\}$$

$$\frac{1}{\sqrt{\pi}}\int_0^\infty \frac{2e^{-z^2}dz}{\sqrt{z^2+\alpha}} = \frac{e^{\alpha}}{\sqrt{\pi}}\int_{\sqrt{\alpha}}^\infty e^{-y^2}dy$$

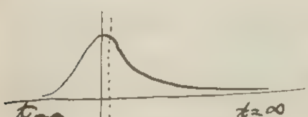
$$\beta z^2+\alpha = y^2$$

$$\int_0^\infty \frac{z\,e^{-z^2}dz}{\sqrt{\beta z^2+\alpha}} = \frac{1}{\beta}\int_{\sqrt{\alpha}}^\infty e^{-\frac{y^2-\alpha}{\beta}}dy = \frac{e^{\frac{\alpha}{\beta}}}{\beta}\int_{\sqrt{\alpha}}^\infty e^{-\frac{y^2}{\beta}}dy$$

$$\frac{1-\alpha}{1+\alpha}-\frac{1+\alpha}{1-\alpha}=\frac{(1-\alpha)^2-(1+\alpha)^2}{1-\alpha^2}$$

$$=\frac{-4\alpha}{1-\alpha^2}$$

Platma $\quad \bar{t} = \frac{b}{c}+\frac{D}{c^2} = \frac{b}{c}\left[1+\frac{D}{bc}\right]$



$$\Delta\tau = \frac{\partial\tau}{\partial\bar{t}}\,\Delta\bar{t}$$

$z=\infty$

~~$\tau = \frac{x_0}{c}\frac{1}{\sqrt{\pi}}\int_{z=-\infty}^{\infty} e^{-z^2}\frac{z\,dz}{\sqrt{z^2+\frac{cx}{D}}}$~~

$$\tau = \frac{x_0}{c\sqrt{\pi}}\left[\int_{-\Delta z}^\infty e^{-z^2}dz - \int_{-\Delta z}^\infty \frac{2e^{-z^2}}{\sqrt{z^2+\frac{cx}{D}}}dz\right]$$

$$=\frac{1}{2}\left\{\frac{}{\left[\int_{-\Delta z}^\infty e^{-z^2}dz+\int_{-\Delta z}^\infty\cdots\right]} - \frac{\int \quad + \int}{\int \quad - \int}\right\}$$

$$=\frac{x_0}{2c\sqrt{\pi}}\,4\cdot\frac{\int_{-\Delta z}^\infty \frac{2e^{-z^2}dz}{\sqrt{\ }}}{\int_{-\Delta z}^\infty e^{-z^2}dz} = \frac{2x_0}{c\sqrt{\pi}}\;\frac{\left[\frac{1}{2\sqrt{\frac{cx}{D}}}-\frac{(\Delta z)^2e^{-(\Delta z)^2}}{2\sqrt{(\Delta z)^2+\frac{cx}{D}}}\right]}{\frac{\sqrt{\pi}}{2}+e^{-(\Delta z)^2}\Delta z}\qquad \cancel{\left[\frac{\sqrt{\pi}}{2}+e^{-(\Delta z)^2}\Delta z\right]}$$

$$\neq \frac{2x_0}{c\sqrt{\pi}}\sqrt{\frac{D}{cx_0}}\left\{1-\underline{\underline{\Delta z}}+\left(\Delta z^2\cdots\right)\right\}$$

$$\tau = \frac{2t_0}{\sqrt{\pi}}\sqrt{\frac{D}{cb}}\left\{1-\underline{\underline{\frac{1}{2}\sqrt{\frac{D}{bc}}}}\cdots\right\}$$

$$z=\frac{x_0-ct}{\sqrt{4Dt}}$$

$$\Delta z = \frac{c\,\Delta t}{\sqrt{4Dt}}$$

$$=\Delta t\cdot\frac{c\sqrt{c}}{2\sqrt{Db}}$$

$$=\frac{D}{c^2}\frac{c}{2}\sqrt{\frac{c}{Db}}$$

$$\Delta z = \frac{1}{2}\sqrt{\frac{D}{bc}}$$

$$\frac{t^+ + t^-}{2} = \frac{x_0}{2c} \left\{ \frac{1-\alpha}{1+\alpha} + \frac{1+\alpha}{1-\alpha} \right\} = \frac{x_0}{c} \, \frac{1+\alpha^2}{1-\alpha^2} \approx \frac{x_0}{c}\left[1+2\alpha^2\right]$$

$$= \frac{b}{c}\left[1 + 2\,\frac{D}{\pi\, b c}\right] = \frac{b}{c} + \frac{2}{\pi}\,\frac{D}{c^2}$$

$$= \frac{b}{c}\left[1 + \frac{2}{\pi}\left(\frac{1}{z_{\text{Teilchen}}}\right)^2\right]$$

~~[illegible]~~

Wenn Teilchen von x_0 ausgehen und man beobachtet Zeit wann sie zum ersten Mal die Null Ebene überschreiten, so muss es darunter auch solche geben welche anfangs nach links gehen und später abermals durch die ~~Null~~ x_0 Ebene treten. Für solche muss die Gesamtzeit eingestellt werden (nicht die Zeit vom zweiten Durchtritt durch die x_0 Ebene!), wenn der allgemeine Durchschnitt der Fallzeiten $= \frac{b}{c}$ sein soll. Dann geschieht automatisch wenn bei einer Skala die Zeit vom Anfang bis Schluss beobachtet wird.

Fletcher Ph. Rev. 4

Drop No 12:

$t^+ = 4.365$
$t^- = 3.078$
$7.443 \quad 3.7215$
$t = 3.672$
$\Delta = 50 = 1.3\,\%$
Theor. $1.7\,\%$

8.349
6.480
$14.829 \quad 7.4145$
7.344
70
0.9%
theor. 0.8%

Drop No 9

4.708
3.109
$7.817 \quad 3.909$
3.795
$\Delta = 104 = 2.8\,\%$
76
28
theor: $2.2\,\%$

No 6:

2.410
3.410
$5.820 \quad 2.910$
2.844
$\Delta = 66 = 2.2\%$
theor $= 1.7\%$

No 5

2.735
1.997
$4.732 \quad 2.366$
2.325
$41 = 1.8\,\%$
theor.: $1.3\,\%$

$$\tau = \frac{2b}{c}\,\frac{J}{1-J^2} = \frac{2b}{c}\,J\left[1 + J^2 + J^4 \dots\right] \qquad J = \frac{2e^{\frac{cb}{D}}}{\sqrt{\pi}}\int_{\sqrt{\frac{cb}{D}}}^{\infty} e^{-y^2}dy =$$

$$= \frac{2b}{c\sqrt{\pi}}\,\frac{\cancel{\sqrt{D}}}{2}\underbrace{\left[1-\frac{z^2}{2}\right]\left[1+\frac{z^2}{\pi}\right]}_{\left(1-z^2\cdot\frac{\pi-2}{2\pi}\right)} \qquad = \frac{1}{\sqrt{\pi}}\sqrt{\frac{D}{cb}}\left[1-\frac{D}{2cb}+\dots\right]$$

$$\tau = \frac{M}{W} = \frac{3H\theta}{N}\,\frac{1}{Wc^2} \qquad \frac{1}{W} = B = \frac{D}{\frac{H\theta}{N}}$$

$$= \frac{3D}{C^2}$$

$$\overline{\sqrt{t}} = x_0\int_0^\infty \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2t\sqrt{\pi D}}\,dt \qquad \overline{\left(\frac{1}{\sqrt{t}}\right)} = x_0\int_0^\infty \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2t^2\sqrt{\pi D}}\,dt$$

$$= \frac{x_0}{2\sqrt{\pi D}}\int_{+\infty}^{-\infty} 2e^{-z^2}dz\,\frac{\left(-1+\frac{z}{\sqrt{}}\right)}{\left(-z+\sqrt{}\right)} = \frac{x_0}{\sqrt{\pi D}}\int_{-\infty}^{+\infty}\frac{e^{-z^2}dz}{\sqrt{z^2+\frac{cx_0}{D}}}$$

$$\overline{\left(\frac{1}{\sqrt{t}}\right)} = \frac{x_0}{\sqrt{\pi D}}\frac{c^2}{D}\int\frac{e^{-z^2}dz}{\sqrt{\dots}}\,\frac{(\dots)}{\left(-z+\sqrt{\dots}\right)^2} = \frac{\cancel{x_0}}{\sqrt{\pi D}}\,\frac{c}{\cancel{D}}\int\frac{e^{-z^2}dz}{\sqrt{}}\,\frac{\left(z+\sqrt{}\right)^2}{\frac{\cancel{x_0}}{\cancel{D}}} = \frac{c}{\sqrt{\pi D}}\int_{-\infty}^{+\infty}\frac{\left(2z^2+\frac{cx_0}{D}\right)}{\sqrt{}}e^{-z^2}dz$$

$$\bar{t} = \frac{x_0}{c} \qquad \bar{t}\,\overline{\left(\frac{1}{\sqrt{t}}\right)} - \overline{\left(\sqrt{t}\right)} = \frac{x_0}{\sqrt{\pi D}}\left\{\int_{-\infty}^{+\infty}\frac{2z^2+\frac{cx_0}{D}-1}{\sqrt{z^2+\frac{cx_0}{D}}}\,e^{-z^2}dz\right\}$$

$$b\,\frac{e^{-\frac{(b-ct)^2}{4Dt}}}{2\sqrt{\pi D t^3}}\,dt\;\frac{e^{-\frac{[b-c(T-t)]^2}{4D(T-t)}}}{2\sqrt{\pi D(T-t)^3}}$$

$$\cancel{(b^2-2bct+c^2t^2)(T-t)+[b^2-2bc(T-t)+c^2(T-t)^2]t=b^2T-2bc(tT-t^2+Tt-t^2)}$$

$$\frac{b^2}{t}-2bc+c^2t+\frac{b^2}{T-t}-2bc+c^2(T-t)=\frac{b^2T}{t(T-t)}-4bc+c^2T$$

$$\int_0^\infty \frac{e^{-\frac{b^2T}{t(T-t)}}}{\sqrt{t(T-t)}^3}\,dt=$$

$$\frac{b\sqrt{T}}{\sqrt{t(T-t)}}=2u\frac{b}{\sqrt{T}}\qquad \frac{T^2}{4}=u^2tT-u^2t^2$$

$$(ut-\tfrac{T}{2})^2=0$$

$$\frac{1}{2}\frac{b\sqrt{T}(T-2t)}{\sqrt{}^3}\,dt=du\qquad t=\frac{T}{2u}$$

$$=\int_0^\infty \frac{e^{-\frac{4u^2b^2}{T}}}{\left[\frac{T^2}{2u}\left(1-\frac{1}{2u}\right)\right]^{3/2}}\frac{T}{2u^2}du=4\int\frac{e^{-\frac{4u^2b^2}{T}}}{T^2(2u-1)^{3/2}}u\,du$$

$$2u-1=z$$
$$2u=z+1$$

$$=\frac{4}{T^2}\int\frac{e^{-\frac{(z+1)^2b^2}{T}}}{z^{3/2}}dz\,(z+1)$$

$$q\frac{\partial U}{\partial q}\qquad \dot q\frac{\partial T}{\partial \dot q}=2T$$

Schaefer u. [illegible] zur Th. d. [illegible] Phys. Z. 10, 630, 1909

Analog für unsere Problem:

$$\frac{\partial^2 u}{\partial t^2} - D\frac{\partial^2 u}{\partial x^2} - c\frac{\partial u}{\partial x} = 0$$

$$\left.\begin{array}{l} x=0 \\ x=\infty \end{array}\right\} u = 0$$

$$u = v \cdot e^{int}$$

$$v^{0,\infty} = 0$$

$$D\frac{\partial^2 v}{\partial x^2} + c\frac{\partial v}{\partial x} + n^2 v = 0 \quad \Big| \; K$$

$$D\frac{\partial^2 K}{\partial x^2} + c\frac{\partial K}{\partial x} = 0 \quad \Big| \; v \qquad \frac{\partial K}{\partial x}\Big|_{\xi-}^{\xi+} = 1 \qquad K^{0,\infty} = 0$$

$$D\int_0^\infty \left(K\frac{\partial^2 v}{\partial x^2} - v\frac{\partial^2 K}{\partial x^2}\right)dx + c\int\left(K\frac{\partial v}{\partial x} - v\frac{\partial K}{\partial x}\right) + n^2\int v\,K\,dx = 0$$

$$\pm \frac{\partial K}{\partial x}\frac{\partial v}{\partial x}$$

$$D\,\cancel{\left(K\frac{\partial v}{\partial x}\right)\Big|_0^\infty} - \cancel{D\frac{\partial v}{\partial x}\Big|_\xi} + D v_\xi = n^2\int_0^\infty v(x)\,K(x,\xi)\,dx + c\int_0^\infty\underbrace{\left[\frac{\partial v}{\partial x}K(x,\xi) - v(x)\frac{\partial K(x,\xi)}{\partial x}\right]}_{\cancel{vK} - \int\frac{\partial v}{\partial x}K}dx$$

$$\lim_{t=0}\int_0^{\frac{x-x_0}{2\sqrt{Dt}}} e^{-z^2}dz = \frac{\sqrt{\pi}}{2} - \frac{e^{-\frac{(x-x_0)^2}{4Dt}}}{\frac{x-x_0}{\sqrt{Dt}}} \quad (x > x_0)$$

$$\lim_{t=0}\int_0^{\frac{x+x_0}{2\sqrt{Dt}}} e^{-z^2}dz = \frac{\sqrt{\pi}}{2} - \frac{e^{-\frac{(x+x_0)^2}{4Dt}}}{\frac{x+x_0}{\sqrt{Dt}}}$$

$$\lim \int + \int = \sqrt{\pi} - \sqrt{Dt}\left(\frac{1}{x-x_0} + \frac{1}{x+x_0}\right) = \sqrt{\pi} - \sqrt{Dt}\cdot\frac{2x}{x^2-x_0^2}$$

$$\lim \int + \int \quad (x < x_0) \quad = \sqrt{Dt}\left(\frac{1}{x-x_0} - \frac{1}{x+x_0}\right) = \sqrt{Dt}\,\frac{2x_0}{x^2-x_0^2} = -\sqrt{Dt}\,\frac{2x_0}{x_0^2-x^2}$$

$$v_\xi = \frac{n^2}{D}\int_0^\infty v(x)\,K(x,\xi)\,dx + 2c\int_0^\infty \frac{\partial v}{\partial x}K(x,\xi)\,dx = \int_0^\infty\left[\frac{n^2}{D}v(x) + 2c\frac{\partial v}{\partial x}\right]K(x,\xi)\,dx$$

$$W = \sqrt{\frac{\beta}{2\pi D(1-e^{-2\beta t})}}\; e^{-\frac{\beta(x-x_0e^{-\beta t})^2}{2D[1-e^{-2\beta t}]}}$$

Bei gegebenem t <u>wahrscheinlichste</u> Änderungsgröße, welche $(x-x_0e^{-\beta t})^2$ zum Minimum macht

also $x = x_0 e^{-\beta t}$

also ist hier die durchschnittliche auch die wahrscheinlichste

$$\frac{\partial W}{\partial x} = -\sqrt{\quad\dots}\;\frac{\beta(x-x_0e^{-\beta t})}{D(1-e^{-2\beta t})}\, e^{-\frac{\beta\cdots}{2D\cdots}}$$

$$\frac{\partial^2 W}{\partial x^2} = \sqrt{\quad\dots}\left[\frac{\beta^2(x-x_0e^{-\beta t})^2}{D^2(1-e^{-2\beta t})^2} - \frac{\beta}{D(1-e^{-2\beta t})}\right]e^{-\frac{\beta}{2D}\cdots}$$

$$\frac{\partial W}{\partial t} = \cancel{\frac{3}{2}\frac{\beta^{3/2}e^{-2\beta t}}{\sqrt{2\pi D}\sqrt{1-e^{-2\beta t}}^{\,3}}}\left\{-\frac{\beta e^{-2\beta t}}{1-e^{-2\beta t}} - \frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})x_0e^{-\beta t}}{1-e^{-2\beta t}} + \frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})^2}{[1-e^{-2\beta t}]^2}e^{-2\beta t}\right\}$$

$$= \left\{\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})}{1-e^{-2\beta t}}\underbrace{\left[\frac{(x-x_0e^{-\beta t})e^{-2\beta t}}{1-e^{-2\beta t}} - x_0e^{-\beta t}\right]}_{\frac{xe^{-2\beta t}-x_0e^{-\beta t}}{1-e^{-2\beta t}}} - \frac{\beta e^{-2\beta t}}{1-e^{-2\beta t}}\right\}\sqrt{\quad}\; e^{\cdots}$$

$$\frac{\partial W}{\partial t} - D\frac{\partial^2 W}{\partial x^2} - x\beta\frac{\partial W}{\partial x} = \left\{\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})(x-x_0e^{+\beta t})e^{-2\beta t}}{(1-e^{-2\beta t})^2} - \frac{\beta e^{-2\beta t}}{1-e^{-2\beta t}}\right. -$$

$$-\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})^2}{(1-e^{-2\beta t})^2} \qquad + \frac{\beta}{1-e^{-2\beta t}}$$

$$\left. + \frac{\beta^2}{D}\frac{x(x-x_0e^{-\beta t})}{1-e^{-2\beta t}}\right\}\sqrt{\quad}\; e^{\cdots}$$

$$= \left[\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})}{(1-e^{-2\beta t})^2}\right]\left\{\cancel{xe^{-2\beta t}} - \cancel{x_0e^{-\beta t}} - \cancel{x} + \cancel{x_0e^{-\beta t}} + \cancel{x} - \cancel{xe^{-2\beta t}}\right\}.$$

$$= \frac{\beta}{1-e^{-2\beta t}}\left[1 - e^{-2\beta t}\right]\sqrt{\quad}\; e^{\cdots} = \cancel{\frac{\ }{2\sqrt{1-e^{-2\beta t}}}\;2 + 2e^{-\beta t} - 3e^{-2\beta t}}$$

$$= \beta W$$

$\frac{\partial}{\partial x}(xW) = W + x\frac{\partial W}{\partial x}$

$\frac{\partial^2}{\partial x^2} = 2\frac{\partial W}{\partial x} + x\frac{\partial^2 W}{\partial x^2}$

$$= \beta\left[\cancel{3} - \frac{2}{1-e^{-2\beta t}}\right] W$$

$$\frac{\partial W}{\partial t} - D\frac{\partial^2 W}{\partial x^2} - \beta\left(x\frac{\partial W}{\partial x} + W\right) = 2\beta\left[1 - \frac{1}{1-e^{-2\beta t}}\right] W$$

$$\log W = \frac{1}{2}\log\left(\frac{\beta}{2\pi D}\right) - \frac{1}{2}\log\left(1-e^{-2\beta t}\right) - \frac{\beta}{2D}\frac{(x-x_0 e^{-\beta t})^2}{1-e^{-2\beta t}}$$

$$\frac{1}{W}\frac{\partial W}{\partial t} = -\frac{\beta e^{-2\beta t}}{1-e^{-2\beta t}}$$

Falls die konstante Energie als determinierendes Moment der Br. B. betrachtet wird, (so wie es für Saiten tut übrigens) würden wir wohl die analoge aktuelle Schwingungsgleichung erhalten:

$$\frac{\partial^2 W}{\partial t^2} - \alpha\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + \beta\frac{\partial}{\partial x}(xW)$$

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und allgemeine Verteilung: $\underbrace{\mathfrak{W} = \int f(x_0)\, W(x, x_0, t)\, dx_0}$

Das ist ermöglicht durch Unabhängigkeit der zusammenwirkenden Partialprozesse. Es wäre aber nicht erlaubt z.B. für W(x) derart anzuwenden, weil hier der Wert von W(x) durch Existenz eines benachbarten x Dinge beeinflusst ist.

Somit ist die Gleichung erfüllt:

$$\frac{\partial W}{\partial t} - D\frac{\partial^2 W}{\partial x^2} - \beta\frac{\partial}{\partial x}(xW) = 0$$

Allgemein: Brownsche Bewegung unter Einwirkung einer Kraft: $F = f(x)$ wird definiert durch

Welches Integral d. DGl.

$$\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + \beta\frac{\partial}{\partial x}\left[W \cdot f(x)\right]$$

welche sich nach d. Ansatz — lösen lassen sollte?

Im Raume:

$$\frac{\partial W}{\partial t} = D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) W + \beta\underbrace{\left\{\frac{\partial}{\partial x}\left[W f(x)\right] + \frac{\partial}{\partial y}\left[W f(y)\right] + \frac{\partial}{\partial z}\left[W f(z)\right]\right\}}$$

$$= -\left(\frac{\partial W}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial W}{\partial y}\frac{\partial U}{\partial y} + \frac{\partial W}{\partial z}\frac{\partial U}{\partial z}\right) - W\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) U$$

Dasselbe bezieht sich auf Diffusion einer Substanz welche von äußeren Kräften beeinflusst wird z.B. $O_2 \leftrightarrow N_2$ in Erdatmosphäre etc.

(Gouy!)

Das ist eigentlich ein Spezialfall der Boltzmann Gleichung der Gastheorie.

$$\frac{\partial F}{\partial t} + \xi\frac{\partial F}{\partial x} + \eta\frac{\partial F}{\partial y} + \zeta\frac{\partial F}{\partial z} + X\frac{\partial F}{\partial \xi} + Y\frac{\partial F}{\partial \eta} + Z\frac{\partial F}{\partial \zeta} = \iint (F_1 F_1' - F F')\, d\omega \ldots$$

welche genau genommen doch wohl so lauten sollte: (?)

$$\frac{\partial F}{\partial t} + \frac{\partial(\xi F)}{\partial x} + \frac{\partial(\eta F)}{\partial y} + \frac{\partial(\zeta F)}{\partial z} + X\frac{\partial F}{\partial \xi} + Y\frac{\partial F}{\partial \eta} + Z\frac{\partial F}{\partial \zeta} = \cdots$$

Durch Mittelbildung (oder Integrierung ?) erhält man nämlich wenn

~~Also setzt sich Kraft zusammengesetzt aus zwei Teilen:~~

$\cancel{f = x}$

Für die Schwankungen der Teilchenzahl muss im Grenzfall genau u. dieselbe Formel gelten wie für Br. B. mit elastischer Kraft, sofern man sich auf geringe Unterschiede δ beschränkt

Also sollte die Diff.-Gl. herauskommen:

$$\frac{\partial W}{\partial t} = a\frac{\partial^2 W}{\partial x^2} + b\left[\frac{\partial W}{\partial x}(x-\nu) + W\right]$$ oder besser:

$$\frac{\partial W(\delta)}{\partial t} = a\frac{\partial^2 W(\delta)}{\partial \delta^2} + b\frac{\partial}{\partial\delta}(W\delta)$$

Das ist wohl nicht der Fall, denn für $\overline{\Delta_x^2}$ erhält man nicht das Analogon der Elast. Formel.

Versuch ob ein lösbares Beispiel sich ergibt durch Annahme $f(x) = a - \frac{b}{x}$

$$\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + a\frac{\partial W}{\partial x} - b\frac{\partial}{\partial x}\left(\frac{W}{x}\right)$$

Falls $a=0$:

Versuch: $\frac{x^2}{t} = z^2$

$x = z\sqrt{t}$

$z = \frac{x}{\sqrt{t}}$

$$\cancel{dW = \frac{\partial W}{\partial x}dx + \frac{\partial W}{\partial t}dt = \frac{\partial W}{\partial z}dz}$$

$$\frac{\partial W}{\partial t} = \frac{dW}{dz}\frac{\partial z}{\partial t} = -\frac12\frac{x}{\sqrt{t^3}}\frac{dW}{dz} = -\frac12\frac{z}{t}\frac{dW}{dz}$$

$$\frac{\partial W}{\partial x} = \frac{dW}{dz}\frac{\partial z}{\partial x} = \frac{1}{\sqrt t}\frac{dW}{dz}$$

$$\frac{\partial^2 W}{\partial x^2} = \frac{d}{dz}\left(\frac{\partial W}{\partial x}\right)\frac{\partial z}{\partial x} = \frac1t\frac{d^2W}{dz^2}$$

$$-\frac12\frac{z}{t}\frac{dW}{dz} = \frac{D}{t}\frac{d^2W}{dz^2} - \frac{b}{z\sqrt t}\frac{1}{\sqrt t}\frac{dW}{dz} + \frac{bW}{z^2 t}$$

$$D\frac{d^2W}{dz^2} + \left(\frac12 z - \frac{b}{z}\right)\frac{dW}{dz} + \frac{b}{z^2}W = 0$$

$$\frac{\partial^2}{\partial x^2}(xW) = x\frac{\partial^2 W}{\partial x^2} + 2\frac{\partial W}{\partial x}$$

$$\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + \beta\frac{\partial}{\partial x}[W\cdot f(x)]$$

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} xW\,dx = D\frac{\partial}{\partial x}(xW)\Big| + \int\Big(\underbrace{\beta x\frac{\partial}{\partial x}[Wf]}_{\beta xWf\big| - \int \beta W f\,dx} - \underbrace{2D\frac{\partial W}{\partial x}}_{-2DW\big|}\Big)\,dx$$

$\therefore$
$$\frac{\partial}{\partial t}\cdot\int_{-\infty}^{+\infty} xW\,dx = -\beta\cdot\int_{-\infty}^{+\infty} W\cdot f(x)\cdot dx$$

(Dasselbe auch direkt ~~aus~~ aus DGl. durch zweimalige Integration nach x)

Der Veranschaulichung der Br. B. und Behandlung ~~als~~ gemäss irreversibler Dynamik hätte man:

$$\frac{dx}{dt} = -\beta f(x)$$

Hier gilt also Wahrscheinlichkeit

$$\frac{\partial(\bar{x})}{\partial t} = -\beta\bar{f}$$

das ist aber nicht dasselbe denn $\bar{f} = \int_{-\infty}^{+\infty} W f(x)\,dx$

braucht nicht identisch zu sein mit $f(\bar{x}) = f\left(\int xW\,dx\right)$

Es ~~wird aber nur~~ muss in dem Falle identisch sein wenn $f(x) = \alpha x$

aber ob auch sonst, ist von vornherein nicht ersichtlich

Für $t = \infty$

$\frac{\partial W}{\partial t} = 0$
$$D\frac{\partial^2 W}{\partial x^2} + \beta\frac{\partial}{\partial x}(Wf) = 0$$

$$D\frac{dW}{dx} = -\beta W f(x) + c$$ ~~[illegible]~~ $c = 0$ denn für $f(x) = 0$ muss $W = \text{const}$

$$W = A e^{-\frac{\beta}{D}\int f(x)\,dx} = A e^{-\frac{\beta}{D}U}$$

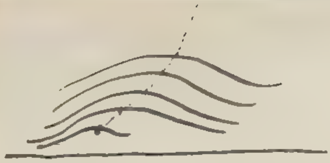
Für $t = 0$

bleibt aus physikalischen Gründen nur $\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2}$ übrig

---

Wie muss der Punkt wandern, für welchen $\frac{\partial W}{\partial x}\Big|_t = 0$, also der Punkt W_{max} ?

$$\frac{d}{dt}(W_m) = ?$$



$W = \varphi(x,t)$

$dW = \frac{\partial\varphi}{\partial x}\,dx + \frac{\partial\varphi}{\partial t}\,dt$

~~$dW = 0$ = für solche dx, dt für welche $\frac{dx}{dt} = -\frac{\frac{\partial W}{\partial x}}{\frac{\partial W}{\partial t}}$~~

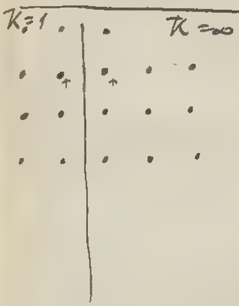
$W = \varphi(x,t)$

$\frac{\partial\varphi}{\partial x} = 0$

$$W = \varphi(x_0 t_0) + \Delta x \underbrace{\left(\frac{\partial\varphi}{\partial x}\right)_0}_{=0} + \Delta t \left(\frac{\partial\varphi}{\partial t}\right)_0 + \frac{1}{2}\Delta x^2 \left(\frac{\partial^2\varphi}{\partial x^2}\right)_0 + \Delta x\,\Delta t\,\frac{\partial^2\varphi}{\partial x\,\partial t} + \frac{\Delta t^2}{2}\frac{\partial^2\varphi}{\partial t^2} + -$$

$$\frac{\partial W}{\partial(\Delta x)} = \Delta x \left(\frac{\partial^2\varphi}{\partial x^2}\right)_0 + \Delta t \left(\frac{\partial^2\varphi}{\partial x\,\partial t}\right)_0 = 0 \quad (\max)$$

$$\left(\frac{\Delta x}{\Delta t}\right)_{\max} = -\frac{\frac{\partial^2\varphi}{\partial x\,\partial t}}{\frac{\partial^2\varphi}{\partial x^2}} = -\frac{D\frac{\partial^3 W}{\partial x^3} + \rho\,\frac{\partial^2}{\partial x^2}(W f(x))}{\frac{\partial^2 W}{\partial x^2}}$$



Sobald Entfernung = ½ Molekeldistanz, bleibt die Anordnung ungeändert so als ob von Wand überhaupt kein Unterschied gäbe

Dagegen für kleinere ~~Abstand~~ Entfernung resultiert scheinbar abstoßende / anziehende } Kraft aber erstere grösser als letztere

grösser

daher im Ganzen eine abstoßende Kraft; es wird also die mittlere Konzentration der + Ionen in den Oberflächenschichten geringer sein als normal

und das müsste in noch höherem Grad für mehrwertige Ionen gelten, da die elektr. Kräfte ∝ e^2

Somit müssten die Randschichten vor Allem Mangel zeigen an mehrwertigen Ionen

Wenn also, wie in Wirklichkeit der Fall ist, die Wandschichten (in Wasser) negativ sind gegen die innern Schichten so heisst das dass darin mehrwertige positive Ionen fehlen.

Das ist schwer begreiflich, ebenso wäre der Einfluss von NaOH etc. Basiten unbegreiflich wenn es nur auf die Wertigkeit ankäme.

Ob aber der Einfluss mehrwertiger Ionen überhaupt in dieser Richtung geht?

Enormer Einfluss von $Th(NO_3)_4$ konstat.; ~~[illegible]~~ wirkt im Sinne einer Umkehrung jener Doppelschicht, also so dass Wand~~[illegible]~~ positiv wird gegen Flüssigkeit; dagegen wäre Ologen zufolge gerade das Umgekehrte zu erwarten.

$$\frac{0{\cdot}2\,\text{mg}}{1\,\text{l}} =$$

$$\begin{array}{r} 14 \\ \underline{48} \\ \underline{62{\cdot}4} \\ 248 \\ \underline{232} \\ 480 \end{array}$$

absolutes Mol.gewicht $= \frac{480}{N}$

auf 0·2 mg entfallen daher $\frac{0{\cdot}2}{480} N$ Th Molek.

$$n = 10^{-3}\cdot\frac{0{\cdot}2}{500}\cdot 6.10^{23} = 0{\cdot}3\,.10^{18}$$

also Abstände der Molek $= \frac{1}{\sqrt[3]{n}} = \sqrt[3]{3{\cdot}3\,.10^{-18}} = 1{\cdot}5\,.10^{-6}$ cm !

Allerdings bewirkt dies nichts betreffs der Abstände in der Grenzschicht selbst!

Experimentum crucis für oder gegen den Einfluss der (Ionentheorie) DK auf die Doppelschicht:

Ist die Wirkung mehrwertiger Ionen zurückzuführen auf die durch die DK der Wand bewirkten Anziehungs und Abstossungskräfte so müsste die Wirksamkeit derselben sich gerade umkehren wenn die DK umgekehrt werden also für den Fall $K_1 > K_2$

müssten Kationen so wirken wie

Anionen im Falle $K_1 < K_2$

Gibt es aber ein ionisierendes Lösungsmittel mit hinreichend kleinem K ?

$$u_{+0} = e^{\frac{cx}{2D}} \int_{-\infty}^{+\infty} \left[\sin\beta x - \frac{2D\beta}{c} \cos\beta x \right] f(\beta)\, d\beta$$

z.B. $f(\beta) = e^{-m\beta}$ $\quad f(-\beta) = e^{-m\beta}$

$$\int_0^\infty e^{-m\beta} \sin\beta x \, d\beta = \frac{x}{x^2+m^2} \qquad \int_0^\infty \beta e^{-m\beta} \cos\beta x \, dx = \frac{1}{x^2+m^2} - \frac{2x^2}{(x^2+m^2)^2}$$

$$= \frac{m^2-x^2}{(x^2+m^2)^2}$$

$$u_0 = e^{-\frac{cx}{2D}} \left[\frac{x}{x^2+\frac{c^2}{4D^2}} - \frac{2D}{c} \frac{\frac{c^2}{4D^2}-x^2}{\left(\frac{c^2}{4D^2}+x^2\right)^2} \right] = e^{-\frac{cx}{2D}} \frac{2D}{c} \frac{\frac{xc}{2D}\left(\frac{c^2}{4D^2}+x^2\right) - \frac{c^2}{4D^2}+x^2}{\left(\frac{c^2}{4D^2}+x^2\right)^2}$$

$$e^{-\frac{cx}{2D}} \left[F(x) - \frac{2D}{c} F'(x) \right] = \varphi(x)$$

wie kann man daraus f bestimmen, wenn φ gegeben ist?

$$F - \frac{2D}{c} F' = e^{+\frac{cx}{2D}} \varphi(x) = \Phi(x)$$

$$y' + ay = X$$
$$z' + az = 0$$

$$\frac{(y'z - z'y)}{z^2} = \frac{X}{z}$$

$$\frac{d}{dx}\left(\frac{y}{z}\right) = \uparrow$$

$$y = z \int \frac{X}{z} dx$$

$$= e^{-ax} \int X \, dx$$

$$F' - \frac{c}{2D} F = - \frac{c}{2D} \Phi$$

$$F = -\frac{c}{2D} e^{\frac{cx}{2D}} \int e^{-\frac{cx}{2D}} \Phi(x) \, dx = \overbrace{-\frac{c}{2D} e^{\frac{cx}{2D}} \int \varphi(x)\, dx}^{\Phi(x)}$$

$$= \int_{-\infty}^{+\infty} \sin\beta x \cdot f(\beta)\, d\beta$$

$$= \frac{1}{2} \int_0^\infty d\alpha \sin\alpha x \int_{-\infty}^{+\infty} \Phi(y) \sin\alpha y \, dy$$

$$f(\beta) = \frac{1}{2} \int_{-\infty}^{+\infty} \Phi(y) \sin\beta y \, dy$$

$$= -\frac{c}{2D} \int_0^\infty e^{\frac{cy}{2D}} \sin\beta y \cdot \int^y \varphi(x)\, dx \cdot dy$$

Tatsächlich nach früher: $\int_0^{+\infty} \varphi(x)\,dx = -\frac{2D}{c} e^{-\frac{cx}{2D}} f(x) \Big|_0^{+\infty} + \frac{2D}{c}\int e^{-\frac{cx}{2D}} f'(x)\,dx \cancel{- \frac{2D}{c}\int e^{-\frac{cx}{2D}} f(x)\,dx} = -\frac{2D}{c} f(0)$

$= -\frac{2D}{c}\int \sin(\beta) f(\beta)\,d\beta = 0$

$\cancel{e^{-\frac{cx}{2D}} \int_{-\infty}^{+\infty} \sin\beta x}$ Probe: $\varphi(x) = -\frac{c}{2D}\int \varphi(x)\,dx + \frac{2D}{A} e^{-\frac{cx}{2D}} \left[\frac{c}{2D} e^{\frac{cx}{2D}} \int \varphi(x)\,dx + e^{\frac{cx}{2D}} \varphi(x)\right]$

stimmt

wenn $u = A e^{-\frac{cx}{D}}$:

$$\int_0^\infty u\,dx = A\int_0^\infty e^{-\frac{cx}{D}}\,dx = \frac{AD}{c}$$

$$u = \frac{C}{D} e^{-\frac{cx}{D}} - \lim \sqrt{\frac{\varepsilon}{\pi}}\, e^{-\frac{(x_0-x)^2}{\varepsilon}} = \varphi(x)$$

$$\int e^{-mx}\sin\beta x\,dx = -\frac{e^{-mx}}{m}\sin\beta x + \frac{\beta}{m}\int e^{-mx}\cos\beta x\,dx$$

$$\int e^{-mx}\cos\beta x\,dx = -\frac{e^{-mx}}{m}\cos\beta x - \frac{\beta}{m}\int e^{-mx}\sin\beta x\,dx$$

$$J = -\frac{e^{-mx}\sin\beta x}{m} - \frac{\beta}{m^2} e^{-mx}\cos\beta x - \frac{\beta^2}{m^2} J$$

$$J = -\frac{[m\sin\beta x + \beta\cos\beta x]\,e^{-mx}}{m^2+\beta^2}$$

$$-\frac{2D}{c} f(\beta) = \int_0^\infty e^{\frac{cy}{2D}} \sin\beta y \int_0^y \varphi(x)\,dx \cdot dy =$$

$$= \cancel{\int_0^y \varphi(x)\,dx \cdot \int e^{\frac{cy}{2D}} \sin\beta y\,dy} \Big|_0^\infty - \int_0^\infty \varphi(y) \frac{\left[\frac{c}{2D}\sin\beta y + \beta\cos\beta y\right] e^{-\frac{c}{2D}y}}{\beta^2 + \frac{c^2}{4D^2}}\,dy$$

$y < x_0$

$$\int_0^y \varphi(x)\,dx = \left[1 - e^{-\frac{cy}{D}}\right] \;\|$$

$$= -e^{-\frac{cy}{D}} \quad | \; y > x_0$$

$$-\frac{2D}{c} f(\beta) = \int_0^\infty e^{\frac{cy}{2D}} \sin\beta y \int_0^y \varphi(x)\,dx\,dy = \int_0^{x_0} e^{\frac{cy}{2D}} \sin\beta y \left[1 - e^{-\frac{cy}{D}}\right] dy - \int_{x_0}^\infty e^{\frac{cy}{2D}} \sin\beta y \cdot e^{-\frac{cy}{D}}\,dy$$

$$= \int_0^{x_0} e^{\frac{cy}{2D}} \sin\beta y\,dy - \int_0^\infty e^{-\frac{cy}{2D}} \sin\beta y\,dy$$

$$= \frac{\left[\frac{c}{2D}\sin\beta x_0 - \beta\cos\beta x_0\right] e^{\frac{cx_0}{2D}} - \beta}{\frac{c^2}{4D^2} + \beta^2}$$

Grenzfall für $x_0=0$:

$$u=\frac{C}{D}e^{-\frac{cx}{D}}+\frac{e^{-\frac{x^2}{4Dt}-\frac{cx}{2D}-\frac{c^2t}{4D}}}{\sqrt{D\pi t}}-\frac{C}{D}\frac{e^{-\frac{cx}{D}}}{\sqrt{\pi}}\int_{-\infty}^{\frac{x-ct}{2\sqrt{Dt}}}e^{-z^2}dz$$

$$=\frac{C}{D}e^{-\frac{cx}{D}}\left[1-\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\frac{x-ct}{2\sqrt{Dt}}}e^{-z^2}dz\right]+\frac{e^{-\frac{(x+ct)^2}{4Dt}}}{\sqrt{\pi D t}}$$

$$\bar{x}=\underbrace{\frac{C}{D}\int_0^\infty x e^{-\frac{cx}{D}}dx}_{-\frac{e^{-\frac{cx}{D}}}{\frac{c}{D}}\left(x+\frac{D}{c}\right)\Big|_0^\infty}-\underbrace{\frac{C}{D}\frac{1}{\sqrt{\pi}}\int_0^\infty x e^{-\frac{cx}{D}}dx\int_{-\infty}^{\frac{x-ct}{2\sqrt{Dt}}}e^{-z^2}dz}+\frac{1}{\sqrt{\pi Dt}}\int_0^{} x e^{-\frac{(x+ct)^2}{4Dt}}dx$$

$$\underbrace{-\frac{e^{-\frac{cx}{D}}}{\frac{c}{D}}\left(x+\frac{D}{c}\right)\int_{-\infty}^{\frac{x-ct}{\sqrt{Dt}}}\Big|_0^\infty}_{+\frac{D^2}{C^2}\int_{-\infty}^{-\frac{c}{2}\sqrt{\frac{t}{D}}}e^{-z^2}dz}\underbrace{+\frac{D}{C}\int_0^\infty e^{-\frac{cx}{D}}\left(x+\frac{D}{c}\right)\frac{e^{-\frac{(x-ct)^2}{4Dt}}}{2\sqrt{Dt}}dx}_{+\frac{D}{C}\int_0^\infty\left(x+\frac{D}{c}\right)\frac{e^{-\frac{(x+ct)^2}{4Dt}}}{2\sqrt{Dt}}dx}$$

~~$\frac{x+ct}{2\sqrt{Dt}}=u$~~

~~$x=2u\sqrt{Dt}-ct$~~

~~$dx=2du\sqrt{Dt}$~~

~~$\frac{D}{C}\int\frac{2u\sqrt{Dt}-ct+\frac{D}{C}}{2\sqrt{Dt}}e^{-u^2}du$~~

$$\bar{x}=\frac{D}{C}-\frac{D}{C}\frac{1}{\sqrt{\pi}}\int_{-\infty}^{-\frac{c}{2}\sqrt{\frac{t}{D}}}e^{-u^2}du+\underbrace{\frac{1}{2\sqrt{\pi Dt}}\int_0^{} x e^{-\frac{(x+ct)^2}{4Dt}}dx-\frac{D}{C}\int_0^\infty\frac{e^{-\frac{(x+ct)^2}{4Dt}}}{2\sqrt{\pi Dt}}dx}$$

$\frac{x+ct}{2\sqrt{Dt}}=u$

$x=2u\sqrt{Dt}-ct$

$dx=2du\sqrt{Dt}$

$$=\int_{+\frac{c}{2}\sqrt{\frac{t}{D}}}^\infty\left[2u\sqrt{Dt}-ct-\frac{D}{C}\right]\frac{e^{-u^2}du}{\sqrt{\pi}}$$

$$=2\sqrt{\frac{Dt}{\pi}}\int_{\frac{c}{2}\sqrt{\frac{t}{D}}}^\infty u e^{-u^2}du+\frac{ct}{\sqrt{\pi}}\int e^{-u^2}du$$

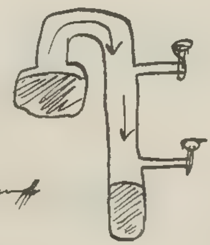
$$\cancel{\bar{x} = \sqrt{\frac{Dt}{\pi}}\, e^{-\frac{c^2t}{4D}} + \frac{ct}{\sqrt{\pi}} \int_{-\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} e^{-u^2} du}$$

$$\cancel{\bar{x} = \frac{D}{c} - \frac{2D}{c} \frac{1}{\sqrt{\pi}} \int_{\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} e^{-u^2} du}$$

$$\bar{x} = \frac{D}{c} - \frac{1}{\sqrt{\pi}}\left(ct + \frac{2D}{c}\right) \int_{\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} e^{-u^2} du + \sqrt{\frac{Dt}{\pi}}\, e^{-\frac{c^2t}{4D}}$$

$\lim_{t=0} \bar{x} = \sqrt{\frac{Dt}{\pi}}$ ebenso wie Brown'sche Bewegung

$\lim_{t=\infty} \bar{x} = \frac{D}{C}$



Methode zur Bestimmung d. Diffusionscoeff.

M. Luft aus Hg Dampf oder anderen Flüssigkeits Dampf

Auch Bestimmung d. Diffusionscoeff von Gummigutt Teilchen:

1). Verteilung im Schwerefeld : $u = \frac{C}{D} e^{-\frac{cx}{D}}$

2). " " bei gleichzeitiger entgegengesetzter convectiver Strömung (in der poröse Zwischenwand verwendet wird):

$$u = \frac{C'}{D} e^{-\frac{c'x}{D}}$$

Oder Diffusionscoeff von Teilchen die spezif. leichter sind als Wasser:

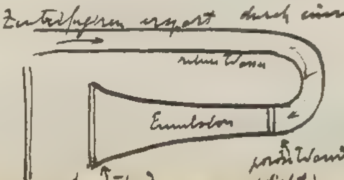
1). Verteilung im Schwerefeld an der Oberfläche

2). " " " bei gleichzeitiger Verdampfung oder Kondensation

was denselben Effekt hat wie Überlagerung einer convectiven Strömung

Dasselbe ginge auch bei Gummigutt teilchen falls sie hinreichend klein sind

Fraktioniertes Zentrifugieren ersetzt durch einmalige Zentrifugierung bei gleichzeitigem Gegenstrom



Querschnitt $q = \frac{A}{x}$

Dabei wirkt der Apparat unabhängig von Rotationsgeschwindigkeit falls Wasserstrom und Zentrifugalkraft verursacht

$$W(n,m) = P\,W(n-1,m) + (1-P)\,W(n-1,m-1)$$

$$W(n,m) - W(n-1,m) = (1-P)\left[W(n-1,m-1) - W(n-1,m)\right]$$

Für grosse n, m Grenzfall:

$$\frac{\partial W}{\partial n} + (1-P)\frac{\partial W}{\partial m} = 0$$

$$W\left[n(P-1)+m\right] = \text{const} = W\left[nP - (n-m)\right]$$

$$\lambda_m \theta = 2950$$

$$\varepsilon = \frac{hc}{\lambda} = \frac{6{,}5\cdot 10^{-27}\cdot 3\cdot 10^{10}}{2950}\cdot\theta = 6{,}6\cdot 10^{-20}\cdot\theta$$

$$E = \frac{mc^2}{2} = \frac{3}{2}k\theta = \frac{3}{2}\,\frac{8\cdot 10^{7}}{6\cdot 10^{23}}\,\theta = 2\cdot 10^{-16}\cdot\theta$$

$$\frac{\varepsilon}{E} = 3{,}3\cdot 10^{-4}$$

also sind Quanten klein im Verh. zur m. kin. En. derselben Temperatur

---

Rotierende Scheibe, Wahrscheinlichkeit gewisser Trefferlagen

Wenn in zwei Kammern geteilt so ist Wahrsch. (· | ·) ebenso gross wie (·· |) + (| ··)

denn

I	II
12	
1	2

2	1
	12

gleichmögliche Fälle

$\frac{2!}{1!\,1!}$ $\quad\frac{2!}{0!\,2!}\quad\frac{2!}{2!\,0!}$

Wenn in drei ~~Kammern~~ Zellen geteilt

123			13	2		13		2	23			3	12		3	1	2
12	3		1	23		1	3	2	2	13			123			13	2
12		3	1	2	3	1		23	2	1	3		12	3		1	23

, ...

Alle drei in einer $\frac{3}{27} = \frac{1}{9}$ $: 3 = \frac{1}{27}$

zwei in einer und eine extra $\frac{18}{27} = \frac{2}{3} : 6 = \frac{1}{9}$

drei extra $\frac{6}{27} = \frac{2}{9} : 1 = \frac{2}{9}$

Dagegen wenn die Zellen individuell betrachtet

$\frac{3!}{3!\,0!\,0!} = 1$ [3 Arten]

$\frac{3!}{1!\,2!\,0!} = 3$ [6 Arten]

$\frac{3!}{1!\,1!\,1!} = 6$ [1 Art]

$$\sum{}'\left[(x_n - x_\kappa)^2 + (y_n - y_\kappa)^2 + (z_n - z_\kappa)^2\right] = S$$ soll Minimum werden während gleichzeitig

$$\sum\left[x_n^2 + y_n^2 + z_n^2\right] = C$$ konstant bleibt

Also $\underbrace{\sum(x_n x_\kappa + y_n y_\kappa + z_n z_\kappa)}_{\cancel{\cos}}$ soll Maximum werden

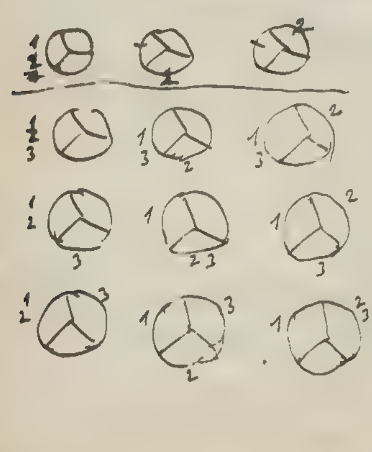
$$\sum \; x_n\delta x_n - x_n\delta x_\kappa - x_\kappa\delta x_n + x_\kappa\delta x_\kappa$$
$$\lambda(x_n\delta x_n \qquad + x_\kappa\delta x_\kappa)$$

$$\sum\left\{\delta x_n\left[x_n(1+\lambda) - x_\kappa\right] + \delta x_\kappa\left[x_\kappa(1+\lambda) - x_n\right]\right\} = 0$$

$$\lambda = \frac{x_\kappa - x_n}{x_n}$$

Es ist also (bei individueller Zelleneinteilung) am wahrscheinlichsten, dass alle Zellen gleich durch Teilchen erhalten, also sollte ein gewisses Teilchenbild eine Wahrscheinlichkeit besitzen unabhängig von der Zelleneinteilung und kontinuierlich variabel. Gibt es nicht eine derartige Wahrsch.-Funktion?

Vielleicht ~~quadratische Abweichung von~~ Quadratsumme der Differenzen?



Andere Form jener Lösung:

$$u = \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x-x_0)^2}{4Dt}} + e^{-\frac{(x+x_0)^2}{4Dt}}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} + \int_{\frac{x+x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}\,dz \cdot \frac{c}{D\sqrt{\pi}}\, e^{-\frac{cx}{D}}$$

Dagegen bei Ogerly für den Fall $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$:

$$u = \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x_0-x)^2}{4Dt}} + e^{-\frac{(x_0+x)^2}{4Dt}} - 2h\, e^{hx}\underbrace{\int_x^{\infty} e^{-hx - \frac{(x_0+x)^2}{4Dt}}\,dx}_{e^{\frac{cx_0}{D}+\frac{c^2 t}{D}}}\right]$$

Versuch: $u = U \cdot e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$

$\frac{\partial u}{\partial x} = \left[-\frac{c}{2D}U + \frac{\partial U}{\partial x}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$

$D\frac{\partial u}{\partial x} + cu = \left[\frac{c}{2}U + D\frac{\partial U}{\partial x}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$

$$D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = \left[D\frac{\partial^2 U}{\partial x^2} - \frac{c^2}{4D}U - \frac{\partial U}{\partial t} + \frac{c^2}{4D}U\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$$

~~Somit erhält man Lösung von~~ $D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$

mit: $D\frac{\partial u}{\partial x} + cu = 0 \;\|\; x = 0$

wenn man die Lösung von

$D\frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial t} = 0$

mit: $D\frac{\partial U}{\partial x} + \frac{c}{2}U = 0 \;\|\; x = 0$

multipliziert mit $e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$

$\frac{\partial u}{\partial x} = -\frac{c}{2D}U + \frac{\partial U}{\partial x}$.

Also erhält man U nach Ogerly durch Einsetzen $h = -\frac{c}{2D}$ und

$$u = \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x_0-x)^2}{4Dt}} + e^{-\frac{(x_0+x)^2}{4Dt}}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} + \frac{1}{2\sqrt{\pi D t}}\,\frac{c}{D}\, e^{-\frac{cx}{D} + \frac{cx_0}{2D} - \frac{c^2 t}{4D}} \underbrace{\int_x^{\infty} e^{+\frac{cx}{2D} - \frac{(x_0-x)^2}{4Dt}}\,dx}_{e^{-\frac{cx}{D}}\int_{\frac{x-x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}dz}$$

$\boxed{\text{stimmt}}$!

Allgemeiner Versuch $u = U V$ $\quad D\frac{\partial^2 u}{\partial x^2} + f\frac{\partial u}{\partial x} + u\frac{\partial f}{\partial x} - \frac{\partial u}{\partial t} = 0$

$$D\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(u f) - \frac{\partial u}{\partial t} = 0$$

$$V\left[D\frac{\partial^2 U}{\partial x^2} + U\frac{\partial f}{\partial x} + f\frac{\partial U}{\partial x} - \frac{\partial U}{\partial t}\right] + 2D\frac{\partial V}{\partial x}\frac{\partial U}{\partial x} + D\frac{\partial^2 V}{\partial x^2}U + f U\frac{\partial V}{\partial x} - U\frac{\partial V}{\partial t} = 0$$

für $f = \text{const}$:

$$V\left[D\frac{\partial^2 U}{\partial x^2} + c\frac{\partial U}{\partial x} - \frac{\partial U}{\partial t}\right] + 2D\frac{\partial V}{\partial x}\frac{\partial U}{\partial x} + D\frac{\partial^2 V}{\partial x^2}U + c U\frac{\partial V}{\partial x} - U\frac{\partial V}{\partial t} = 0$$

$$D\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t} \qquad c\frac{\partial}{\partial x}(UV) + 2D\frac{\partial U}{\partial x}\frac{\partial V}{\partial x} = 0 \quad ?$$

$$D\frac{\partial^2 U}{\partial x^2} = \frac{\partial U}{\partial t}$$

oder:

Allgemein erhält man Lösung von $D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$

mit Grenzbedingung I). $u = 0$

für $x = 0$: II). $\frac{\partial u}{\partial x} = 0$

III). Wärme

$\frac{\partial u}{\partial x} + h u = 0$

indem man die Lösung von

$$D\frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial t} = 0$$

mit Grenzbedingungen

$$\left.\begin{array}{l} \text{I). } u = 0 \\ \text{II). } U + \frac{2D}{c}\frac{\partial U}{\partial x} = 0 \\ \text{III). } \left(h - \frac{c}{2D}\right)U + \frac{\partial U}{\partial x} = 0 \end{array}\right\} \text{ mit } e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} \text{ multipliziert}$$

$$W(x_0 x)_{(k+1)\tau}\,dx = \int_{-\infty}^{+\infty} W(x_0\alpha)_{k\tau}\, W(\alpha x)_\tau\, d\alpha\, dx$$

$$W(\alpha x)_\tau = \frac{1}{2\sqrt{\pi D t}}\, e^{-\frac{\left(x-\alpha-\frac{\beta}{\alpha}\right)^2}{4Dt}}\, dx$$ oder auch $e^{-\frac{\left(x-\alpha-\frac{\beta}{x}\right)^2}{4Dt}}$ $\beta = \varepsilon$

$$W(x_0\alpha)_t = \frac{1}{2\sqrt{\pi D t}}\, e^{-\frac{\left(\alpha-x_0-\frac{\beta}{x_0}\right)^2}{4Dt}}\, d\alpha$$

$$W(x_0 x)_{2\tau}\, dx = \left(\frac{1}{2\sqrt{\pi D t}}\right)^2 e^{-\frac{\left(x_0+\frac{\beta}{x_0}\right)^2 - 2\alpha\left(x_0-\frac{\beta}{x_0}\right)+\alpha^2+x^2-2\alpha x - \frac{2\beta x}{\alpha}+\alpha^2+\frac{\beta^2}{\alpha^2}+2\beta}{4Dt}}$$

$$\lim_{\beta=0} P = 1 - \frac{2}{\sqrt{\pi}}\left[\beta - \frac{\beta^3}{3}\right] + \frac{1}{\beta\sqrt{\pi}}\left[\frac{\beta^2}{1} - \frac{\beta^4}{2}\right]$$

$$= 1 - \frac{2\beta}{\sqrt{\pi}} + \frac{2\beta^3}{3\sqrt{\pi}} + \frac{\beta}{\sqrt{\pi}} - \frac{\beta^3}{2\sqrt{\pi}} = 1 - \frac{\beta}{\sqrt{\pi}} + \frac{\beta^3}{6\sqrt{\pi}}$$

$$t \sim \frac{1}{\beta^2} \qquad z = e^{-\frac{1}{\beta^2}}$$

$\omega = 2n\pi$

$$-\lg z = \frac{1}{\beta^2}$$

$$P = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{1}{\sqrt{\lg z}}} e^{-y^2}dy + \frac{\sqrt{\lg z}}{\sqrt{\pi}}\left[1 - e^{+\frac{1}{\beta^2}}\right]$$

$$U = i\omega = 4\pi\left(1 - \frac{r}{\sqrt{r^2+a^2}}\right)\overbrace{2\varepsilon n}^{i} = 4\varepsilon\omega\left(1-\frac{r}{\sqrt{a^2+r^2}}\right)$$

$$F = 4\pi i\left[\frac{1}{\sqrt{a^2+r^2}} - \frac{1}{2}\frac{r^2}{\sqrt{}^3}\right] = 4\pi i\,\frac{2a^2+r^2}{\sqrt{a^2+r^2}^{3/2}}$$

$$W = i t^{\prime\prime} M$$

$$M = \frac{a^2\pi\, 4\pi}{r}\left(1+\frac{a^2}{2r^2}\right) \qquad = \frac{4\pi i}{r}\left[1+\frac{2a^2}{r^2}\right]\left[1-\frac{3}{2}\frac{a^2}{r^2}\right]$$

$$= \frac{4\pi^2 a^2}{r} \qquad = \frac{4\pi i}{r}\left[1+\frac{a^2}{2r^2}\right]$$

Drei-atom. -Debye Model H_2

$\frac{e^2}{r} = 6.10^{-14}$

$$x = \frac{(4{\cdot}7)^2 10^{-20}}{6.10^{-14}} = 4.10^{-6}$$

$f_{(r)} = \cancel{15} \cdot 6 e d^2$

$$\frac{d^2}{dr^2}\left(2\frac{e^2 d^2}{r^3}\right) = 24\frac{e^2 d^4}{r^5} = 6.10^{-14}$$

$$r^5 = 24 . 4.10^{-6} . (0{\cdot}3)^4 . 10^{-32} = 10^{-38} = 100.10^{-40}$$

$$r = \sqrt[5]{100} . 10^{-8} = 2.10^{-8}$$

$f_{(r)} = 6\frac{e^2}{r^4} d^2$

$$U_{(r)} = \frac{3}{2}\frac{e^2 d^2}{r^3} = 6.10^{-14}$$

$$r^3 = \frac{3}{2}\frac{e^2}{6.10^{-14}} . d^2 = 6.10^{-6} . 0{\cdot}09 . 10^{-16}$$
$$= \frac{1}{2} . 10^{-22} = 50 . 10^{-24}$$

$$\underline{\underline{r = 4.10^{-8}}}$$

$$W = (2\sum_n n)^2 \frac{4\pi^2 a^2}{r} = 4\pi^2 . \frac{10^{-16}}{4} . 4 . \left(\frac{4{\cdot}7 . 10^{-10}}{3 . 10^{10}}\right)^2 . \frac{4}{6} . 10^{32} \frac{1}{r}$$

$$= 10^{-15} \cdot \frac{16 . 3{\cdot}2}{6} . 10^{-8} \frac{1}{r}$$

$$r = \frac{10^{-15} . \frac{8 . 3{\cdot}2}{3} . 10^{-8}}{6.10^{-14}} = 10^{-9} . 1{\cdot}5$$

18.18
$\underline{174}$
324

Also sind magnet. Anziehungskräfte bei kleinen Distanzen viel geringer wie die elektr. Abstossung, doch muss wieder letztere in grösseren Distanzen überwiegen, da die gegenseitige magnet. Energie abnimmt wie $\frac{1}{r}$, die elektrostat. wie $\frac{1}{r^5}$

~~zugleich [illegible] wenn mit (Anzahl d. Elektronen im Ringe)~~^{2} ~~wächst Anziehung~~

Bei einatomigen Molek. wird wohl der Ring immer der wirksamen magn. Kraft angepasst welche von d. benachbarten ~~Ring~~ Atom ausgeübt wird.

Born & Karman Ph. Z. 13, 298 1912

$$m\ddot{u}_n = \alpha(u_{n+1} - 2u_n + u_{n-1}) \qquad -m\nu^2 = \alpha\left[e^{i\varphi} + e^{-i\varphi} - 2\right]$$

$$u_n = U e^{i(\nu t + n\varphi)} \qquad\qquad = -4\alpha \sin^2\frac{\varphi}{2}$$

$$\boxed{na = \lambda = \frac{2\pi a}{\varphi}} \qquad \nu = \nu_0 \sin\frac{\varphi}{2} \qquad \nu_0 = 2\sqrt{\frac{\alpha}{m}}$$

Wellengeschw. $\omega = \frac{\nu\lambda}{2\pi} = \frac{\nu_0\lambda}{2\pi}\sin\frac{a\pi}{\lambda} = \frac{a\nu}{2\arcsin\frac{\nu}{\nu_0}}$

Also ν hat Maximalwert ν_0 Grenzfrequenz wobei  und $\omega = \frac{a\nu_0}{\pi}$ $\left(\neq \frac{2}{\pi}c\right.$

Minimalwert $\nu = 0$ und $\omega = \frac{a\nu_0}{2}$

p. 305:

$$T = \frac{m}{2}\left[\cdots \dot{u}_{-2}^2 + \dot{u}_{-1}^2 + \dot{u}_0^2 + \dot{u}_1^2 + \cdots\right]$$

$$V = \frac{\alpha}{2}\left[\cdots (u_{-1}-u_0)^2 + (u_0-u_1)^2 + \cdots\right] = \alpha\left[\cdots u_{-1}^2 + u_0^2 + u_1^2 + u_2^2 + \cdots - u_{-1}u_0 - u_0u_1 - u_1u_2 \cdots\right]$$

Normalkoordinaten

$$U_n = \cdots k_{-1n}u_{-1} + k_{0n}u_0 + k_{1n}u_1 + k_{2n}u_2 + \cdots$$

so dass

$$T = \frac{m}{2}\left[\cdots \dot{U}_{-2}^2 + \dot{U}_{-1}^2 + \dot{U}_0^2 + \dot{U}_1^2 + \cdots\right]$$

$$V = \frac{\alpha}{2}\left[\cdots p_{-1}U_{-1}^2 + p_0 U_0^2 + p_1 U_1^2 + \cdots\right]$$

Dann jede Koordinate unabhängig von übrigen.

$$m\ddot{U}_n + \alpha p_n U_n = 0$$

$$U_n = A_n \sin(\nu_n t + \vartheta_n) \quad ||| \quad \nu_n = \sqrt{p_n\frac{\alpha}{m}}$$

Auffindung der Normalkoordinaten:

Da U sich linear aus u zusammensetzen, muss auch gelten:

$$m\ddot{U}_n = \alpha(U_{n+1} - 2U_n + U_{n-1})$$

und wenn U_n wirklich Normalkoordin. sind, so heisst das, dass jede unabhängig von d. übrigen schwingen kann also

$U_n = \cancel{\ldots} \; A_n \sin(\nu_n t + \varepsilon_n)$ also

$\ddot U_n = -\nu_n^2 U_n = -p_n \frac{\alpha}{m}$

(33) $\therefore \; -p_n U_n = U_{n+1} - 2U_n + U_{n-1}$ dies Gleichungssystem besitzt im Allg. keine Lösung (für beliebige p) dazu ist nötig dass Det.

$$\begin{vmatrix} \ldots & & & & & & \\ \ldots 0 & -1 & 2-p & -1 & 0 & 0 & \ldots \\ \cdot\; 0 & 0 & -1 & 2-p & -1 & 0 & \ldots \\ \cdot\; 0 & 0 & 0 & -1 & 2-p & -1 & -0 \\ & & \ldots & & & & \end{vmatrix} = 0$$

Wurzeln davon $p_1, p_2, p_3 \ldots$

Wenn diese in (33) eingesetzt so folgen n verschiedene Lösungssysteme welche den Relationen erfüllen [illegible]

$$\sum_m k_{mn} k_{mn'} = 0 \qquad (n \neq n')$$

und bei entsprechender Wahl d. willkürlichen Faktoren:

$$\sum_m k_{mn}^2 = 1$$

dann sind dies gerade die gesuchten Koeffizienten der k für U

Anzahl der in einem Frequenzintervall $d\nu$ befindlichen Normalschwingungen:

$$N(\nu)\, d\nu = N L \frac{d\varphi}{2\pi} \qquad \left(\nu = \nu_0 \sin\frac{\varphi}{2}\right)$$

L = Länge der Saite

N = Anzahl Massenpunkte pro Längeneinheit

$$u = \frac{c}{D}\frac{e^{-\frac{cx}{D}}}{\sqrt{\pi}}\int_{\frac{x+x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}dz + \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x-x_0)^2}{4Dt}} + e^{-\frac{(x+x_0)^2}{4Dt}}\right]e^{-\frac{c(x-x_0)}{2D}-\frac{c^2t}{4D}}$$

$x = x_0$:

$$u = \frac{c}{D}\frac{e^{-\frac{cx_0}{D}}}{\sqrt{\pi}}\int_{\frac{2x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}dz + \frac{1}{2\sqrt{\pi D t}}\left[1 + e^{-\frac{x_0^2}{Dt}}\right]e^{-\frac{c^2t}{4D}}$$

$$z = \frac{x+x_0-ct}{2\sqrt{Dt}}$$

$$\frac{1}{\sqrt{\pi}}\int_0^{\infty}\frac{e^{-\frac{(\xi-\alpha t)^2}{4Dt}}d\xi}{2\sqrt{Dt}} - \cancel{\int_0^{\infty}\frac{e^{-\frac{(\xi+\alpha t)^2}{4Dt}}d\xi}{2\sqrt{Dt}}} = \frac{1}{\sqrt{\pi}}\left[\int_{-\frac{\alpha t}{2\sqrt{Dt}}}^{\infty} e^{-u^2}du - \int_{\frac{\alpha t}{2\sqrt{Dt}}}^{\infty} e^{-u^2}du\right] = \frac{1}{\sqrt{\pi}}\frac{\alpha t}{2\sqrt{Dt}}$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\frac{\alpha t}{2\sqrt{Dt}}} e^{-u^2}du =$$

$$W(x,r) = 1 - 2\int_r^{\infty}\frac{e^{-\frac{\xi^2}{4Dt}}}{2\sqrt{\pi D t}}d\xi = 1 - \frac{2}{\sqrt{\pi}}\int_{\frac{r}{2\sqrt{Dt}}}^{\infty} e^{-z^2}dz = 1 - \frac{2}{\sqrt{\pi}}\frac{e^{-\frac{r^2}{4Dt}}}{\frac{r}{\sqrt{Dt}}} = 1 - \frac{2}{\pi}\sqrt{\frac{Dt}{r}}\,e^{-\frac{r^2}{4Dt}}$$

$$W^n = e^{-\frac{2n}{\pi}\sqrt{\frac{Dt}{r}}\left(e^{-\frac{r^2}{4Dt}}\right)}$$

$$W(x) \int \frac{e^{-\frac{(\xi+\eta-\alpha t)^2}{4Dt}} \, d\eta \, d\xi}{\sqrt{\pi D t}}$$

$$\frac{\partial W}{\partial t} = D \frac{\partial^2 W}{\partial x^2} + \beta \frac{\partial}{\partial x}\left[W f\right]$$

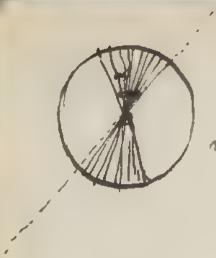
$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty} x^2 W \, dx = D \int \underbrace{x^2 \frac{\partial^2 W}{\partial x^2} \, dx}_{\cancel{x^2 \frac{\partial W}{\partial x}} - 2\int \underbrace{x \frac{\partial W}{\partial x} dx}_{\cancel{xW} - \int \underbrace{W dx}_{=1}}} + \beta \int \underbrace{x^2 \frac{\partial}{\partial x}(Wf) \, dx}_{\cancel{x^2 Wf} - 2\int \underbrace{x (Wf)}\, dx}$$

$$\frac{\partial}{\partial t}(\overline{x^2}) = 2D - 2\beta \int_{-\infty}^{+\infty} x \, W(x) \, f(x) \, dx \qquad \frac{\partial}{\partial t}(\overline{x}) = -\beta \int_{-\infty}^{\infty} W f \, dx$$

$$\frac{\partial}{\partial t}\overline{(x-x_0)^2} = \frac{\partial(\overline{x^2})}{\partial t} - 2x_0 \frac{\partial(\overline{x})}{\partial t} = 2D - 2\beta \int_{-\infty}^{+\infty} x . W . f . dx + 2x_0 \beta \int W f \, dx$$

$$= 2D + 2\beta \int_{-\infty}^{+\infty} (x_0 - x) \, W(x) \, f(x) \, dx$$

$$= 2D - 2\beta \, \overline{\left[(x_0 - x) f(x)\right]}$$



Wahrscheinlichkeit gewisser Trefferlagen auf der rotierenden Scheibe:

relat. Wahrsch. dass zwei Treffer in A keiner in B : $\frac{2!}{2!0!} = 1 \quad = \frac{1}{4}$

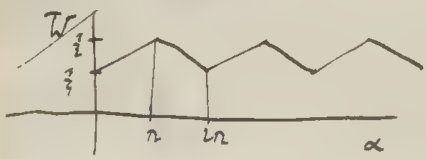
" ein Treffer in A einer in B : $\frac{2!}{1!1!} = 2 \quad = \frac{1}{2}$

" keine Treffer in A, zwei in B : $\frac{2!}{0!2!} = 1 \quad = \frac{1}{4}$

Kann man nicht daraus die Wahrsch. eines Winkelabstands α, unabhängig von der Lage der Zellenteilung ~~ab~~ ableiten? Indem man die Zelle rotieren lässt

$$W = 2\left(\frac{1}{2}\frac{\alpha}{2\pi} + \frac{1}{4}\frac{\pi-\alpha}{2\pi}\right) = \frac{\alpha}{\pi}\left(\frac{1}{2}-\frac{1}{4}\right) + \frac{1}{4} = \frac{1}{4}\left(1+\frac{\alpha}{\pi}\right) \qquad \alpha < \pi$$

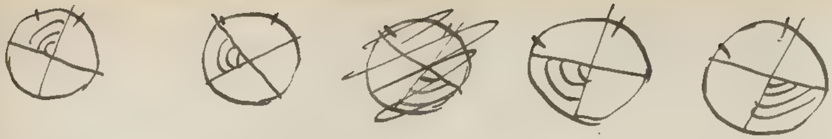
$$W = 2\left(\frac{1}{4}\frac{\alpha-\pi}{2\pi} + \frac{1}{2}\frac{2\pi-\alpha}{2\pi}\right) = \frac{\alpha}{\pi}\left(\frac{1}{4}-\frac{1}{2}\right) + 1 - \frac{1}{4} = 1 - \frac{1}{4}\left(1+\frac{\alpha}{\pi}\right) \qquad \alpha > \pi$$



Dabei würde aber doch W von der Zahl der Zellen abhängen, denn würde man z.B. 4 Zellen anbringen, so wäre z.B.

W. für 2 in A, 0 B, 0 C, 0 D $= \frac{2!}{2!0!0!0!} = 1 = \frac{1}{16}$

A	B	C	D		
2	0	0	0	$= \frac{2!}{2!0!0!0!} = 1$	$= \frac{1}{16}$
1	1	0	0	$= \frac{2!}{1.} = 2$	$= \frac{1}{8}$
1	0	1	0	= 2	
1	0	0	1	= 2	
0	2	0	0	= 1	
0	1	1	0	= 2	
0	1	0	1	= 2	
0	0	1	1	= 2	
0	0	2	0	= 1	
0	0	0	2	= 1	



$$W = 4\left[\frac{\alpha}{2\pi}\,\frac{1}{8} + \left(\frac{\pi}{2}-\alpha\right)\frac{1}{2\pi}\,\frac{1}{16}\right] = \frac{\alpha}{4\pi} + \frac{1}{16} - \frac{\alpha}{8\pi} = \frac{1}{16}\left(1+\frac{2\alpha}{\pi}\right)$$

Offenbar ist W abhängig von Zellenteilung und es hat gar keinen Sinn von der ~~[illegible]~~ Wahrscheinlichkeit einer gewissen Trefferanordnung zu sprechen, ohne die Zellenteilung zu definieren. An und für sich ist jede Trefferanordnung gleich wahrscheinlich?

Andrerseits wenn es sich z.B. um Verwechseln weisser und schwarzer Kugeln handelt, so entspricht das unzweifelhaft einem Wachsen der Wahrscheinlichkeit (als einem reellen Vorgang). Wie ist dieser Begriff aber zu definieren ohne Bezug auf eine (willkürliche) Zellenteilung?

Eindimensionaler Vorgang: ●●●○○   ~~[illegible]~~ sorge ist weiss

~~gleichberechtigt~~ sind alle einzelnen Permutationen der individuellen Elemente; darunter sind aber viele gleichwertig wenn wir die Individualität nicht unterscheiden

gleichberechtigt ist jede bestimmte Kombination

z.B.: ●●● ○○○   und: ●●●●●●

Zu untersuchen auf einem [illegible]

Dagegen wenn wir eine Statistik der Abstände von einem ~~[illegible]~~ weiss bis zum nächsten weiss untersuchen, wird die Formel $\left. e^{-\nu}\frac{\nu^n}{n!}\right|_{\nu=1} = \frac{1}{n!\,e} = W(n)$ die Wahrsch. eines n-fachen Abstandes bedeuten ? (falls durchschnittlich ebensoviel weiss wie schwarz)

Bei kontinuierlicher Wahrscheinlichkeit z.B.: wenn Scintillationen auf der t Axe registriert werden

$W(t)\,dt = k\,e^{-kt}\,dt$

Hiermit lässt sich die Wahrsch. verschiedener Distanzen abschätzen; aber Frage ob eine bestimmte Verteilung wahrsch. oder unwahrsch. ist?

$W(t_1)\,dt_1\; W(t_2)\,dt_2\; W(t_3)\,dt_3 \ldots = k^n e^{-k(t_1+t_2+\ldots t_n)}\,dt_1\,dt_2 \ldots dt_n$

also kommt es nur auf die Summe $t_1+t_2+\ldots t_n$ an und die Verteilung ist ganz gleichgültig!

[illegible]

Nadzwyczajne wydanie

„CZAS" wychodzi codziennie o godzinie 8-tej wieczór.
Numer pojedynczy 25 gr.

PRENUMERATA MIESIĘCZNIE WYNOSI:

Krakowie bez odnoszenia do domu . . . . zł. 5.40
Krakowie z odnoszeniem do domu . . . . zł. 6.—
prowincję z przesyłką pocztową . . . . zł. 6.—
anicę z przesyłką pocztową . . . . zł. 10.—

Za każdą zmianę adresu dolicza się zł. 0.50.

lamacje niezapieczętowane nie podlegają opłacie pocztowej.
Listów nieopłaconych nie przyjmuje się.

renumeratę przyjmują: Adm istracja „Czasu", wszystkie urzędy pocztowe, wszystkie miejsco i zamiejscowe Biura dzienników.

Redakcja r sów nie zwraca.

C

KO
Telefon Reda
Adres Redakcji
Godziny biur

Z Bezpartyjnego Bloku

12 żydów — 12 socjalistów —

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$\frac{1}{\varepsilon}\frac{r}{m} \equiv \frac{q}{n} \equiv \frac{q^2}{\cancel{m}} \equiv \frac{\cancel{q}\,K}{\varepsilon m^2} \equiv 1$

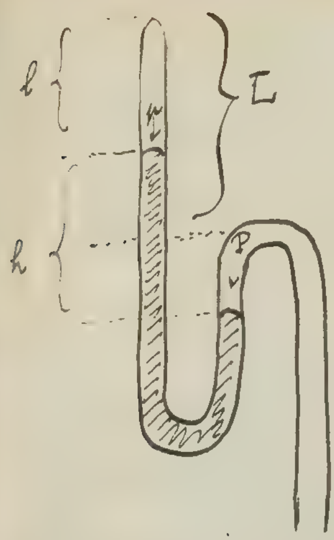
$m = \frac{1}{10}$

$v = \frac{1}{10}$

$r \equiv \varepsilon m$

$q \equiv \cancel{n} \equiv \sqrt{m}$

$K = \frac{\varepsilon m^2}{q} = \varepsilon m^{3/2}$



$$pv = R\vartheta$$

$$p\,l = c\theta \qquad \Big| \qquad \cancel{p_0 l_0 = c\theta_0}$$

$$P = p + h \qquad \Big| \qquad \cancel{P_0 = p_0 + h_0}$$

$$l + \frac{h}{2} = L$$

$$p\left(L - \frac{h}{2}\right) = c\theta$$

$$P = p + h$$

$$P = h + \frac{c\theta}{L - \frac{h}{2}} \qquad \Big\| \qquad P_0 = h_0 + \frac{c\,\theta_0}{L - \frac{h_0}{2}}$$

$$2PL - P\frac{h}{\cancel{2}} = 2hL - \frac{h^2}{\cancel{2}} + 2c\theta$$

$$h^2 \pm h(P + 2L) = 2(c\theta - PL)$$

$$h = \frac{P + 2L}{2} \pm \sqrt{\left(\frac{P+2L}{2}\right)^2 + 2(c\theta - PL)}$$

$$P = h + \frac{\theta}{\theta_0}(P_0 - h_0)\,\frac{L - \frac{h_0}{2}}{L - \frac{h}{2}}$$

$$c = \frac{(P_0 - h_0)\left(L - \frac{h_0}{2}\right)}{\theta_0}$$

$$h = \frac{P + 2L}{2} - \sqrt{\left(\frac{P - 2L}{2}\right)^2 + 2\frac{\theta}{\theta_0}(P_0 - h_0)\left(L - \frac{h_0}{2}\right)}$$

$$= L + \frac{P}{2} - \sqrt{\left(\frac{P}{2} - L\right)^2 + \frac{\theta}{\theta_0}(P_0 - h_0)(2L - h_0)}$$

[illegible] ∂

$$h = L + \frac{P}{2} - \sqrt{\left(\frac{P}{2} - L\right)^2 + (P_0 - h_0)(2L - h_0)}$$

$\rho\left(u\frac{\partial u}{\partial x}+\dots\right) = \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\mu}{\rho}\Delta^2 u$

. . . .

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

~~[illegible]~~ m u
n x
α μ
b p
r ρ

Podobieństwo dynamiczne

$$\frac{m^2}{n} = \frac{b}{rn} = \frac{\alpha}{r}\frac{m}{n^2}$$

$$m^2 = \frac{b}{r} = \frac{\alpha m}{rn}$$

$$\boxed{m^2 = \frac{b}{r} \quad \Big| \quad m = \frac{\alpha}{rn}}$$

$n=1$ $\quad m = \frac{\alpha}{r}$ $\quad b = rm^2 = \frac{\alpha^2}{r}$

Symetrya prądu i tworzenie się promieni będzie zatem zależeć (dla różnych cieczy) od $b=\frac{\alpha^2}{r}$

wtedy: $m=\frac{\alpha}{r}$

|| Tem większa asymetrya im większe α

|| z drugiej strony podobieństwo wtedy ~~[illegible]~~

przy stałem b r $m/1$ $\alpha \infty n$ t.j. tworzenie promieni

tem łatwiejsze im większy otwór | jeśli równe stałe |

Przy stałem α, r:

$m=\sqrt{b}$ $\quad n=\frac{1}{\sqrt{b}}$

Podobne przy zwiększeniu ciśnienia a zmniejszeniu rozmiarów

$$\frac{b}{h}\frac{m^2}{n} \equiv \frac{b}{n} \equiv \beta\frac{n}{n^2} \equiv \frac{b}{h}$$

$$\frac{mb}{n} \equiv \beta\frac{m^2}{n^2} \equiv \gamma\frac{h}{n^2}$$

$$m^2 = h = n\ ; \quad \beta = \gamma\ ; \quad nb = m\beta$$

$$\frac{1}{\eta}\frac{d\eta}{d\theta} = \frac{1}{\eta}\left(\frac{\partial\eta}{\partial\theta}\right)_{v=\text{const}} + \frac{1}{\eta}\left(\frac{\partial\eta}{\partial v}\right)_{\theta=\text{const}}\frac{\partial v}{\partial\theta} = \frac{1}{\eta}\frac{\partial\eta}{\partial\theta} + \frac{1}{\eta}\frac{\partial\eta}{\partial p}\,\frac{\frac{1}{v}\frac{\partial v}{\partial\theta}}{\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_{\theta=\text{const}}}\ .$$

$$= \frac{1}{\sqrt{T}} - \frac{\alpha}{\beta}\cdot\frac{1}{\eta}\frac{\partial\eta}{\partial p}$$

Co do $\left(\frac{\partial\eta}{\partial v}\right)_{\theta=\text{const}}$:

Pod założeniem kul różnią się rozmaite gazy tylko co do wielkości cząstki σ i mas μ

Porównajmy dwa gazy o wielkościach σ_1 σ_2 .
(o równej ilości cząsteczek) $N_1 = N_2$ μ_1 μ_2

Ruchy ich będą dynamicznie podobne, jeśli: ~~$\frac{\rho_1}{\ }\ \frac{\ }{\rho_2} = \sigma_1^3 : \sigma_2^3$~~

$$V_1 : V_2 = \sigma_1^3 : \sigma_2^3 \qquad \text{I).}$$

$$\rho_1 : \rho_2 = \frac{\mu_1}{\sigma_1^3} : \frac{\mu_2}{\sigma_2^3}$$

i jeśli $c_1 : c_2 = \sigma_1 : \sigma_2$

zatem $T_1 : T_2 = \mu_1 c_1^2 : \mu_2 c_2^2 = \mu_1\sigma_1^2 : \mu_2\sigma_2^2$ II).

Wtedy $\xrightarrow{\text{I}}$ $\xleftarrow{\text{II}}$

$$\frac{\partial u_1}{\partial y_1} = \frac{\partial u_2}{\partial y_2}$$

$$X_{y_1} : X_{y_2} = \mu_1 c_1^2 : \mu_2 c_2^2$$

$$\omega_1 : \omega_2 = \sigma_1^2 : \sigma_2^2$$

$$\eta_1 : \eta_2 = \frac{X_{y_1}}{\omega_1} : \ldots = \frac{\mu_1 c_1}{\sigma_1^2} : \frac{\mu_2 c_2}{\sigma_2^2} = \frac{\mu_1}{\sigma_1} : \frac{\mu_2}{\sigma_2}$$

podobieństwo $V_1 \circ dN_1$ $\quad dN_1 : dN_2 = \sigma_1^3 : \sigma_2^3 = V_1 : V_2$

$V_2 \circ dN_2$

$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial v}\right)_1 : \left(\frac{1}{\eta}\frac{\partial\eta}{\partial v}\right)_2 = \frac{1}{\sigma_1^3} : \frac{1}{\sigma_2^3} \quad \Big\| \quad \left(\frac{v}{\eta}\frac{\partial\eta}{\partial v}\right)_1 : \left(\quad\right)_2 = 1 : 1$$

$$\frac{T_1}{\eta_1} : \frac{T_2}{\eta_2} = \sigma_1^3 : \sigma_2^3$$

$$v \cdot \frac{1}{\eta}\frac{\partial\eta}{\partial v} = \frac{\frac{1}{\eta}\frac{d\eta}{d\theta} - \frac{1}{T}}{\alpha}$$

$\eta = fc.(v, \theta)$ $\quad$ Normalsubstanz: σ_0, μ_0, η_0

$$\eta_{\theta=\theta_0} = fc.(v, \theta_0) = \varphi(v)$$

$\eta(v,\theta)$ porównać z $\eta(v,\theta_0)$ $\Big\|$: $\eta(v,\theta) : \eta(v,\theta_0) = \sqrt{\theta} : \sqrt{\theta_0}$

$$\eta(v,\theta) = \sqrt{\frac{\theta}{\theta_0}}\,\eta(v,\theta_0) = \sqrt{\frac{\theta}{\theta_0}}\,\varphi(v)$$

$$\varphi(v) = \sqrt{\frac{\theta_0}{\theta}}\,\eta(v,\theta)$$

Przy gęstości $\quad \rho_1 = \overbrace{\frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^3}^{\alpha}\rho_2$

i temperaturze: $\quad \theta_1 = \overbrace{\frac{\mu_1}{\mu_2}\left(\frac{\sigma_1}{\sigma_2}\right)^2}^{\beta}\theta_2$

będą lepkości w stosunku: $\quad \eta_1 = \underbrace{\frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)}_{\gamma}\eta_2$

$\cancel{\sqrt{\alpha\gamma} = \frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^0}$

$\cancel{\beta\sqrt{\alpha\gamma} = \left(\frac{\mu_1}{\mu_2}\right)^2}$ $\quad \cancel{\alpha\beta = \left(\frac{\mu_1}{\mu_2}\right)^2\frac{\sigma_2}{\sigma_1}}$

$\cancel{\frac{\alpha}{\beta} = \frac{\sigma_1}{\sigma_2}}$

$$\frac{\alpha}{\gamma} = \left(\frac{\sigma_2}{\sigma_1}\right)^2$$

$$\sqrt{\alpha\gamma} = \frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^2$$

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{\alpha\gamma^3}}{\alpha}$$

$$\beta = \sqrt{\frac{\gamma^3}{\alpha}}\cdot\frac{\gamma}{\alpha} = \sqrt{\frac{\gamma^5}{\alpha^3}}$$

$$\eta_1\left[\frac{\mu_1}{\mu_2}\left(\frac{6_2}{6_1}\right)^3\rho_2,\ \frac{\mu_1}{\mu_2}\left(\frac{6_1}{6_2}\right)^2\theta_2\right] = \frac{\mu_1}{\mu_2}\frac{6_2}{6_1}\cdot\eta_2[\rho_2,\theta_2]$$ $\gamma^5 = \beta^2\alpha^3$ 134

$$\underbrace{\eta_1[\alpha\rho_2,\ \beta\theta_2]} = \gamma\cdot\eta_2[\rho_2,\theta_2] = \sqrt[5]{\beta^2\alpha^3}\cdot\eta_2[\rho_2,\theta_2]$$

$$= \sqrt{\beta}\cdot\eta_1[\alpha\rho_2,\ \theta_2] = \sqrt[5]{\beta^2\alpha^3}\cdot\eta_2[\rho_2,\theta_2]$$

$$\eta_1[\alpha\rho_2,\ \theta_2] = \sqrt[10]{\frac{\alpha^6}{\beta}}\cdot\eta_2[\rho_2,\theta_2] \qquad \sqrt[10]{\frac{\alpha^6}{\beta}} = \sqrt{\frac{\mu_1}{\mu_2}}\cdot\left(\frac{6_2}{6_1}\right)^2$$

ważne przy dowolnej temperaturze, ale przy oznaczonym stosunku objętości

$$\eta_2[\rho_2,\theta_2] = \sqrt{\frac{\mu_2}{\mu_1}}\left(\frac{6_1}{6_2}\right)^2\cdot\eta_1\left[\frac{\mu_1}{\mu_2}\left(\frac{6_2}{6_1}\right)^3\rho_2;\ \theta_2\right]$$

$$\eta_2[\rho_2,\theta_2] = \varepsilon\cdot\eta_1[\alpha\rho_2,\ \theta_2]$$

$$\eta_2[\rho_1\ \theta] = \varepsilon\ \eta_1[\alpha\rho_1,\ \theta]$$

Wartości Sachsa: Densel$_{200}$: $\frac{1}{\eta}\frac{\partial\eta}{\partial p} = 0{\cdot}000930$ (atm.)

$\frac{1}{v}\frac{\partial v}{\partial p} = 0{\cdot}000091$

$\eta_{10} = 42{\cdot}4$ } 73·9

$\eta_{30} = 31{\cdot}5$

$10{\cdot}9 : 37 = 0{\cdot}295 : 20$

35     $0{\cdot}0147$

$\frac{1}{\eta}\frac{d\eta}{d\theta} = 0{\cdot}0147$

$\alpha_{20} = 0{\cdot}00123$

$$\frac{\alpha}{\beta}\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{0{\cdot}00123\cdot 0{\cdot}000093}{0{\cdot}000091} = 0{\cdot}0123\cdot\frac{93}{91}$$

25   124

$0{\cdot}00000129$ 40

$0{\cdot}0000511$

$0{\cdot}0011763$

$0{\cdot}0012274$

$$\frac{\partial}{\partial T}(\sqrt{T}) = \frac{1}{2}\frac{1}{\sqrt{T}} = \frac{1}{2}\frac{1}{293} = \frac{1}{586} = .166$$

4

$-\boxed{0{\cdot}0148}$

$+0{\cdot}0017$

$-0{\cdot}0131$! $\frac{1}{\eta}\frac{d\eta}{d\theta}$ obliczo

$\boxed{(C_2 (\partial_\vartheta)_2 O}$

$\frac{1}{\eta}\frac{\partial \eta}{\partial p} = 0{\cdot}000730$

$\frac{1}{v}\frac{\partial v}{\partial p} = 0{\cdot}000173$

$\alpha = 0{\cdot}00155$

$\frac{d}{\beta}\frac{1}{\eta}\frac{\partial \eta}{\partial \vartheta} = \frac{0{\cdot}00155 . \cancel{0{\cdot}000}73}{\cancel{0{\cdot}000}173} = 0{\cdot}006_{54}$

1903
8633
0536
2380
8156

0000035, 20
00007
148
0·00155

47
151·3
0·00156

$\eta = A \frac{t_0 - t}{t - T}$ $\quad t_0 = 194$

$\eta_1 t_1 - \eta_1 T = A(t_0 - t_1)$ | η_2 | $t_0 - t_2$

$\eta_2 t_2 - \eta_2 T = A(t_0 - t_2)$ | η_1 | $t_0 - t_1$

$\cancel{\eta_1 t_1 - \eta_2 t_2} \quad \eta_1 \eta_2 (t_1 - t_2) = A\{\eta_2(t_0 - t_1) - \eta_1(t_0 - t_2)\}$

$\eta_1 t_1 (t_0 - t_2) - \eta_2 t_2 (t_0 - t_1) = [\eta_1 (t_0 - t_2) - \eta_2 (t_0 - t_1)] T$

$t_0 = 193{\cdot}5$

$t_1 = 2{\cdot}4 \quad \eta_1 = 2{\cdot}871$

$t_2 = 18{\cdot}4 \quad \eta_2 = 2{\cdot}392$

2·392 . 191·1 − 2·871 . 175·1

3788
2812
6600

4571

4581
2433
~~2...~~
7014

5028
4571
45·7

0·3788
0·4581
1·2041
2·0410
1·6599
0·3811

A = 2·405

$\bar{0}.4581$
$\bar{1}.3802$
2.2433
$\bar{3}.0816$
24

$\bar{0}.3788$
1.2648
2.2812
$\bar{3}.9248$

$\bar{3}.8575$
-1.6599
2.1976
$\frac{59}{17}$:20

$T = -157.6$

12070

84100
12070
72030

$$\eta = 2.405 \, \frac{193.5 - t}{t + 157.6}$$

$$\lg \eta = \lg A + \lg(t_0 - t) - \lg(t - T)$$

$$\frac{1}{\eta}\frac{d\eta}{dt} = -\frac{1}{t_0 - t} - \frac{1}{t - T} = -\frac{\cancel{t} - T + t_0 - \cancel{t}}{(t_0 - t)(t - T)}$$

$t_0 - T = \left.\begin{matrix} 193.5 \\ +157.6 \end{matrix}\right\} = 351.1$

$t_0 - t = 173.5$
$t - T = 177.6$

23925
2495
48875

5454
48875
05665

$= 0.0114$
2 Molez. (Hydrzellen)

19.3
18.9
$0.4 : 19.1 = \frac{0.021}{20} =$ 0.001 Rellstab (?) Diss. Bonn 1868

$\left.\begin{matrix} 14.5 \\ 11.7 \end{matrix}\right\}$ 26.2
$2.8 : 13.1 = \frac{0.214}{20} =$ 0.0107 Pribram & Handl
18

$$P = p + a\rho^2 = R\theta\,\rho\, f(\rho) \qquad f(0)=1$$

$$\cancel{2a\rho = R\theta\left[f(\rho)+\rho f'(\rho)\right]}$$ — ~~dla $\rho=0$ niemożliwe~~

VdW:

$\frac{\rho}{f(\rho)} = \frac{1}{\frac{1}{\rho}-b}$

$f = \frac{1}{1-b\rho}$

$$1 = \left\{R\theta\left[f(\rho)+\rho f'(\rho)\right] - 2a\rho\right\}\frac{\partial\rho}{\partial p}$$

$$2a\rho\,\frac{\partial\rho}{\partial\theta} = R\,\rho f(\rho) + R\theta\left[f(\rho)+\rho f'(\rho)\right]\frac{\partial\rho}{\partial\theta}$$

$$\frac{1}{\rho^2} = \left\{R\theta\,\frac{f+\rho f'}{\rho} - 2a\right\}\frac{1}{\rho}\frac{\partial\rho}{\partial p}$$

$$2a\,\frac{1}{\rho}\frac{\partial\rho}{\partial\theta} = \frac{Rf}{\rho} + R\theta\,\frac{f+\rho f'}{\rho}\,\frac{1}{\rho}\frac{\partial\rho}{\partial\theta}$$

$$\frac{1}{\rho^2} = \left\{R\theta\,\frac{f+\rho f'}{\rho} - 2a\right\}\beta \qquad \Big|\ \alpha$$

$$-\frac{Rf}{\rho} = -\left\{R\theta\,\frac{f+\rho f'}{\rho} - 2a\right\}\alpha \qquad \Big|\ \beta$$

$$\frac{\alpha}{\rho^2} - \frac{Rf\beta}{\rho} = 0$$

$$\frac{\alpha}{\beta} = Rf\rho$$

$$\alpha = +Rf\,\rho\cdot\beta = \frac{p+a\rho^2}{\theta}\cdot\beta$$

$$\alpha_0 = \frac{a\rho_0^2}{\theta}\beta_0 \qquad \left[\!\left[\ a = \frac{\alpha_0}{\beta_0}\,\frac{\theta}{\rho_0^2}\right.\right.$$

powinno być niezależne od temperatury

przy absol. zero: ściśliwość $= 0$

$(C_2H_5)_2O$: $\beta_{14^\circ} = 0{\cdot}000168$ | $\alpha_{14} = 0{\cdot}00153$

$\beta_{100^\circ} = 0{\cdot}000560$ | $\alpha_{100} = 0{\cdot}002976$

0_3 13489
0_2 131074
135088
0_2 279651
− 10347
$0{\cdot}0_2$17618

13489
13107
13509
40105
− 10347
$0{\cdot}0_2$29758 (Hirn)
29911 (Kopp extrap.)
31865 (Pierre extrap.)

0·13489
6554
3377
23420
− 3449
1·19971

$$\frac{\rho_{100^\circ}}{\rho_{14^\circ}} = \frac{1{\cdot}0021}{1{\cdot}1997} \cdot 0{\cdot}7366$$

$$\frac{0{\cdot}00153}{0{\cdot}000168} \cdot \frac{287}{(0{\cdot}7366)^2} \cdot (1{\cdot}0021)^2$$

0·8673 − 1

0·1847 − 3
2·4579
0·0018
0·6444 − 1
− 0·9598 + 5
0·6846 + 3

0·2253 − 4
0·7345 − 1
0·9598 − 5

$a_{14^\circ} = 4840$ (Atmosph.)

$$\frac{2976}{560} \cdot \frac{373}{(0{\cdot}7366)^2} (1{\cdot}1997)^2$$

4737
5717
0·1784
0·2238
− 0·4827
0·7411 + 3

7482
7345
4827

$a_{100^\circ} = 5510$ (Atmosph.)

ważne tylko przy zawiadamianiu gęstości pary nasyconej 100°, ponieważ α_0 / β_0

vdW: $a = \frac{\vartheta^2}{\pi}\,\frac{27}{64}\,R^2 = \frac{27}{64}\,\frac{\vartheta^2}{\pi}\left(\frac{p_0}{\rho_0\vartheta_0}\right)^2\left(\frac{\mu_0}{\mu}\right)^2$

$= p_0 \cdot \frac{27}{64}\,\frac{\vartheta^2}{\pi}\,\frac{p_0}{\rho_0^2\vartheta_0^2}\left(\frac{\mu_0}{\mu}\right)^2$

$\vartheta = 193{\cdot}3$
$\underline{273}$
$466{\cdot}3$

$\pi = 37{\cdot}1$

$(C_2H_5)_2O = \left.\begin{matrix}24\\5\end{matrix}\right\}\ \frac{29.}{58}$ $\quad N_2 \ldots 29$
$\underline{16}$
74

$\frac{27.}{64}\,\frac{(466)^2}{37{\cdot}1}\,\frac{1}{(0{\cdot}001293 \cdot 273)^2}\left(\frac{29}{74}\right)^2$
1116

0·4314	0·8062
1·3368	1·5694
0·9248	0·2233 − 6
2·6930	0·8724
− 0·2097 + 1	1·7384
3·4833	5·2097 − 6

$a = 3040$ Atm [w punkcie krytycznym]

$\rho_k = \frac{1}{3b}$

$\frac{2a^2}{(1-b\rho)^3}$

$\frac{2(1-b\rho)^3}{\rho^2\, 2\, b^2}$ $\quad \frac{\left(\frac{2}{3}\right)^3}{\frac{1}{9}} = \frac{8}{3}$

$\frac{\partial p}{\partial \rho} = \left| R\theta\left[f + \rho f'\right] = 2a\rho \right\| = 0$

$\frac{\partial^2 p}{\partial \rho^2} = \left| R\theta\left[2f' + \rho f''\right] = 2a \right\| = 0$

$\frac{f + \rho f'}{2f' + \rho f''} = \rho$

$f = \rho f' + \rho^2 f'' \Big|_{\text{kryt.}}$

w punkcie krytycznym

$\frac{R\theta_k}{p_k v_k} = \left(\frac{R\theta}{p}\right)\rho_k$

$= \frac{2a\rho^2}{[f + \rho f']\ldots}$

$= \frac{R\theta}{-a\rho + R\theta f} = \frac{1}{f - \frac{a\rho}{R\theta}}$

$= \frac{1}{f - \frac{f + \rho f'}{2}} = \frac{2}{f - \rho f'} = \frac{2}{\rho^2 f''}$

$c = c_{gaz} + \frac{a\rho^2 \alpha}{\rho} = c_{gaz} + a\alpha\rho$

86636
5991

0·818
0·1847-3
6·869 5

6232
5991-1
2223

Nyn. Eter $a_{14°} = 4840 \cdot 10^6$

$\boxed{V. dW. ;\ a\rho^2 = 1430\ Atm}$

$\alpha_{14} = 0·00153$

$\rho_{14} = \frac{0·7366}{1·0021} = 0·7351$

$\frac{4840 \cdot 10^6 \cdot 0·00153 \cdot 0·7351}{(0·7351)^2 \cdot 42 \cdot 10^6} = \frac{4·84 \cdot 0·153}{4·2 \cdot (0·7351)} = 0·444$
$+ c_{gaz}$ $\Big\} = c$ obliczone

c obserw. $= 0·56$

c_{gaz}? gdyby 1atomowy:

$c_p - c_v = AR$

$c_v(k-1) = AR$

$c_v = \frac{AR}{k-1}$

$c_{Hg} = 0·0246 \cdot \frac{3}{5}$
$= 0·01476$
$\mu_{Hg} = 200$
$\mu_{..} = 74$

$0·0147 \cdot \frac{200}{74} = 0·040$

podaną przy różnicę obliczone 0·12

$c_{gaz} = 0·37$! Winkelmann p. 391

6·8659-1
+ 0·8664-1
6·7323
− 6·6232
0·1091-1

0·000854·14
3316
0·01186 : 1·08

$0·1286$
$+ c_v\,gaz$ $\Big\} = c$ obliczone

$c_v\,gaz_{14°} = 0·3455$
+ 0·0110
0·3565

c obserw. $= 0·529$
14° + 0·008
0·537

c obliczone	c obserw.
0·485 >	0·537

Hg. $\beta = 0{\cdot}00000295$

$\alpha_{0^\circ} = 0{\cdot}0001818$ $\quad \alpha_{100^\circ} = 0{\cdot}0001822$

$v_0 = \frac{1}{13{\cdot}59}$ $\quad v_{100} = 1{\cdot}0182$

$$a = \frac{\alpha}{\beta}\frac{\theta}{\rho^2} = \frac{0{\cdot}000182}{0{\cdot}00000295}\cdot\frac{273.}{(13{\cdot}59)^2}$$

2601
4362
6963
7362
9601

4698
2664
7362

2265

$a = 91{\cdot}2$ (Atm.)

$a\rho^2 = 16830$ Atm.

$a\alpha\rho_{0^\circ} =$

9601
1332
2596
3529

9601
1332
2605
3538
-6078
3460

$\frac{91.13{\cdot}6.0{\cdot}00018}{42}$

2254 322

$:42 = 0{\cdot}005367$

2218
$:42$

31686

$0{\cdot}005331$

$C_{v\,gaz} = 0{\cdot}0246 \cdot \frac{3}{5} = 0{\cdot}01476$

$C_{liq} = 0{\cdot}03333$ (0°) $\quad 0{\cdot}03262$ (100°)

wzg. ilościowo wcale nie ma zgodności, tylko znak dobry.

Podobieństwo równych gazów:

Różnice pod względem: μ, σ, a

$$p + a\rho^2 = R\theta\rho f(\rho)$$

$n : n_1 =$

Podobieństwo istnieje jeżeli $\sigma_1^3 : \sigma^3 = \frac{\mu_1}{\rho_1} : \frac{\mu}{\rho}$

wtedy jeszcze różnica w do masy i prędkości (temp.)

$$p + a\rho^2 : p_1 + a_1\rho_1^2 = \frac{\mu c^2}{\sigma^3} : \frac{\mu_1 c_1^2}{\sigma_1^3} = \frac{\theta}{\sigma^3} : \frac{\theta_1}{\sigma_1^3}$$

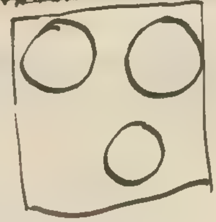
$$\theta : \theta_1 = \mu c^2 : \mu_1 c_1^2 \qquad = \theta\left(\frac{\rho}{\mu}\right) : \theta_1\left(\frac{\rho_1}{\mu_1}\right)$$

$$= \frac{\mu c^2}{\sigma^3}$$

$$p + a_i\rho = R\theta\rho f\left(\frac{\rho\sigma_i^3}{\mu_i}\right)$$

~~Względne równanie dla ...~~

$n = n_1$



1

$$\cancel{p_1 + a_k\rho_1 = \frac{R}{\mu_k}\theta_1\rho_1 f\left(\frac{\rho}{\sigma_1^3}\right) = \ldots \cdot \frac{\theta_1}{\theta}\frac{\rho_1}{\rho} = \frac{\theta_1\rho_1}{\theta\rho}\frac{\mu_i}{\mu_k}}$$

$$p_1 + a_k\rho_1 = \underbrace{(p + a_i\rho)}_{R\theta\rho f\left(\frac{\rho\sigma_i^3}{\mu_i}\right)} \frac{\theta_1}{\theta}\left(\frac{\rho_1}{\rho}\right)\left(\frac{\mu}{\mu_1}\right) = R\theta_1\rho\left(\frac{\rho_1}{\rho}\right) f\left(\rho_1\sigma_k^3\right)$$

$$\rho_1\left(\frac{\mu}{\mu_1}\right)^3\left(\frac{\sigma}{\sigma_1}\right)^2$$

$$\frac{\rho_1}{\rho} = \left(\frac{\sigma}{\sigma_1}\right)^3\frac{\mu_1}{\mu}$$

$$= R\theta_1\rho\left(\frac{\rho_1}{\rho}\right)\left(\frac{\mu}{\mu_1}\right) f\left(\frac{\rho_1\sigma_k^3}{\mu_1}\right) = R\theta_1\rho_1 f\left(\frac{\rho_1\sigma_k^3}{\mu_1}\right)\cdot\left(\frac{\mu}{\mu_1}\right)$$

$$\cancel{= R\theta_1\rho_1\left(\frac{\mu}{\mu_1}\right)^{1/3+2/3} f\left(\frac{\rho_1\sigma_k^3}{\mu_1}\right)}$$

~~...~~ p^+

$$p + a_i \rho^2 = $$

$$= \frac{r}{\mu_i}\, \theta \rho \cdot \varphi\left(\frac{\rho\, b_i^3}{\mu_i}\right)$$

$$R = \frac{r}{\mu_1}$$

$$f(\rho) = \varphi\left(\frac{\rho\, b^3}{\mu}\right)$$

$$f'(\rho) = \frac{b^3}{\mu}\, \varphi'$$

$$f''(\rho) = \frac{b^6}{\mu^2}\, \varphi''\left(\frac{\rho\, b^3}{\mu}\right)$$

$$\beta = \frac{1}{\rho^2\left[R\theta\, \frac{f+\rho f'}{\rho} - 2a\right]} = \frac{1}{R\theta\, \rho f + R\theta \rho^2 f' - 2a\rho^2}$$

$$= \frac{1}{R\theta\, \rho^2 f' - R\theta\, \rho f + 2p}$$

$$= \frac{1}{\rho^2\left[\frac{p+a\rho^2}{\rho f}\, \frac{f+\rho f'}{\rho} - 2a\right]} = \frac{}{\cancel{p + a\rho^2 +}}$$

$$= \frac{1}{p + a\rho^2 + \rho(p+a\rho^2)\frac{f'}{f} - 2a\rho^2} = \frac{1}{p\left(1 + \frac{\rho f'}{f}\right) + a\rho^2\left(\frac{\rho f'}{f} - 1\right)}$$

$$= \frac{1}{p - a\rho^2 + (p + a\rho^2)\frac{\rho f'}{f}}$$

Jsou-li $p \gg a\rho^2$: $\beta = \frac{1}{p\left(1 + \rho\frac{f'}{f}\right)}$ $\qquad p\beta = \frac{1}{1 + \rho\frac{f'}{f}}$

$$\alpha = \frac{Rf \cdot \theta_\rho}{\rho\left\{R\theta\, \frac{f+\rho f'}{\rho} - 2a\right)\cdot\theta_\rho} = \frac{p + a\rho^2}{\theta\left\{(p+a\rho^2)\left(1+\rho\frac{f'}{f}\right) - 2a\rho^2\right\}}$$

$$= \frac{p + a\rho^2}{\theta\left\{p - a\rho^2 + (p+a\rho^2)\,\rho\frac{f'}{f}\right\}}$$

Jeśli $p \gg a\rho^2$: $\quad \alpha\theta = \beta p$

$$\alpha : \beta = p : \theta$$

$$\alpha = \frac{1}{\theta\left(1+\rho\frac{f'}{f}\right) - \underbrace{\frac{2a\rho^2}{\rho R f}}_{= \frac{2R\theta\rho f - 2p}{R\rho f}}} = \frac{1}{\theta\left(\rho\frac{f'}{f} - 1\right) + \frac{2p}{R\rho f}}$$

Dla bardzo wielkich p: $\quad p = R\theta\rho f$

$$\beta = \frac{1}{p + R\theta\rho^2 f'}$$

$p\beta \overset{=3000\text{ atm}}{} = 0{\cdot}078$ więc główna część jest $\cdot\ \rho\frac{f'}{f}$

$p = R\theta\rho f \qquad \cancel{\log p = \log R\theta + \log\rho + \log f}$

Czemu dla rozmaitych cieczy podobne $\beta_{p=\infty}$?

$$\rho\frac{\partial p}{\partial\rho} = (R\theta f + R\theta\rho f')\rho$$

$$\frac{1}{\beta} = p\left(1 + \rho\frac{f'}{f}\right) = p\left(1 + \frac{\rho\sigma^3}{\mu}\cdot\frac{\varphi'\left(\frac{\rho\sigma^3}{\mu}\right)}{\varphi\left(\frac{\rho\sigma^3}{\mu}\right)}\right) = p\ \mathrm{fc.}\left(\frac{\rho\sigma^3}{\mu}\right)$$

Jinde $\frac{\rho f'}{f} \gg 1$:

$$\beta = \frac{1}{(p+a\rho^2)\frac{\rho f'}{f}} = \frac{1}{R\rho\theta f\,\frac{\rho f'}{f}} = \frac{1}{R\rho^2\theta f'} = \frac{1}{\frac{\mu}{\mu}\rho^2\theta\frac{b^3}{\mu}\varphi'}$$

$$\alpha = \frac{1}{\theta\,\frac{\rho f'}{f}} =$$

Zavme:

$$\alpha : \beta = \frac{1}{\theta} : \frac{1}{p+a\rho^2} = p+a\rho^2 : \theta$$

$$\alpha\theta = \beta\,(p+a\rho^2)$$

P. ex. C_2H_5OH (Amagat)

$p=1$ | $\alpha_{10°} = 0{\cdot}001080$

$\beta = 0{\cdot}0000919$

$\rho = 0{\cdot}80606$

$$a = \frac{280 \,.\, 0{\cdot}00108}{0{\cdot}000092 \,.\, (0{\cdot}807)^2} = 5052\ (\text{atm}).$$

4472   9633
0334   8138
4806   7771
−7771
7035

9844

$p=3000$ $\rho = 0{\cdot}9647$

7035
9688
6723

$a\rho^2 = 4702$

$p+a\rho^2 = 7702$

$\beta = < 0{\cdot}0000300$

$\frac{\beta(p+op')}{\theta} = \frac{0{\cdot}231}{280} = 0{\cdot}0008$

$\alpha_{3000} = 0{\cdot}000535$

Pountitiee Amagat

$\alpha_{2500\,Atm,\,280} = 0{\cdot}00113$

$\beta \quad = 0{\cdot}000106$

$\rho \quad = 0{\cdot}849$

$\frac{\alpha\theta - \beta p}{\beta \rho^2} = a$

$\frac{0{\cdot}00113 \cdot 280}{226}$ $\quad \frac{0{\cdot}000106 \cdot 2500}{0{\cdot}265}$

904

0·3164

0·2650

0·0514 : 0·764 = |7110 / 8831 / 8279|

$0{\cdot}0673 \cdot 10^4 = 673 = a$

1500 Atm: $\rho = 0{\cdot}724$

$\beta = 0{\cdot}000197$

8597.2
7194
8280
5474
1500
2047 Atm.

3112
2945
6057
− 4472
1585

1233

$\alpha = 0{\cdot}00144$ Atm.

$\alpha_{Atm.\,1500} = 0{\cdot}00181$

$\rho_{100} = 0{\cdot}1328$

2466
8280
0746

1187
100
111·88

$\alpha = 0{\cdot}00478$

$\theta = 280$

6794
4472
1266

1266
− 0488
0778

$\beta = 0{\cdot}01196$

?

$$\frac{p_1}{\rho_1} + a^2\rho_1 = R\theta$$
$$\frac{p_2}{\rho_2} + a^2\rho_2 = R\theta$$

jeśli $\rho_2 = 2\rho_1$

$$\frac{p_1 v_1}{p_2 v_2} = \frac{R\theta - a^2\rho_1}{R\theta - a^2\rho_2} = 1 + a^2\frac{(\rho_2-\rho_1)}{R\theta}$$
$$= 1 + a^2\frac{\rho^2}{p}$$

$$\underline{673.(0{\cdot}00129)^2} = \begin{matrix}2212\\8280\\\hline 0492\end{matrix} = 0{\cdot}00112 \qquad = 1{\cdot}00112 \text{ oblicz.}$$
$$1{\cdot}00141 \text{ obserw.}$$

$$pv = R\theta f - a^2\rho$$

$$\frac{\partial}{\partial p}(pv) = R\theta f' - a^2 = 0 \quad \text{dla minim.}$$

$$f' = \frac{a^2}{R\theta}$$

wtedy byłoby $\beta = \dfrac{1}{R\theta\rho f - a\rho^2} = \dfrac{1}{p}$

$$\alpha\theta = 1 + \frac{a\rho^2}{p}$$

$$pv = \alpha(p_0v_0)$$

$$v + p\frac{\partial v}{\partial p} = 0$$

$$\frac{1}{v}\frac{\partial v}{\partial p} = -\frac{1}{p}$$

to się rozumie samo przez się

$$\alpha\theta = \beta(p + a\rho^2)$$
$$= \left(\frac{1}{p} - \frac{1}{\eta}\frac{\partial\eta}{\partial p}\right)(p + a\rho^2)$$

$$\frac{pv}{p_0v_0} = \eta(p)$$

$$v + p\frac{\partial v}{\partial p} = p_0v_0 \cdot \frac{\partial\eta}{\partial p}$$

$$-\frac{1}{v}\frac{\partial v}{\partial p} = \frac{1}{p} - \frac{p_0v_0}{pv}\frac{\partial\eta}{\partial p}$$

$$\beta = \frac{1}{p} - \frac{p_0v_0}{pv}\frac{\partial\eta}{\partial p} = \frac{1}{p} - \frac{1}{\eta}\frac{\partial\eta}{\partial p}$$

Air Witkowski

$\theta = 0$: $p = 1$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = -0{\cdot}00051$

$p = 50$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{-0{\cdot}00035}{0{\cdot}9754}$

$p = 100$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = 0$

$p = 127$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{-0{\cdot}0003}{0{\cdot}9730}$

$\theta = -103{\cdot}5$ $p = 1$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} =$

$p = 50$ $\alpha =$ 487

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{-000027}{0{\cdot}4839}$

~~$p =$~~ 100 $\alpha =$ 579

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{0{\cdot}00029}{0{\cdot}3881}$

W każdym razie według tej hipotezy c_v powinno być niezależne od temperatury i od ciśnienia, co jest (zupełnie w) sprzeczności z rezultatami Witkowskiego. zmienności $\frac{\partial c_v}{\partial v}$ wskazują na $\frac{\partial c_v}{\partial \theta}$

Już z tej przyczyny teorya ta nie może być ścisła bo nawet przy nieskończenie miękkości kul zgniecenie c_v powinno być równe zeru (podczas gdy Rokam ... -

Compressibilität fester Körper:		$\alpha = 3.$	$\rho =$	c_a
Pb (Amagat)	0·00000276	0·0000292	11·3	0·00164
Cu "	0· 0857	168	8·7	0·00195
(Regnault)	0· 123			
Stahl (Amagat)	0·00000068	11	7·8	0·00152
SiO_2 (Voigt)	0· 2675			
Topas "	0·00000061			

$$a = \frac{\alpha\theta}{\beta\rho^2} \qquad c_a = a\alpha\rho = \frac{\alpha^2\theta}{42\cdot\beta\rho}$$

876
0·9425

64410 - 6
1·0531
0·4941 - 5

1·8850 - 10
2·4472
0·3322 - 6
- 0·4941 + 5
0·8381 - 2
- 1·6232
0·2149 - 3

504
7024

1·4048 - 10
2·4472
0·8520 - 7
5627
2893

9395
6232
5627

0828
4472
5300
3478
~~0433~~
1822 152

~~6414~~
~~8921~~
~~6232~~
~~5567~~

8325
8921
6232
3478

$c_{Pb} = \frac{0{\cdot}0314}{164} = 5\%$

$c_{Cu} = \frac{0{\cdot}094}{195} = 2\%$

$c_{Fe} = \frac{0{\cdot}113}{152} = 1{\cdot}4\%$

$c_{Hg} = \frac{0{\cdot}0333}{54} = 16\%$

więc u ciał stałych wpływ nieznaczny wyłomaniami R-hasło nie ponuszono.

Air Wrth. $\theta = -103{\cdot}5$
$p = 40{\cdot}88$

c_p ~~0·371~~
c_v = ~~0·172~~ } k = 2·16

$c_p = 0{\cdot}345$
$c_v \quad \frac{0{\cdot}189}{0{\cdot}156}$ $\quad k = 1{\cdot}83$

$\frac{p}{\rho}\frac{\rho_0}{p_0} = 0{\cdot}5098$

$\rho = \frac{40{\cdot}88}{0{\cdot}5098} \; 0{\cdot}001293$

6115
1116
7231
−7074
0·0157 − 1
2·8280
1·8437
157 − 1
0·8594

$a = 673$

$\rho = 0{\cdot}104$

$\alpha = 0{\cdot}00463$

$p = 40{\cdot}88$
$\Delta p^2 = +7{\cdot}235$
48·116

6656
6822
3478
− 6232
7246
− 0157 −1
7089

= ~~0·005304~~

$= 0{\cdot}0516$

tyle przypadało by na pracę zewnętrzną teoretycznie, podczas gdy w rzeczy. 0·156!

a więc by 20 razy tyle wyszło —

$\theta = 0°$

$p = 96.8$

$c_p = 0.282$

$c_v = 0.177$

$\overline{0.105}$

$k = 1.60$

$\alpha =$

$\theta = -35°$

$p = \cancel{\ \ }\ 65.5$

$\alpha = 0.004486$

$c_p = 0.294$

$\underline{190}$

0.104

$\rho = \dfrac{65.5 \cdot 0.001293}{0.8184}$

8162
1116
————
9278
− 9130
————
0148

$\rho = 0.1035$

65.5
7.3
————
72.8

1.8621
6518
————
5139
− 6380
————
8759

6232
0148
————
6380

Δ = 0.07515

aby dostać 0'104 musiałoby $\underline{a}$ być $\underline{4}$ razy tak duże!

podaje 0.104

C_6H_6 $\quad c = 0.3834 + 0.001043\,\theta$

$\alpha = 0.00117626 + 0.0000025551\,\theta + 0.0_7 2419\,\theta^2$

$\rho_0 = 0.899$

$\beta = 0.000059$

$a = \dfrac{\alpha\theta}{\beta\rho^2}$

$c_a = a\alpha\rho$

$\dfrac{\partial c_a}{\partial\theta} = a\left[\dfrac{\partial\alpha}{\partial\theta}\rho + \alpha\dfrac{\partial\rho}{\partial\theta}\right] = a\rho\left[\dfrac{\partial\alpha}{\partial\theta} - \alpha^2\right]$

$\frac{\partial \alpha}{\partial \theta} = 0{\cdot}256 \,.\, 10^{-5}$

$\alpha^2 = \frac{0{\cdot}0139 \quad 10^{-5}}{0{\cdot}242 \,.\, 10^{-5}}$

$\frac{\alpha\theta}{\rho} = \begin{array}{r} 0704 \\ 4362 \\ \hline 5066 \\ -\,7709 \\ \hline 7357 \\ +\,3838 \\ \hline 1195 \end{array}$

$a = \frac{5440}{(0{\cdot}9)^2}$

$\frac{\partial c_a}{\partial \theta} = \frac{\overline{1316{:}9}\;\; 0{\cdot}01462}{42} =$

$= \underline{0{\cdot}00035}$

$\begin{array}{r} 7356 \\ 0704 \\ \hline 8060 \\ 5775 \\ \hline 2285 \end{array}$

$c_a = \frac{5440}{0{\cdot}9} \cdot \frac{0{\cdot}001176}{42} =$

$\boxed{c_a = 0{\cdot}169}$

$\beta = \frac{A}{B+p}$

$\frac{A}{\cancel{A}} = 133\,B$

$B\,(133 - 65{\cdot}4) = 65{\cdot}4 \,.\, 950$

$67{\cdot}6$

$B = 920$

$\frac{1}{\rho}\frac{\partial \rho}{\partial p} = \frac{a}{b+p}$

$\lg \rho = a \lg (b+p)$

$\rho = \rho_0 (b+p)^a$

$\beta = \frac{133}{1 + \frac{p}{920}}$

77
92

$p = 250: \quad \frac{133 \,.\, 920}{1170} = 104{\cdot}9 \quad | \; 108{\cdot}8$

$133 = 117 = 114$
16
4

$p = 450: \quad \frac{133 \,.\, 920}{1370} = 890 \quad \| \; 83{\cdot}5$

$p = 2750: \quad \frac{133 \,.\, \cancel{920}}{1+3} = 33{\cdot}1 \quad \| \; 31{\cdot}7$

$$\cancel{4\pi \quad 5.10^{-6} \text{ cm}}$$

$$r = 2.10^{-6}\text{ cm}$$

$$p = 8.10^{-18} \cdot \frac{4}{3} \cdot \cancel{3{\cdot}14} \cdot 10 = \frac{1}{3} \cdot 10^{-15} \text{ g.}$$

$\frac{1}{2} \quad 10^{-7}$

$$\frac{p}{p_{O_2}} = (40)^3 \cdot 10 = 64.10^4$$

$10 \text{ Km} = 10^6 \text{cm}$. $2 \text{ cm} = h$

$$6\cancel{\pi}\mu\cancel{a}u = \frac{4\cancel{\pi}}{3} a^{\cancel{2}} \rho g$$

$$u = \frac{2}{9}\frac{a^2 \rho g}{\mu} = \frac{2}{9}\frac{4.10^{-12}.10^4}{0{\cdot}010} \neq 10^{-6}$$

$$= \frac{\partial \eta}{\partial x}\left\{\frac{v}{u^4}V^2\frac{\partial v}{\partial \eta} - \frac{V^2}{u^3}\frac{\partial u}{\partial \eta}\right\} = \frac{V^2}{u^4}\left\{v\frac{\partial v}{\partial \eta} - u\frac{\partial u}{\partial \eta}\right\}$$

$$\frac{\partial^2 \eta}{\partial y^2} = \frac{v^2}{u^2}\frac{\partial^2 \eta}{\partial x^2} + -\frac{V^2}{u^4}\left\{v\frac{\partial v}{\partial \eta} - u\frac{\partial u}{\partial \eta}\right\}\cdot\frac{\partial \eta}{\partial x}$$

$$v^2\frac{\partial}{\partial \xi}\left(\frac{u}{v}\right) + \frac{1}{2u}\frac{\partial(V^2)}{\partial \eta}\cdot\frac{\partial \eta}{\partial x} = 0$$

$$\frac{\partial v}{\partial \rho} + \frac{\partial u}{\partial \zeta} = \zeta(\Delta)^2 u = \zeta\left(\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \zeta^2}\right)$$

$$\frac{\partial v}{\partial \zeta} - \frac{\partial u}{\partial \rho} = \zeta(\Delta)^2 v = \zeta\left(\frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 v}{\partial \zeta^2}\right)$$

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 u}{\partial \zeta \partial \rho} = \zeta\left(\frac{\partial^3 u}{\partial \rho^3} + \frac{\partial^3 u}{\partial \zeta^2 \partial \rho}\right)$$

$$\frac{\partial^2 v}{\partial \zeta^2} - \frac{\partial^2 u}{\partial \zeta \partial \rho} = \frac{\partial^2 v}{\partial \rho^2} + \frac{\partial v}{\partial \zeta^2} + \zeta\left(\frac{\partial^3 v}{\partial \rho^2 \partial \zeta} + \frac{\partial^3 v}{\partial \zeta^3}\right)$$

$+$

$$\frac{\partial^3 u}{\partial \rho^3} + \frac{\partial^3 u}{\partial \zeta^2 \partial \rho} + \frac{\partial^3 v}{\partial \rho^2 \partial \zeta} + \frac{\partial^3 v}{\partial \zeta^3} = 0$$

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial \zeta^2}\right)\left(\frac{\partial u}{\partial \rho} + \frac{\partial v}{\partial \zeta}\right) = 0$$

$$\frac{\partial^2 v}{\partial \rho \partial \zeta} + \frac{\partial^2 u}{\partial \zeta^2} = \left(\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \zeta^2}\right) + \zeta\left(\frac{\partial^3 u}{\partial \rho^2 \partial \zeta} + \frac{\partial^3 u}{\partial \zeta^3}\right)$$

$$\frac{\partial^2 v}{\partial \rho \partial \zeta} - \frac{\partial^2 u}{\partial \rho^2} = \zeta\left(\frac{\partial^3 v}{\partial \rho^3} + \frac{\partial^3 v}{\partial \rho \partial \zeta^2}\right)$$

$$\frac{\partial^3 u}{\partial \rho^2 \partial \zeta} + \frac{\partial^3 u}{\partial \zeta^3} - \left(\frac{\partial^3 v}{\partial \rho^3} + \frac{\partial^3 v}{\partial \rho \partial \zeta^2}\right) = 0$$

2 ctem $\frac{\partial u}{\partial \rho} = -\frac{\partial v}{\partial \zeta} + f(\rho, \zeta)$

$R\, f(\rho + \zeta)$

φ

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial \zeta^2}\right)\left(\frac{\partial u}{\partial \zeta} - \frac{\partial v}{\partial \rho}\right) = 0$$

$$\frac{\partial u}{\partial \zeta} = \frac{\partial v}{\partial \rho} + R\, f(\rho + \zeta)$$

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial \rho^2} &= -\frac{\partial^2 v}{\partial \zeta \partial \rho} + \frac{\partial \varphi}{\partial \rho} \\ \frac{\partial^2 u}{\partial \zeta^2} &= \frac{\partial^2 v}{\partial \rho \partial \zeta} + \frac{\partial \psi}{\partial \zeta} \end{aligned}\right\} \quad \frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \zeta^2} = \frac{\partial \varphi}{\partial \rho} + \frac{\partial \psi}{\partial \zeta}$$

$$\left.\begin{aligned} \frac{\partial^2 v}{\partial \rho^2} &= \frac{\partial^2 u}{\partial \zeta \partial \rho} - \frac{\partial \psi}{\partial \rho} \\ \frac{\partial^2 v}{\partial \zeta^2} &= -\frac{\partial^2 u}{\partial \zeta \partial \rho} + \frac{\partial \varphi}{\partial \zeta} \end{aligned}\right\} \quad \frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 v}{\partial \zeta^2} = \frac{\partial \varphi}{\partial \zeta} - \frac{\partial \psi}{\partial \rho}$$

$p = x^2 - y^2$

$q = 2xy$

$p = x^2 - \left(\frac{q}{2x}\right)^2$

$x^4 - px^2 = \frac{q^2}{4}$

$x = \sqrt{\frac{p}{2} \pm \sqrt{\frac{p^2}{4} + \frac{q^2}{4}}}$

$x = \sqrt{\frac{p + \sqrt{p^2+q^2}}{2}}$

$y = \sqrt{\frac{-p + \sqrt{q^2+p^2}}{2}}$

$p = \left(\frac{q}{2y}\right)^2 - y^2$

$y^4 + py^2 = \left(\frac{q}{2}\right)^2$

$x^2 + 3y^2 = \frac{p + \sqrt{\ }}{2} \quad \frac{-3p + 3\sqrt{\ }}{2}$

$= -p + 2\sqrt{p^2+q^2}$

$\frac{\partial u}{\partial x} = \frac{1}{4}\left[\frac{4y}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2}\right]$

$= \frac{1}{2}\frac{y}{x^2+y^2}\left[1 - \frac{2xy}{x^2+y^2}\right]$

$= \frac{1}{2}\frac{y(x-y)^2}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial y} = -\frac{1}{4}\left[\frac{2x}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2} - \frac{x}{x^2+y^2}\right]$

$\frac{\partial u}{\partial y} = -\frac{1}{4}\left\{\frac{2y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2}\right\} = \frac{1}{2}\frac{x^2y + y^3 - 2x^2y}{(x^2+y^2)^2}$

$= \frac{1}{2}\frac{y^3 - x^2y}{(x^2+y^2)^2}$

$p^2 + q^2 = \frac{1}{x^2+y^2}$

$x = \frac{p}{p^2+q^2}$

$y = -\frac{q}{p^2+q^2}$

$\frac{\partial v}{\partial x} = \frac{1}{4}\left\{\frac{2y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2} + \frac{y}{x^2+y^2}\right\}$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{4}\left[\frac{x^3 + xy^2 + 2x^2y + y^3 - 8xy^2}{x^2+y^2}\right.$

140

$\frac{y}{x^2+y^2} + \frac{(y^2+x^2)2y - (y^2-x^2)2y}{(y^2+x^2)^2} = \frac{y^3 + x^2y}{}$

$= \frac{3}{x^2+y^2} \qquad \frac{\partial u}{\partial y} = \frac{y^3 - 3x^2y}{(y^2+x^2)^2}$

$+ \frac{(y^2+x^2)2x - 2x(y^2-x^2)}{(y^2+x^2)^2} + xy^2 + x^3$

$= \frac{-3xy^2 + x^3}{(y^2+x^2)^2} \qquad \frac{\partial v}{\partial x}$

$\frac{2x}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2} - \frac{x}{x^2+y^2}$

$\frac{\partial v}{\partial y} = \frac{x^3 - 3xy^2}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial x} = \frac{3y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2} + \frac{y}{x^2+y^2}$

$= \frac{3y^3 - x^2y}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{-2y^3 - 2x^2y}{(y^2+x^2)^2} = \frac{-2y}{y^2+x^2}$

$$v = \frac{1}{4}\left[-p \cosh p \sin\xi + \xi \sinh p \cos\xi\right]$$

$$\frac{\partial v}{\partial p} = \frac{1}{4}\left[-\cosh p \sin\xi - p \sinh p \sin\xi + \xi \cosh p \cos\xi\right]$$

$$\frac{\partial v}{\partial \xi} = \frac{1}{4}\left[-p \cosh p \cos\xi + \sinh p \cos\xi - \xi \sinh p \sin\xi\right]$$

$$\frac{\partial^2 v}{\partial p^2} = \frac{1}{4}\left[-2\sinh p \sin\xi - \cancel{p \cosh p \sin\xi} + \cancel{\xi \sinh p \cos\xi}\right]$$

$$\frac{\partial^2 v}{\partial \xi^2} = \frac{1}{4}\left[+\cancel{p \cosh p \sin\xi} - 2\sinh p \sin\xi - \cancel{\xi \sinh p \cos\xi}\right]$$

$$\Delta^2 v = \cancel{\tfrac{1}{4}}\left[- \sinh p \sin\xi\right.$$

$$-p \sinh p \sin\xi = \xi \cosh p \cos\xi \ ?$$

$$-p\, x\, \operatorname{tg}\xi = \xi\, y \operatorname{ctg}\xi \ ?$$

$$\operatorname{tg}^2\xi = -\frac{\xi y}{p x}$$

$$\frac{\partial \xi}{\partial x} = \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} = $$

$$= \frac{\partial p}{\partial y}$$

$$(\xi x \mp p y) = v$$

$$\frac{\partial v}{\partial x} = x \frac{\partial p}{\partial y} + \xi \mp y \frac{\partial p}{\partial x}$$

$$\frac{\partial^2 v}{\partial x^2} = 2\frac{\partial p}{\partial y} + x \frac{\partial^2 p}{\partial x \partial y} \mp y \frac{\partial^2 p}{\partial x^2}$$

$$\frac{\partial v}{\partial y} = -x \frac{\partial p}{\partial x} \mp p \mp y \frac{\partial p}{\partial y}$$

$$\frac{\partial^2 v}{\partial y^2} = -x \frac{\partial^2 p}{\partial x \partial y} \mp 2 \frac{\partial p}{\partial y} \mp y \frac{\partial^2 p}{\partial y^2}$$

$$v = \frac{1}{4}(\xi x + p y)$$

$$v = \frac{1}{4}\left[p \sinh p \cos \xi + \xi \cosh p \sin \xi\right]$$

$$\frac{\partial v}{\partial p} = \frac{1}{4}\left[\sinh p \cos\xi + p \cosh p \cos \xi + \xi \sinh p \sin \xi\right] \qquad \frac{\partial v}{\partial \xi} = \frac{1}{4}\left[-p \sinh p \sin\xi + \cosh p \sin\xi + \xi \cosh p \cos\xi\right]$$

$$\frac{\partial^2 v}{\partial p^2} = \frac{1}{4}\left[2\cosh p \cos \xi + p \sinh p \cos\xi + \xi \cosh p \sin \xi\right] \qquad \frac{\partial^2 v}{\partial \xi^2} = \frac{1}{4}\left[-p \sinh p \cos\xi + 2\cosh p \cos\xi - \xi \cosh p \sin\xi\right]$$

$$\frac{\partial^2 v}{\partial p^2} + \frac{\partial^2 v}{\partial \xi^2} = \cosh p \cos \xi$$

$$\underbrace{\frac{\partial^2 u}{\partial p^2} + \frac{\partial^2 u}{\partial \xi^2}}_{} = \frac{e^{p} + e^{-p}}{2}\,\frac{e^{i\xi} + e^{-i\xi}}{2} = \frac{e^{p+i\xi} + e^{-p+i\xi} + e^{p-i\xi} + e^{-p-i\xi}}{4}$$

$$16\frac{\partial^2 u}{\partial\alpha\,\partial\beta} = e^{\alpha} + e^{-\alpha} + e^{\beta} + e^{-\beta}$$

$$16\frac{\partial u}{\partial \beta} = \int\left[\cdots\cdots\right]\partial\alpha$$

$$= e^{\alpha} - e^{-\alpha} + \alpha e^{\beta} + \alpha e^{-\beta} + f.\beta$$

$$16 u = \beta\left(e^{\alpha} - e^{-\alpha}\right) + \alpha\left(e^{\beta} - e^{-\beta}\right) + \Phi(\beta) + \Psi(\alpha)$$

$$16 u = (p - i\xi)\underbrace{\left(e^{p+i\xi} - e^{-p-i\xi}\right)} + (p + i\xi)\underbrace{\left(e^{p-i\xi} - e^{-p+i\xi}\right)} + \cdots$$

$$= p\left(e^{p} - e^{-p}\right)\cos\xi + i\left(e^{p} + e^{-p}\right)\sin\xi \qquad \left(e^{p} - e^{-p}\right)\cos\xi - i\left(e^{p} + e^{-p}\right)\sin\xi$$

$$16 u = 2\left[p\left(e^{p} - e^{-p}\right)\cos\xi + \left(e^{p} + e^{-p}\right)\xi \sin\xi\right] + p\,\cancel{\left(e^{p} - e^{-p}\right)\cos\xi + \left(e^{p} + e^{-p}\right)\sin\xi}$$

$$u = \frac{1}{4}\left[p \sinh p \cos\xi + \cosh p\, \xi \sin\xi\right] + \Phi(p) + \Psi(\xi)$$

[illegible]

$x+iy = \sinh(\mu+i\xi)$

$= \dfrac{e^{\mu+i\xi}-e^{-\mu-i\xi}}{2}$

$x = \dfrac{e^{\mu}-e^{-\mu}}{2}\cos\xi = \sinh\mu\cos\xi$

$y = \dfrac{e^{\mu}+e^{-\mu}}{2}\sin\xi = \cosh\mu\sin\xi$

$\dfrac{x^2}{\sin^2h\mu}+\dfrac{y^2}{\cos^2h\mu}=1$

$\dfrac{y^2}{\sin^2\xi}-\dfrac{x^2}{\cos^2\xi}=1$

$x+iy = \cosh(\mu+i\xi)$

$= \dfrac{e^{\mu+i\xi}+e^{-\mu-i\xi}}{2}$

$x = \dfrac{e^{\mu}+e^{-\mu}}{2}\cos\xi = \cosh\mu\cos\xi$

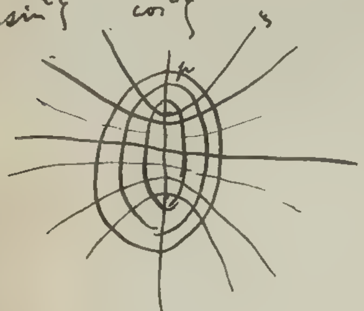
$y = \dfrac{e^{\mu}-e^{-\mu}}{2}\sin\xi = \sinh\mu\sin\xi$

$\dfrac{x^2}{\cos^2h\mu}+\dfrac{y^2}{\sin^2h\mu}=1$

$\dfrac{x^2}{\cos^2\xi}-\dfrac{y^2}{\sin^2\xi}=1$

$1 = \sinh\mu\,\dfrac{\partial\mu}{\partial y}\sin\xi+\cosh\mu\cos\xi\,\dfrac{\partial\xi}{\partial y}$

$0 = \cosh\mu\,\dfrac{\partial\mu}{\partial y}\cos\xi-\sinh\mu\sin\xi\,\dfrac{\partial\xi}{\partial y}$



$y=0: \quad \xi=0$

$x=0: \; y<1: \mu=0$

$y>1: \; \xi=-\frac{\pi}{2}, +\frac{\pi}{2}$

$\dfrac{\partial\mu}{\partial y} = \dfrac{\sinh\mu\sin\xi}{\sin^2h\mu\sin^2\xi+\cos^2h\mu\cos^2\xi}$

$\dfrac{\partial\xi}{\partial y} = \dfrac{\cos h\mu\cos\xi}{\dots}$

$\cosh\mu = \dfrac{e^{\mu}+e^{-\mu}}{2}$

$\sinh\mu = \dfrac{e^{\mu}-e^{-\mu}}{2}$

$\cos^2h\mu = \dfrac{e^{2\mu}+e^{-2\mu}}{4}+\dfrac{1}{2} = \dfrac{\cosh 2\mu+1}{2}$

$\sin^2h\mu = \dfrac{e^{2\mu}+e^{-2\mu}}{4}-\dfrac{1}{2}$

$= \dfrac{\cosh 2\mu-1}{2}$

$\cos^2h\mu-\sin^2h\mu = 1$

$u = \frac{1}{4}(\mu x \pm \xi y)$

$\dfrac{\partial u}{\partial x} = \frac{1}{4}\left(\mu + x\dfrac{\partial\mu}{\partial x} \pm y\dfrac{\partial\xi}{\partial x}\right)$

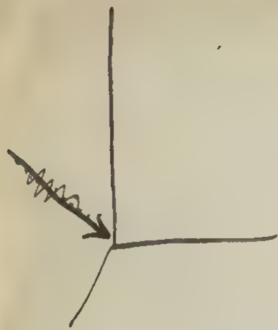
$\dfrac{\partial u}{\partial y} = \frac{1}{4}\left(x\dfrac{\partial\mu}{\partial y} \pm \xi \pm y\dfrac{\partial\xi}{\partial x}\right)$

$\dfrac{\partial^2 u}{\partial x^2} = \frac{1}{4}\left(2\dfrac{\partial\mu}{\partial x} + x\dfrac{\partial^2\mu}{\partial x^2} \pm \cancel{y\dfrac{\partial^2\xi}{\partial x\partial y}}\right)$

$\dfrac{\partial^2 u}{\partial y^2} = \frac{1}{4}\left(x\dfrac{\partial^2\mu}{\partial y^2} \mp 2\dfrac{\partial\mu}{\partial x} \cancel{\mp y\dfrac{\partial^2\xi}{\partial x\partial y}}\right)$

$\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2} = 1$

$$A \sin \alpha\left[t - \frac{x\cos\lambda + y\cos\mu + z\cos\nu}{a}\right]$$



$6 = 6' + 6''$

$6_1 = 6_1$

$x=0: \quad u = u_1$

rot $\frac{\partial u}{\partial t} = \frac{\partial u_1}{\partial t}$ rot: $\boxed{a^2 \frac{\partial 6}{\partial x} = a_1^2 \frac{\partial 6_1}{\partial x}}$

$p = p_1$

$p_0(1 + k6) = p_0(1 + k_1 6_1)$ $\qquad \boxed{k6 = k_1 6_1}$

$$k A' \sin\alpha'\left(t - y\frac{\cos\mu'}{a}\right) + k A'' \sin\alpha''\left(t - y\frac{\cos\mu''}{a}\right) = k_1 A_1 \sin\alpha_1\left(t - y\frac{\cos\mu_1}{a_1}\right)$$

$\alpha' = \alpha'' = \alpha_1$

$\frac{\cos\mu'}{a} = \frac{\cos\mu''}{a} = \frac{\cos\mu_1}{a_1}$

$\mu = \mu''$ $\qquad \sin\lambda' : \sin\lambda_1 = a : a_1$

$a^2 \frac{\partial 6}{\partial x} = a_1^2 \frac{\partial 6_1}{\partial x}$

$a A' \cos\lambda' + a A'' \cos\lambda'' = a_1 A_1 \cos\lambda_1$

$$\left.\begin{aligned} [A' - A''] \cos\lambda' \frac{\sin\lambda'}{\sin\lambda_1} &= A_1 \cos\lambda_1 \\ k[A' + A''] &= k_1 A_1 \end{aligned}\right\}$$

$$\frac{\partial u}{\partial t} + \cancel{u\frac{\partial u}{\partial x} + \dots} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \dots = 0$$

$$\frac{\partial v}{\partial t} + \quad = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \cancel{u\frac{\partial \rho}{\partial x} + \dots} = 0$$

$$\frac{\partial w}{\partial t} + \quad = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \dots \Bigg\} \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \qquad \frac{\partial \rho}{\partial t} = -\rho_0\left(\frac{\partial u}{\partial x} + \dots\right) \Big| \frac{\partial}{\partial t}$$

$$\rho_0\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t} + \dots\right) = -\frac{\partial^2 \rho}{\partial t^2} = -\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \right)$$

$$\frac{\partial^2 u}{\partial y \partial t} - \frac{\partial^2 v}{\partial x \partial t} = 0$$

$\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = $ const $= 0$ jeśli początkowo spoczynek

zatem ruch potencyalny

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p \qquad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$$

$$\cancel{\frac{1}{p_0}\frac{\partial p}{\partial x} = k\,\rho}$$

$$\log p - \log p_0 = k(\log \rho - \log \rho_0)$$

$$\frac{1}{p_0}\frac{\partial p}{\partial x} = \frac{k}{\rho_0}\frac{\partial \rho}{\partial x}$$

$$\frac{1}{p_0}\frac{\partial^2 p}{\partial x^2} = \frac{k}{\rho_0}\frac{\partial^2 \rho}{\partial x^2}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{k\,p_0}{\rho_0}\left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial z^2}\right)$$

$$\rho = \rho_0(1+\sigma)$$

$$\frac{\partial \rho}{\partial t} = \rho_0\frac{\partial \sigma}{\partial t} \quad \dots \quad \dots \text{etc}$$

$$\frac{\partial^2 \sigma}{\partial t^2} = \frac{k\,p_0}{\rho_0}\Delta^2 \sigma$$

$$\frac{\partial^2 \sigma}{\partial t^2} = \frac{k\,p_0}{\rho_0}\frac{\partial^2 \sigma}{\partial x^2}$$

ſ

$c = \sqrt{p\,\varepsilon} = \sqrt{1{\cdot}4\ \frac{[illegible]}{[illegible]}}$

1461

1516

1117

5198

$331\ \frac{m}{s}$

$a' = a\left(1 - \frac{1}{[illegible]\sqrt{n}}\right)$ $\qquad v = \sqrt{\frac{\mu}{\rho}} + \left(\sqrt{T} - [illegible]\right)\sqrt{\frac{k}{c\rho}}$

Low

	[illegible]	[illegible]
$n = 256$	$v = 327{\cdot}2[illegible]$	$320{\cdot}[illegible]$
$n = 1024$	$328{\cdot}6[illegible]$	[illegible]

$a = 330{\cdot}58\ \frac{m}{s}$

[illegible] $N = \frac{[illegible]}{[illegible]}$

I [illegible] $N = \frac{[illegible]}{[illegible]} \frac{\pi r}{4}$

II [illegible]

III [illegible]

[illegible]

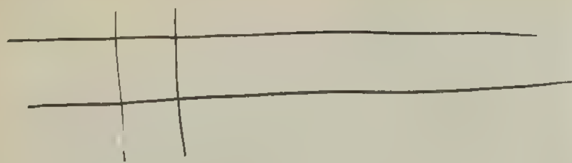
a Dla [illegible]

[illegible]

Fale dla wody 0·00005

Ag 0·0000037

$$\cancel{\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}}$$



$$\rho \frac{\partial u}{\partial t}\Delta x = p \ldots - \ldots \qquad q\,\Delta x\,\frac{\partial \rho}{\partial t}\,\Delta t = (\rho u) \text{ —— }$$

$u = 0$

$\sigma = 0$

$$\sigma = (A \sin \beta x + B \cos \beta x)(M \sin \alpha t + N \cos \alpha t)$$

$$= f_1(x + at) + f_2(x - at)$$

$$a = \sqrt{k \frac{p_0}{\rho_0}} = \sqrt{kR\theta}$$

$$\cancel{\sigma\, r} \qquad u = \frac{\partial \varphi}{\partial x}$$

$$\cancel{\sigma = \frac{f(x-at)}{r}}$$

$$\frac{\partial}{\partial r}(f r) = f + r\frac{\partial f}{\partial r}$$

$$\frac{\partial^2}{\partial r^2} \quad = 2\frac{\partial f}{\partial r} + r\frac{\partial^2 f}{\partial r^2}$$

$$\sigma = f(r, t)$$

$$\frac{\partial \sigma}{\partial x} = \frac{\partial f}{\partial r}\frac{x}{r}$$

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{\partial^2 f}{\partial r^2}\frac{x^2}{r^2} + \frac{\partial f}{\partial r}\frac{1}{r} - \frac{\partial f}{\partial x}\frac{x^2}{r^3}$$

$$\Delta^2 \sigma = \frac{\partial^2 f}{\partial r^2} + 2\frac{\partial f}{\partial r}\frac{1}{r} = \cancel{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)} = \frac{1}{r}\frac{\partial^2}{\partial r^2}(f r)$$

$$\cancel{\frac{\partial^2 (f r)}{\partial t^2} = a^2 \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)} \qquad \frac{\partial^2}{\partial t^2}(f r) = \frac{\partial^2}{\partial r^2}(f r)$$

$$f(r) = \varphi(r \pm at)$$

$$\sigma = \frac{\varphi(r \pm at)}{r}$$

$$u = \frac{\log(x^2+y^2)}{2} + \frac{2y^2}{y^2+x^2} - 1 \qquad \frac{\partial u}{\partial x} = \frac{x}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2}$$

$$v = -\frac{2xy}{x^2+y^2} + \operatorname{arctg}\frac{y}{x} \qquad \frac{\partial v}{\partial y} = \frac{-x}{x^2+y^2} + \frac{4xy^2}{(x^2+y^2)^2}$$

$$\left.\begin{matrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \end{matrix}\right\} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$$

$$u = \frac{1}{4}\left[-\frac{\log(\rho^2+\zeta^2)}{2} + \frac{2\zeta^2}{\rho^2+\zeta^2} - 1\right]$$

$$v = \frac{1}{4}\left[\frac{2\zeta\rho}{\rho^2+\zeta^2} - \operatorname{arctg}\frac{\zeta}{\rho}\right]$$

$$\boxed{\begin{aligned}
\frac{\partial u}{\partial \rho} &= \frac{1}{4}\left[-\frac{\rho}{\rho^2+\zeta^2} - \frac{4\rho\zeta^2}{(\rho^2+\zeta^2)^2}\right] \\
\frac{\partial u}{\partial \zeta} &= \frac{1}{4}\left[\frac{3\zeta}{\rho^2+\zeta^2} - \frac{4\zeta^3}{(\rho^2+\zeta^2)^2}\right] \\
\frac{\partial v}{\partial \rho} &= \frac{1}{4}\left[\frac{3\zeta}{\rho^2+\zeta^2} - \frac{4\zeta\rho^2}{(\rho^2+\zeta^2)^2}\right] \\
\frac{\partial v}{\partial \zeta} &= \frac{1}{4}\left[\frac{\rho}{\rho^2+\zeta^2} - \frac{4\rho\zeta^2}{(\rho^2+\zeta^2)^2}\right]
\end{aligned}}$$

$$\frac{\partial^2 u}{\partial \rho^2} = \frac{\rho^4 + 12\rho^2\zeta^2 - 5\zeta^4}{4(\rho^2+\zeta^2)^3}$$

$$\frac{\partial^2 u}{\partial \zeta^2} = \frac{3\rho^4 - 12\rho^2\zeta^2 + [illegible]\zeta^4}{4(\rho^2+\zeta^2)^3}$$

$$\frac{\partial^2 v}{\partial \rho^2} = \frac{2\rho^3\zeta - 14\rho\zeta^3}{4(\rho^2+\zeta^2)^3}$$

$$\frac{\partial^2 v}{\partial \zeta^2} = \frac{-10\rho^3\zeta + 6\rho\zeta^3}{4(\rho^2+\zeta^2)^3}$$

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \zeta^2} = \frac{4\rho^4 - 4\zeta^4}{4(\rho^2+\zeta^2)^3} = \frac{\rho^2-\zeta^2}{(\rho^2+\zeta^2)^2} = \frac{\frac{\partial \rho}{\partial x}}{\left(\frac{\partial \rho}{\partial x}\right)^2 + \left(\frac{\partial \rho}{\partial y}\right)^2}$$

$$\frac{\partial v}{\partial \rho} + \frac{\partial u}{\partial \zeta} = \frac{1}{4}\left[\frac{6\zeta}{\rho^2+\zeta^2} - \frac{4\zeta\cancel{(\rho^2+\zeta^2)}}{(\rho^2+\zeta^2)^{\cancel{2}}}\right] = \frac{1}{2}\,\frac{\zeta}{\rho^2+\zeta^2}$$

$$\frac{\partial v}{\partial \zeta} - \frac{\partial u}{\partial \rho} = \frac{1}{4}\left[\frac{2\rho}{\rho^2+\zeta^2}\right] = \frac{1}{2}\,\frac{\rho}{\rho^2+\zeta^2}$$

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 v}{\partial \zeta^2} = -\frac{8\rho^3\zeta - 8\rho\zeta^3}{4(\quad)^3} = -\frac{2\rho\zeta}{(\rho^2+\zeta^2)^2} = \frac{-\frac{\partial \rho}{\partial y}}{\left(\frac{\partial \rho}{\partial x}\right)^2 + \left(\frac{\partial \rho}{\partial y}\right)^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$$

$$\begin{cases} \dfrac{\partial \varphi}{\partial x} = -\dfrac{\partial \xi}{\partial y} \\[2mm] \dfrac{\partial \varphi}{\partial y} = \dfrac{\partial \xi}{\partial x} \end{cases}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \varphi}\frac{\partial \varphi}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \varphi^2}\left(\frac{\partial \varphi}{\partial x}\right)^2 + 2\frac{\partial^2 u}{\partial \varphi \partial \xi}\frac{\partial \varphi}{\partial x}\frac{\partial \xi}{\partial x} + \frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial u}{\partial \varphi}\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial \varphi^2}\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right] + 2\frac{\partial^2 u}{\partial \varphi \partial \xi}\left[\cancel{\underbrace{\frac{\partial \varphi}{\partial x}\frac{\partial \xi}{\partial x} + \frac{\partial \varphi}{\partial y}\frac{\partial \xi}{\partial y}}_{=0}}\right] + \frac{\partial^2 u}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right] +$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial \varphi^2} \qquad \cancel{\frac{\partial^2 v}{\partial \varphi \partial \xi}} \qquad \frac{\partial^2 v}{\partial \xi^2}$$

$$\Delta^2 u = \left(\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial \xi^2}\right)\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right] = \frac{\partial \varphi}{\partial x} \;\Big\| = \left(\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial \xi^2}\right)\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right] = -\frac{\partial \xi}{\partial y}$$

$$\Delta^2 v = \left(\frac{\partial^2 v}{\partial \varphi^2} + \frac{\partial^2 v}{\partial \xi^2}\right)\left[\qquad\qquad\right] = \frac{\partial \varphi}{\partial y} = \left(\frac{\partial^2 v}{\partial \varphi^2} + \frac{\partial^2 v}{\partial \xi^2}\right)\left[\qquad\qquad\right] = \frac{\partial \xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial \xi^2} = \frac{\frac{\partial \varphi}{\partial x}}{\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2} = \frac{-\frac{\partial \xi}{\partial y}}{\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2} \;\Big\|\; \frac{\partial^2 v}{\partial \varphi^2} + \frac{\partial^2 v}{\partial \xi^2} = \frac{\frac{\partial \varphi}{\partial y}}{\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2} = \frac{\frac{\partial \xi}{\partial x}}{\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \varphi}\frac{\partial \varphi}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial \varphi}{\partial y}$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial \varphi}\frac{\partial \varphi}{\partial y} - \frac{\partial v}{\partial \xi}\frac{\partial \varphi}{\partial x} = -\frac{\partial u}{\partial \varphi}\frac{\partial \varphi}{\partial x} - \frac{\partial u}{\partial \xi}\frac{\partial \varphi}{\partial y} \qquad -\frac{\partial \varphi}{\partial y}$$

$$\frac{\partial v}{\partial x} = \zeta + \frac{\partial u}{\partial y} = \zeta + \frac{\partial u}{\partial \varphi}\frac{\partial \varphi}{\partial y} - \frac{\partial u}{\partial \xi}\frac{\partial \varphi}{\partial x} \qquad = \frac{\partial v}{\partial \varphi}\frac{\partial \varphi}{\partial x} + \frac{\partial v}{\partial \xi}\frac{\partial \varphi}{\partial y} \qquad \frac{\partial \varphi}{\partial x}$$

$$\zeta\frac{\partial \varphi}{\partial x} - \frac{\partial u}{\partial \xi}\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right] = \frac{\partial v}{\partial \varphi}\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right]$$

$$\frac{\partial v}{\partial \varphi} = \zeta\frac{\frac{\partial \varphi}{\partial x}}{\left(\frac{\partial \varphi}{\partial x}\right)^2 + (\cdot)} - \frac{\partial u}{\partial \xi}$$

$$U = cx\left(1+\frac{a^3}{2r^3}\right) = c\, r\cos\theta\left(1+\frac{a^3}{2r^3}\right)$$

$$\frac{\partial U}{\partial r} = c\cos\theta - \frac{c a^3}{r^3}\cos\theta \qquad {}^{a=r}=0$$

$$\frac{\partial U}{\partial \theta} = -c\, r\sin\theta\left(1+\frac{a^3}{2r^3}\right)$$

$$\frac{\partial U}{\partial x} = c\left(1+\frac{a^3}{2r^3}\right) - \frac{3c x^2 a^3}{2r^5}$$

$$\lim \frac{\partial U}{\partial x} = c$$

$$i = \lambda\frac{\partial U}{\partial x} \qquad c = \frac{i}{\lambda}$$

$$\frac{\partial\varphi}{\partial t} = \kappa\Delta^2\varphi$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = r^2\frac{\partial\varphi}{\partial t}$$

$$r\frac{\partial^2\varphi}{\partial r^2} + 2\frac{\partial\varphi}{\partial r} = r\frac{\partial\varphi}{\partial t}$$

$$\frac{\partial}{\partial r}(r\varphi) = \varphi + r\frac{\partial\varphi}{\partial r}$$

$$\frac{\partial^2}{\partial r^2} = 2\frac{\partial\varphi}{\partial r} + r\frac{\partial^2\varphi}{\partial r^2}$$

$$\frac{\partial^2(r\varphi)}{\partial r^2} = \frac{\partial(r\varphi)}{\partial t}$$

$$\frac{r^2}{4t}$$

$$\cancel{\varphi = r\alpha t + f}$$

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\partial\varphi}{\partial t}$$

$$r^2\varphi = \psi$$

$$\frac{\partial^2}{\partial r^2}(r^2\varphi) = \frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{\partial}{\partial r}(2r\varphi)$$

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial r^2} - 2\frac{\partial}{\partial r}\left(\frac{\psi}{r}\right)$$

$$= \left(v\frac{\partial^2 u}{\partial\xi^2} - u\frac{\partial^2 v}{\partial\xi^2}\right) + \left(\frac{\partial\eta}{\partial x}\right)^2\left(\frac{v}{u^2}\frac{\partial^2 u}{\partial\eta^2} - \frac{1}{u}\frac{\partial^2 v}{\partial\eta^2}\right) +$$

$$+\left(v\frac{\partial u}{\partial\xi} - u\frac{\partial v}{\partial\xi}\right)\frac{\xi}{V^2} + \left(v\frac{\partial u}{\partial\eta} - u\frac{\partial v}{\partial\eta}\right)\left\{\frac{1}{u^2}\frac{\partial^2\eta}{\partial x^2} + \frac{\partial\eta}{\partial x}\cancel{\left[\frac{1}{u}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{v}{u^3}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right]}\right\} =$$

$$\left[\frac{1}{u}\frac{\partial v}{\partial y} + \frac{v}{u^2}\frac{\partial v}{\partial x} - \frac{v}{u^2}\frac{\partial u}{\partial y} - \frac{v^2}{u^3}\frac{\partial u}{\partial x}\right] =$$

$$\cancel{\left[-\frac{1}{u}\frac{\partial u}{\partial x} - \frac{v^2}{u^2}\frac{\partial u}{\partial x}\right]}$$

$$= \frac{1}{u}\left[\frac{\partial v}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial y}\right] + \frac{v}{u^2}\left[\frac{\partial v}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial x}\right] - \frac{v}{u^2}\left[\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y}\right] -$$

$$-\frac{v^2}{u^3}\left[\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x}\right] =$$

$$= \frac{\partial v}{\partial\xi}\left[-\frac{u}{u} + \frac{v^2}{u^2}\right] + \frac{\partial v}{\partial\eta}\left[\frac{v}{u^2}\frac{\partial\eta}{\partial x} + \frac{v}{u^2}\frac{\partial\eta}{\partial x}\right] - \frac{\partial u}{\partial\xi}\left[-\frac{v}{u} + \frac{v^3}{u^3}\right] - \frac{\partial u}{\partial\eta}\left[\frac{v^2}{u^3}\frac{\partial\eta}{\partial x} + \frac{v^2}{u^3}\frac{\partial\eta}{\partial x}\right]$$

$$v\frac{\partial u}{\partial\xi} - u\frac{\partial v}{\partial\xi} + \left(\frac{\partial u}{\partial\eta} + \frac{v}{u}\frac{\partial v}{\partial\eta}\right)\frac{\partial\eta}{\partial x} = 0$$

$$= \frac{1}{u}\left(v\frac{\partial u}{\partial\xi} - u\frac{\partial v}{\partial\xi}\right) + \frac{v^2}{u^3}\left(u\frac{\partial v}{\partial\xi} - v\frac{\partial u}{\partial\xi}\right) + 2\frac{\partial\eta}{\partial x}\cdot\frac{v}{u^3}\left(u\frac{\partial v}{\partial\eta} - v\frac{\partial u}{\partial\eta}\right)$$

$$= \frac{u^2-v^2}{u^3}\underbrace{\left(v\frac{\partial u}{\partial\xi} - u\frac{\partial v}{\partial\xi}\right)}_{-\frac{\partial\eta}{\partial x}\left(\frac{\partial u}{\partial\eta} + \frac{v}{u}\frac{\partial v}{\partial\eta}\right)} + 2\frac{v}{u^3}\frac{\partial\eta}{\partial x}\left(u\frac{\partial v}{\partial\eta} - v\frac{\partial u}{\partial\eta}\right)$$

$$= \frac{\partial\eta}{\partial x}\left\{-\frac{1}{u}\frac{\partial u}{\partial\eta} - \cancel{\frac{v}{u^2}\frac{\partial v}{\partial\eta}} + \cancel{\frac{v^2}{u^3}\frac{\partial u}{\partial\eta}} + \frac{v^3}{u^4}\frac{\partial v}{\partial\eta} + 2\frac{v}{u^2}\frac{\partial v}{\partial\eta} - 2\frac{v^2}{u^3}\frac{\partial u}{\partial\eta}\right\} =$$

$$\frac{\partial\eta}{\partial y} = -\frac{\frac{\partial\xi}{\partial x}}{\frac{\partial\xi}{\partial y}}\frac{\partial\eta}{\partial x}$$

$$\cancel{\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial x} + \left(\frac{\partial u}{\partial\eta} - \frac{\partial v}{\partial\eta}\frac{\frac{\partial\xi}{\partial x}}{\frac{\partial\xi}{\partial y}}\right)\frac{\partial\eta}{\partial x} + \frac{\partial v}{\partial\xi}\frac{\partial\xi}{\partial y} = 0} \qquad \frac{v}{u} = -\frac{\frac{\partial\Psi}{\partial x}}{\frac{\partial\Psi}{\partial y}}$$

N. p. $\xi = \Psi$ $\qquad$ Ψ = const = linii prądu

$$v = +\frac{\partial\Psi}{\partial x} = \frac{\partial\xi}{\partial x} \qquad \frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$$

$$u = -\frac{\partial\Psi}{\partial y} = -\frac{\partial\xi}{\partial y} \qquad \left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2 = u^2+v^2 = V^2$$

$$\frac{\partial\eta}{\partial y} = \frac{v}{u}\frac{\partial\eta}{\partial x}$$

$$\left(\frac{\partial\eta}{\partial x}\right)^2 + \left(\frac{\partial\eta}{\partial y}\right)^2 = \left(1+\frac{v^2}{u^2}\right)\left(\frac{\partial\eta}{\partial x}\right)^2 = \frac{V^2}{u^2}\left(\frac{\partial\eta}{\partial x}\right)^2$$

$$\frac{\partial^2\eta}{\partial y^2} = \frac{\partial\eta}{\partial x}\,\frac{-v\frac{\partial u}{\partial y} + u\frac{\partial v}{\partial y}}{u^2} + \frac{v}{u}\underbrace{\frac{\partial^2\eta}{\partial x\partial y}}_{}$$

$$= \frac{v}{u}\frac{\partial^2\eta}{\partial x^2} + \frac{\partial\eta}{\partial x}\,\frac{-v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x}}{u^2}$$

$$= \frac{\partial\eta}{\partial x}\left[\frac{1}{u}\left(\frac{\partial v}{\partial y} + \frac{v}{u}\frac{\partial v}{\partial x}\right) - \frac{v}{u^2}\left(\frac{\partial u}{\partial x}\frac{v}{u} + \frac{\partial u}{\partial y}\right)\right] + \frac{v^2}{u^2}\frac{\partial^2\eta}{\partial x^2}$$

$$\frac{\partial x}{\partial\xi} = \frac{\left\{\begin{aligned}&\left[\frac{\partial^2 u}{\partial\xi^2}\frac{\partial\eta}{\partial y} - \frac{\partial^2 v}{\partial\xi^2}\frac{\partial\eta}{\partial x}\right]\left[\left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2\right] + \left[\frac{\partial^2 u}{\partial\eta^2}\frac{\partial\eta}{\partial y} - \frac{\partial^2 v}{\partial\eta^2}\frac{\partial\eta}{\partial x}\right]\left[\left(\frac{\partial\eta}{\partial x}\right)^2 + \left(\frac{\partial\eta}{\partial y}\right)^2\right]\\ &+\left[\frac{\partial u}{\partial\xi}\frac{\partial\eta}{\partial y} - \frac{\partial v}{\partial\xi}\frac{\partial\eta}{\partial x}\right]\left[\frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2}\right] + \left[\frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y} - \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial x}\right]\left[\frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial y^2}\right]\end{aligned}\right\}}{\frac{\partial\xi}{\partial x}\frac{\partial\eta}{\partial y} - \frac{\partial\xi}{\partial y}\frac{\partial\eta}{\partial x}}$$

$$= \frac{\begin{aligned}&V^2\left(\frac{\partial^2 u}{\partial\xi^2}\frac{v}{u} - \frac{\partial^2 v}{\partial\xi^2}\right) + \left[\frac{\partial^2 u}{\partial\eta^2}\frac{v}{u} - \frac{\partial^2 v}{\partial\eta^2}\right]\frac{V^2}{u^2}\left(\frac{\partial\eta}{\partial x}\right)^2 +\\ &+\left(\frac{\partial u}{\partial\xi}\frac{v}{u} - \frac{\partial v}{\partial\xi}\right) + \left(\frac{\partial u}{\partial\eta}\frac{v}{u} - \frac{\partial v}{\partial\eta}\right)\left\{\frac{V^2}{u^2}\frac{\partial^2\eta}{\partial x^2} + \frac{\partial\eta}{\partial x}\left[\frac{1}{u}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{v}{u^2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right]\right\}\end{aligned}}{\left(\frac{v^2}{u} + u\right)}$$

$x + iy = f.(\xi + i\eta)$

$$\left(\frac{\partial f}{\partial \xi}\right)^2 + \left(\frac{\partial f}{\partial \eta}\right)^2 = \left[\frac{\partial f}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial f}{\partial y}\frac{\partial x}{\partial \eta}\right]^2 + \left[\frac{\partial f}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial f}{\partial y}\frac{\partial x}{\partial \xi}\right]^2$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]\left[\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial x}{\partial \eta}\right)^2\right]$$

Jeśli $\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial x}{\partial \eta}\right)^2 = 1$

$$\left(\frac{\partial x}{\partial \eta}\right)^2 = \left(\frac{\partial y}{\partial \xi}\right)^2$$

$x + iy = \cos\xi + i\sin\xi$

~~Np. $x = \sin\xi$, $y = \cos\xi$~~

W ogólnym przypadku, podstawiając zamiast x, y, inne zmienne: ξ, η:

$$\frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial^2 u}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right] + \frac{\partial^2 u}{\partial \eta^2}\left[\left(\frac{\partial \eta}{\partial x}\right)^2 + \left(\frac{\partial \eta}{\partial y}\right)^2\right] + 2\frac{\partial^2 u}{\partial \xi \partial \eta}\left[\frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial \xi}{\partial y}\frac{\partial \eta}{\partial y}\right]$$

$$+ \frac{\partial u}{\partial \xi}\left[\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right] + \frac{\partial u}{\partial \eta}\left[\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right]$$

$$\frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial^2 v}{\partial \xi} \quad — \quad - \quad -- \quad ----$$

$$\frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x} + \frac{\partial v}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial y} = 0$$

Jeśli nowe spółrzędne tworzą system ortogonalny:

$$\frac{\frac{\partial \xi}{\partial x}}{\frac{\partial \xi}{\partial y}} = -\frac{\frac{\partial \eta}{\partial y}}{\frac{\partial \eta}{\partial x}}$$

wtedy odpada wyrażenie

Równanie wyrażone w odniesieniu do współrzędnych p, ξ

$$f(x+iy) = \xi + ip$$

$$\frac{\partial^2\psi}{\partial p^2} + \frac{\partial^2\psi}{\partial \xi^2} = \frac{\xi}{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2}$$

$$u = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial y}$$

$$v = \frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial x}$$

$$\Delta\psi = 0$$

Ponieważ z φ zawsze sprzężone musi być χ tego rodzaju że: $\varphi + i\chi = f.(x+iy)$ itd. zatem $\frac{\partial\varphi}{\partial x} = \frac{\partial\chi}{\partial y}$ więc dobierając do ψ odpowiednią funkcję χ: $\Delta^2\chi = 0$ można także napisać:

$$u = -\frac{\partial(\psi+\chi)}{\partial y}$$

$$v = \frac{\partial(\psi+\chi)}{\partial x}$$

(albo też χ opuścić zaznaczając, że jako ψ bierze się najogólniejszą całkę.)

Przy ~~...~~ ścianach stałych $\frac{\partial}{\partial x}(\psi+\chi)=0$; $\frac{\partial}{\partial y}(\psi+\chi)=0$;

zatem tam $\Psi = \psi+\chi$ musi więc wartości maksym(minim)alne

Wogóle tylko takie funkcje ~~...~~ Ψ które wartości maksymalne nie osiągają w pojedyńczych punktach lecz ~~...~~ na krzywych.

Jeśli Ψ wyrażone przez p, ξ: $\Psi = f.(p, \xi)$

~~..~~ Maksym. lub Minim. Ψ musi albo ~~dla~~ (p m, ξ dowolne)

albo ~~dla~~ (p dowolne, ξ m.)

albo dla $p = \varphi(\xi)$

$$\frac{\partial p}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial \xi} = \cancel{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\left(\frac{\partial x}{\partial \xi}\right)^2}$$

$$= \frac{\partial^2 u}{\partial x^2}\left[\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial x}{\partial \eta}\right)^2\right] + \frac{\partial^2 u}{\partial y^2}\left[\left(\frac{\partial y}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2\right] +$$

$$2\frac{\partial^2 u}{\partial x \partial y}\left(\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \xi} + \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \eta}\right) + \frac{\partial u}{\partial x}\left(\frac{\partial^2 x}{\partial \xi^2} + \frac{\partial^2 x}{\partial \eta^2}\right) + \frac{\partial u}{\partial y}\left(\frac{\partial^2 y}{\partial \xi^2} + \frac{\partial^2 y}{\partial \eta^2}\right)$$

$$\frac{\partial p}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial \eta} = \frac{\partial^2 v}{\partial x^2}\left[\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial x}{\partial \eta}\right)^2\right] + \frac{\partial^2 v}{\partial y^2}\left[\left(\frac{\partial y}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2\right] +$$

$$\frac{\partial u}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \xi} + \frac{\partial v}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \eta} = 0$$

N. p. $x + iy = f(\xi + i\eta) = \overset{x}{\varphi}(\xi,\eta) + i\,\overset{y}{\varphi}(\xi,\eta)$

$$\left\{\begin{aligned} \frac{\partial x}{\partial \xi} &= \frac{\partial y}{\partial \eta} \\ \frac{\partial x}{\partial \eta} &= -\frac{\partial y}{\partial \xi} \end{aligned}\right\}$$

I. $\frac{\partial p}{\partial x}\frac{\partial x}{\partial \xi} - \frac{\partial p}{\partial y}\frac{\partial x}{\partial \eta} = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\left[\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial x}{\partial \eta}\right)^2\right]$

II. $\frac{\partial p}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial p}{\partial y}\frac{\partial x}{\partial \xi} = \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]\left[\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial x}{\partial \eta}\right)^2\right]$

III. $\cancel{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\frac{\partial x}{\partial \xi}} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial \eta}\right)\frac{\partial x}{\partial \eta} = 0$

Transformacya

III) (Niemożliwa oprócz jeśli $f = 0$ lub $\frac{\partial x}{\partial \eta} = \frac{\partial y}{\partial \xi} = 0$

Zatem słuszne tylko dla ruchu niewirowego ~~[illegible]~~.

$$\frac{\partial p_1}{\partial x} = \Delta^2 u$$

$$\frac{\partial p_1}{\partial y} = \Delta^2 v$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial p_1}{\partial \xi} = \cdots$$

$$x = \varphi(\xi, \eta)$$

$$y = \psi(\xi, \eta)$$

$$\frac{\partial x}{\partial \xi} = \frac{\partial y}{\partial \eta}$$

$$\frac{\partial x}{\partial \eta} = -\frac{\partial y}{\partial \xi}$$

$$1 = f'\left(\frac{\partial \xi}{\partial x} + i\frac{\partial \eta}{\partial x}\right)$$

$$\frac{\partial p_1}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial p_1}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial u}{\partial \eta}\frac{\partial^2 \eta}{\partial x^2} + 2\frac{\partial^2 u}{\partial \xi \partial \eta}\frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial x} +$$

$$+ \frac{\partial^2 u}{\partial \eta^2}\left(\frac{\partial \eta}{\partial x}\right)^2 + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial u}{\partial \eta}\frac{\partial^2 \eta}{\partial x^2}$$

$$= \frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial^2 u}{\partial \eta^2}\left(\frac{\partial \eta}{\partial x}\right)^2 + 2\frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \Delta\frac{\partial u}{\partial \eta}\frac{\partial^2 \eta}{\partial x^2} + 2\frac{\partial^2 u}{\partial \xi \partial \eta}\frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial x}$$

Jakie związki φ, ψ ażeby w nowym układzie ruch był możliwy

$$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \xi}$$

$$\frac{\partial^2 u}{\partial \xi^2} = \frac{\partial^2 u}{\partial x^2}\left(\frac{\partial x}{\partial \xi}\right)^2 + \frac{\partial u}{\partial x}\frac{\partial \frac{\partial x}{\partial \xi}}{\partial x}\frac{\partial x}{\partial \xi} +$$

$$+ 2\frac{\partial^2 u}{\partial x \partial y}\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \xi} + \frac{\partial u}{\partial y}\frac{\partial \frac{\partial y}{\partial \xi}}{\partial x}\frac{\partial x}{\partial \xi}$$

$$+ \frac{\partial^2 u}{\partial y^2}\left(\frac{\partial y}{\partial \xi}\right)^2 + \frac{\partial u}{\partial y}\frac{\partial \frac{\partial y}{\partial \xi}}{\partial y}\frac{\partial y}{\partial \xi} + \frac{\partial u}{\partial x}\frac{\partial \frac{\partial x}{\partial \xi}}{\partial y}\frac{\partial y}{\partial \xi}$$

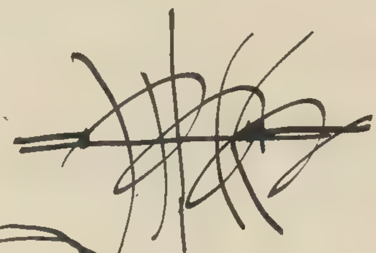
$$= \frac{\partial^2 u}{\partial x^2}\left(\frac{\partial x}{\partial \xi}\right)^2 + \frac{\partial^2 u}{\partial y^2}\left(\frac{\partial y}{\partial \xi}\right)^2 + 2\frac{\partial^2 u}{\partial x \partial y}\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \xi} + \frac{\partial u}{\partial x}\frac{\partial^2 x}{\partial \xi^2} + \frac{\partial u}{\partial y}\frac{\partial^2 y}{\partial \xi^2}$$

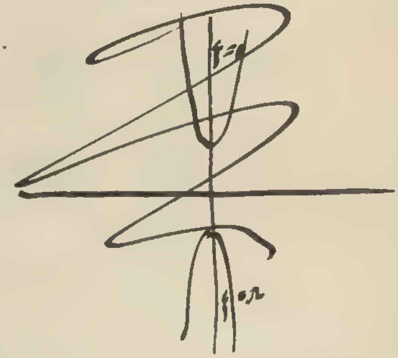
$$\cancel{x+iy = \cos(\xi+i\eta) = \frac{e^{i\xi-\eta}+e^{-i\xi+\eta}}{2}} = \frac{(\cos\xi + i\sin\xi)e^{-\eta} + (\cos\xi - i\sin\xi)e^{\eta}}{2}$$

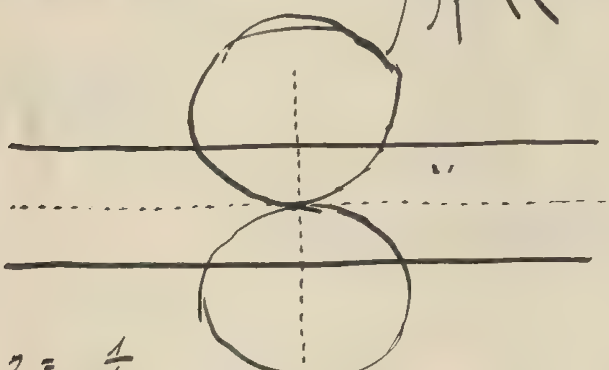
$$\cancel{y = \cos\xi\cosh\eta}$$

$$\cancel{x = \sin\xi\sinh\eta}$$

$$\cancel{\frac{x^2}{\cos^2\xi}} - \cancel{\frac{y^2}{\sin^2\xi}} = 1$$







$$z = \frac{1}{\zeta}$$

$$x+iy = \frac{1}{\xi+i\eta} = \frac{\xi-i\eta}{\xi^2+\eta^2}$$

$$\frac{\eta}{\xi^2+\eta^2} = c$$

$$\eta = c(\xi^2+\eta^2)$$

$$x = \frac{\xi}{\xi^2+\eta^2} \quad\Big|\quad \xi = \frac{x}{x^2+y^2}$$

$$y = -\frac{\eta}{\xi^2+\eta^2} \quad\Big|\quad \eta = -\frac{y}{x^2+y^2}$$

$$d\left(\frac{\partial x}{\partial \xi}\right) = \frac{\partial\left(\frac{\partial x}{\partial \xi}\right)}{\partial}$$

$$v = \frac{\partial\varphi}{\partial y} + \underbrace{\frac{x}{4}\left[-\operatorname{tg}\xi + 2\xi\right]}$$

$$u = \frac{\partial\varphi}{\partial x} + \frac{\cos h\rho}{4}\left[\cos\xi + 2\xi\sin\xi\right]$$

$$\frac{\sin h\rho}{4}\left[-\sin\xi + 2\xi\cos\xi\right]$$

$v=0$ dla $\rho = 0$ lub

$$2\xi = \operatorname{tg}\xi$$

$$\xi = c \qquad u = \frac{\cos h\rho}{4}\left[\cos\xi + \frac{\sin^2\xi}{\cos\xi}\right] = \frac{\cos h\rho}{4\cos\xi} = \frac{\cos h\rho}{4\cos c}$$

$$= \frac{y}{4\cos c\sin c}$$

Superponując: $u = \frac{-y}{4\cos c\sin c} = \frac{-y}{2\sin 2c}$

otrzyma się przylegania ~~na~~ równej hiperboli : $\xi = c$

Można superponować $v = v_1 + ax \qquad u = u_1 - by$

$$\xi = \xi_1 + a + b$$

$$v = \sin h\rho\left[-\frac{\sin(\xi-a)^{+b} + 2(\xi-a)^{+b}\cos(\xi-a)^{+b}}{4} + a\cos(\xi-a)\right]$$

$v=0$ dla: $\operatorname{tg}(\xi-(a+b)) = 2(\xi-a) + 4a = 2(\xi+a+b)$

$$\xi-(a+b) = c$$

$$\frac{-x^2}{\cos^2(\xi-(a+b))} + \frac{y^2}{\sin^2(\xi-(a+b))} = 1$$

$$u = \frac{\cos h\rho}{4}\left[\cos(\xi-a) + 2(\xi-a)\sin(\xi-a)\right]$$

$$= \frac{y}{4\sin^2(\xi-a)}\left[\cos\xi \;\cdots\;\right]$$

~~$\lim c = \frac{\pi}{2}$~~
~~$\lim(a+b) = \infty$~~
~~$\lim b = \frac{\pi}{4}$~~

$$b = \frac{\operatorname{ctg} c}{4} + \frac{c}{2}$$

$$\frac{\partial p_1}{\partial x} = \Delta^2 u_1 \qquad \frac{\partial p_2}{\partial x} = \Delta^2 u_2$$

$$\frac{\partial p_1}{\partial y} = \Delta^2 v_1 \qquad \frac{\partial p_2}{\partial y} = \Delta^2 v_2$$

$u_1 + u_2 = u$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \qquad \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0$$

$$\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y} = \zeta_1 \qquad \frac{\partial v_2}{\partial x} - \frac{\partial u_2}{\partial y} = \zeta_2$$

$$\frac{\partial(v_1+v_2)}{\partial x} - \frac{\partial(u_1+u_2)}{\partial y} = \zeta_1 + \zeta_2 = \zeta$$

$\zeta_2 = \frac{\pi}{4}$

$p_2 = 0$

$x = f_1(p_1, \zeta_1)$

$x = f_1(p_1\ \zeta - \frac{\pi}{4})$

. . . .

$u_1 = \varphi_1(p_1, \zeta - \frac{\pi}{4})$

$u = u_2 + \varphi_1(p_1, \zeta - \frac{\pi}{4})$

. . .

$$\boxed{\begin{gathered}\frac{x^2}{\sin^2 p} + \frac{y^2}{\cos^2 p} = 1 \\ \frac{-x^2}{\cos^2(\zeta - \frac{\pi}{4})} + \frac{y^2}{\sin^2(\zeta - \frac{\pi}{4})} = 1\end{gathered}}$$

$$\frac{\partial u}{\partial x} = \frac{\sinh p}{4}\left[\cos\zeta + 2\zeta\sin\zeta\right]\frac{\partial p}{\partial x} + \frac{\cosh p}{4}\left[+\sin\zeta + 2\zeta\cos\zeta\right]\frac{\partial p}{\partial y}$$

$$= \frac{1}{2}\,\frac{\sinh p\cosh p\left[\cos^2\zeta + \cancel{2\zeta\sin\zeta\cos\zeta} - \sin^2\zeta - \cancel{2\zeta\sin\zeta\cos\zeta}\right]}{\cosh 2p + \cos 2\zeta}$$

$$= \frac{1}{4}\,\frac{\sinh 2p\,\cos 2\zeta}{\cosh 2p + \cos 2\zeta}$$

$$\frac{\partial v}{\partial y} = \frac{\cosh p}{4}\left[-\sin\zeta + 2\zeta\cos\zeta\right]\frac{\partial p}{\partial y} - \frac{\sinh p}{4}\left[+\cos\zeta + 2\zeta\sin\zeta\right]\frac{\partial p}{\partial x}$$

$$= -\frac{\sinh p\cosh p\left[-\sin^2\zeta + \cancel{2\zeta\sin\zeta\cos\zeta} + \cos^2\zeta - \cancel{2\zeta\sin\zeta\cos\zeta}\right]}{2\,(\cosh 2p + \cos 2\zeta)}$$

$$= -\frac{\sinh 2p\,\cos 2\zeta}{4(\cosh 2p + \cos 2\zeta)} \qquad\qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial y} = \frac{\sinh p}{4}\left[\cos\zeta + 2\zeta\sin\zeta\right]\frac{\partial p}{\partial y} - \frac{\cosh p}{4}\left[\sin\zeta + 2\zeta\cos\zeta\right]\frac{\partial p}{\partial x}$$

$$= -\frac{\sin\zeta\sinh^2 p\,(\cos\zeta + 2\zeta\sin\zeta) - \cos\zeta\cosh^2 p\,(\sin\zeta + 2\zeta\cos\zeta)}{2\,(\cosh 2p + \cos 2\zeta)}$$

$$= -\frac{\sin\zeta\cos\zeta\,\overbrace{(\sinh^2 p + \cosh^2 p)}^{\cosh 2p}}{4\,(\cosh 2p + \cos 2\zeta)} - \frac{\zeta}{2}$$

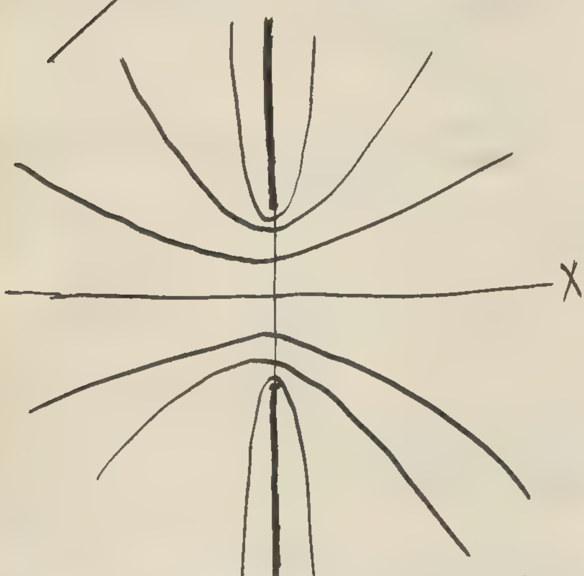
$$x+iy = \sin\left(-\frac{\pi}{4}+\xi+i\mu\right)$$

$$\cancel{x = \cos\left(-\frac{\pi}{4}+\xi\right)\sinh\mu = \frac{\cos\xi+\sin\xi}{\sqrt{2}}\sinh\mu}$$

$$\cancel{y = \sin\left(-\frac{\pi}{4}+\xi\right)\cosh\mu = \frac{\sin\xi-\cos\xi}{\sqrt{2}}\cosh\mu}$$

$$\cancel{u = \frac{\partial\varphi}{\partial x} + \frac{\cosh\mu}{4}\left[\cos\left(-\frac{\pi}{4}+\xi\right) + 2\left(-\frac{\pi}{4}+\xi\right)\sin\left(-\frac{\pi}{4}+\xi\right)\right]}$$

$$\cancel{v = \frac{\partial\varphi}{\partial y} + \frac{\sinh\mu}{4}\left[-\sin\left(-\frac{\pi}{4}+\xi\right) + 2\left(-\frac{\pi}{4}+\xi\right)\cos\left(-\frac{\pi}{4}+\xi\right)\right]}$$



$$\frac{\partial v}{\partial x} = \frac{\cosh\mu}{4}\left[-\sin\xi+2\xi\cos\xi\right]\frac{\partial\mu}{\partial x} + \frac{\sinh\mu}{4}\left[\cos\xi-2\xi\sin\xi\right]\frac{\partial\mu}{\partial y}$$

$$= \underline{\cos\xi\cosh^2\mu}\left[\underline{-\sin\xi+2\xi\cos\xi}\right] - \underline{\sin\xi\sinh^2\mu}\left[\cos\xi-2\xi\sin\xi\right]$$

$$= -\frac{\sin\xi\cos\xi\cosh 2\mu}{4(\cos\quad -)} + \frac{\xi}{2}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \xi$$

$$\frac{\partial\psi}{\partial x} = -\frac{\sin\zeta \sinh\mu}{4} + \zeta\frac{\cos\zeta \sinh\mu}{2}$$

$$= \frac{\sinh\mu}{4}\left[-\sin\zeta + 2\zeta\cos\zeta\right] \qquad = 0 \text{ dla } \mu = 0 \text{ i dla } \zeta = 0$$

$$u = \frac{\partial\varphi}{\partial x} + \underbrace{\frac{\cosh\mu}{4}\left[\cos\zeta + 2\zeta\sin\zeta\right]}$$

$$u = \frac{\partial\varphi}{\partial x} + \frac{y}{4}\left[\operatorname{ctg}\zeta + 2\zeta\right]$$

$x=0$:

$\zeta = \pm\frac{\pi}{2}$ | lub $\mu = 0$, $y = \sin\zeta$

$u = \pm y\frac{\pi}{4}$ | lub: $u = \frac{\sqrt{1-y^2}}{4} + \frac{y}{2}\arcsin y$

$u = \pi\frac{\cosh\mu}{4}$

$y=0$:

$\zeta = 0$

$u = \frac{\cosh\mu}{4}$

Zatem ~~...~~ φ_x

superponując $u = \mp y\frac{\pi}{4}$

$$v = \frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial x} = \frac{\partial\varphi}{\partial y} + \frac{\sinh\mu}{4}\left[-\sin\zeta + 2\zeta\cos\zeta\right]$$

$$= \frac{\partial\varphi}{\partial y} + \frac{x}{4}\left[-\operatorname{tg}\zeta + 2\zeta\right]$$

$x=0$:

$v = \mp\frac{\sinh\mu}{4} = \frac{\sqrt{...y^2-1}}{4}$ | lub $v=0$

$y=0$:

$v=0$

~~$\mu x + \zeta y = \alpha$~~

~~$-\mu y + \zeta x = \beta$~~

~~$x=0$: $\alpha = \pm\frac{\pi y}{2}$ lub $\alpha = \zeta y$~~

~~$\beta = -\mu y$~~

~~$= -\mu\cosh\mu$~~

~~$\frac{1}{\sinh\mu}$~~

Nie czyni zadość $\Delta^2\varphi = 0$!

~~$\varphi = ... \cosh^2\mu$~~

~~$\varphi = \frac{\sinh 2\mu - 2\mu}{4}$~~

$$\frac{\partial\varphi}{\partial x} = \frac{\cosh 2\mu - 1}{2}\cdot\frac{\partial\mu}{\partial x} = \sinh^2\mu\,\frac{\cos\zeta\cosh\mu}{\cosh^2\mu - \sin^2\zeta}$$

$$(= \sinh^2\mu + \cos^2\zeta)$$

~~$\frac{\partial f}{\partial\mu} = \sinh^2\mu = \frac{\cosh 2\mu - 1}{2}$~~

~~$f = \frac{\sinh 2\mu - 2\mu}{4}$~~

~~$= \frac{\sinh\mu\cosh\mu - \mu}{2}$~~

$x=0$, $=0$

$$\frac{\partial\varphi}{\partial y} = \sinh^2\mu\,\frac{\sin\zeta\sinh\mu}{\sinh^2\mu + \cos^2\zeta} \;\Big|\; = \pm\cosh\mu$$

$$\frac{\partial\psi}{\partial y} = \mp\frac{1}{8}\left[+\frac{(\cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta)\cos\zeta\cosh\mu}{\cancel{} - \sin^2 h\mu} + \right.$$

$$\left. + 2\zeta\sinh 2\mu \sin\zeta\sinh\mu \right]$$

$$= -\frac{1}{4\,\cancel{(\zeta\cosh\mu)}\,(\cosh 2\mu + \cos 2\zeta)}\left[(\cosh 2\mu + \cos 2\zeta)\cos\zeta\cosh\mu + \right.$$

$$\left. + 4\zeta\left(\sin\zeta\cos^2\zeta\cosh\mu + \sin\zeta\sin^2 h\mu\cosh\mu\right)\right]$$

$$= -\frac{1}{4}\left[\cos\zeta\cosh\mu\right] - \zeta\sin\zeta\cosh\mu\left(\frac{\cos^2\zeta + \sin^2 h\mu}{\cos 2\zeta + \cosh 2\mu}\right)_{=\frac{1}{2}}$$

$$\frac{\partial\psi}{\partial y} = -\frac{\cos\zeta + 2\zeta\sin\zeta}{4}\cdot\cosh\mu = [\text{illegible}]$$

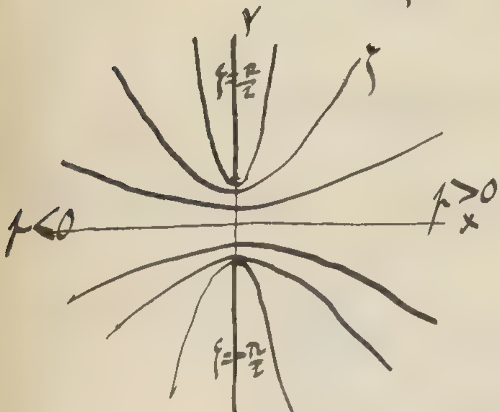
$$\frac{\partial\psi}{\partial x} = -\frac{(\cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta)\sin\zeta\sinh\mu + 2\zeta\sinh 2\mu\cos\zeta\cosh\mu}{4(\cosh 2\mu + \cos 2\zeta)}$$

$$= -\frac{\sin\zeta\sinh\mu}{4} + \zeta\,\frac{\sinh\mu\cosh^2\mu\cos\zeta - \sinh\mu\sin^2\zeta\cos\zeta}{\cosh 2\mu + \cos 2\zeta}$$

$$\frac{\partial\psi}{\partial x} = \frac{\sin^2\zeta - \cosh^2\mu}{16\sinh\mu}\left[\frac{-\sin^2\zeta + \cosh^2\mu}{\sin\zeta} - 4\zeta\,\frac{\cos^2\zeta - \sinh^2\mu}{\cos\zeta}\right]$$

$x = \cos\zeta\sinh\mu$ ‖ $\dfrac{x^2}{\sinh^2\mu} + \dfrac{y^2}{\cosh^2\mu} = 1$

$y = \sin\zeta\cosh\mu$ ‖ $-\dfrac{x^2}{\cos^2\zeta} + \dfrac{y^2}{\sin^2\zeta} = 1$



$x=0:\quad \zeta = \pm\frac{\pi}{2}$ bzw. $\mu = 0$

$y=0:\quad \zeta = 0,$

$$x^2+y^2 = \cos^2\zeta\sinh^2\mu + \sin^2\zeta\cosh^2\mu = (1-\sin^2\zeta)\sinh^2\mu + \sin^2\zeta\cosh^2\mu =$$
$$= \sinh^2\mu + \sin^2\zeta = \cosh^2\mu - \cos^2\zeta$$

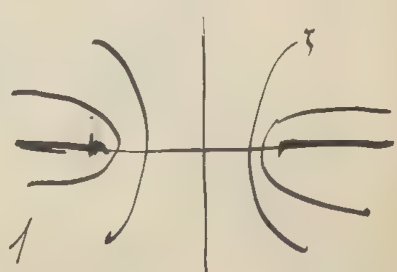
$$\frac{xy}{4} = \sin 2\zeta\,\sinh 2\mu$$

$$x+iy = \cos(\zeta + i\mu) = \frac{e^{i\zeta-\mu} + e^{-i\zeta+\mu}}{2}$$

$x = \cos\zeta\cosh\mu$

$y = -\sin\zeta\sinh\mu$

$\dfrac{x^2}{\cosh^2\mu} + \dfrac{y^2}{\sinh^2\mu} = 1$ ‖ $\dfrac{x^2}{\cos^2\zeta} - \dfrac{y^2}{\sin^2\zeta} = 1$



$$u = \frac{\partial\psi}{\partial x} \qquad - \frac{\partial\psi}{\partial y}$$

$$\sin^2\zeta \, \frac{\cosh 2\mu - 1}{2} + \cos^2\zeta \, \frac{\cosh 2\mu + 1}{2} = \frac{\cosh 2\mu + \cos 2\zeta}{2}$$

$$= \sin^2\zeta(\sinh^2\mu - 1) + (1 - \sin^2\zeta)\cosh^2\mu$$

$$= \cosh^2\mu - \sin^2\zeta$$

$$= \sinh^2\mu + \cos^2\zeta$$

$$\frac{\partial\psi}{\partial y} = -\frac{\frac{\partial\psi}{\partial\zeta}}{\frac{\partial\mu}{\partial x}} + \frac{\frac{\partial\psi}{\partial\mu}}{\frac{\partial\mu}{\partial y}} = \frac{(\sin^2\zeta \sinh^2\mu + \cos^2\zeta \cosh^2\mu)}{8}$$

$$\left[-\frac{\cosh 2\mu + 2\zeta \sin 2\zeta + \cos 2\zeta}{\cos\zeta \cosh\mu} \pm \frac{2\zeta \sinh 2\mu}{\sin\zeta \sinh\mu}\right]$$

$$= -\frac{\cosh 2\mu + \cos 2\zeta}{\cos\zeta\cosh\mu} - 2\zeta\,\frac{2\sin^2\zeta \cancel{\cos\zeta} + 2 \cancel{\sinh\mu}\cosh^2\mu \cancel{\cos\zeta}}{\sin\zeta \cancel{\cos\zeta\cosh\mu}\cosh\mu}$$

$$= -\frac{\cosh 2\mu + \cos 2\zeta}{16\cosh\mu}\left[\frac{\cosh 2\mu + \cos 2\zeta}{\cos\zeta} + 4\zeta\,\frac{\sin^2\zeta + \cosh^2\mu}{\sin\zeta}\right]$$

$$= \frac{\sin^2\zeta - \cosh^2\mu}{16\cosh\mu}\left[\frac{-\sin^2\zeta + \cosh^2\mu}{\cos\zeta} + 4\zeta\,\frac{\sin^2\zeta + \cosh^2\mu}{\sin\zeta}\right]$$

$$\frac{\partial\psi}{\partial x} = \frac{\frac{\partial\psi}{\partial\zeta}}{\frac{\partial\mu}{\partial y}} + \frac{\frac{\partial\psi}{\partial\mu}}{\frac{\partial\mu}{\partial x}} = \frac{\sin^2\zeta - \cosh^2\mu}{16}$$

$$\underbrace{\left[-\frac{\cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta}{\sin\zeta\sinh\mu} + \frac{2\zeta\sinh 2\mu}{\cos\zeta\cosh\mu}\right]}$$

$$\frac{-\sin^2\zeta + \cosh^2\mu}{\sin\zeta\sinh\mu} + 2\zeta\,\frac{2\sinh^2\mu \cancel{\cosh\mu} \cancel{\sin\zeta} - 2\cancel{\cosh\mu}\cancel{\sin\zeta}\cos^2\zeta}{\cancel{\sin\zeta}\cos\zeta\sinh\mu\cancel{\cosh\mu}}$$

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$$\psi = \int \frac{e^{\rho}+e^{-\rho}}{8i}\, d\rho \underbrace{\int (\alpha-\rho)(e^{\alpha}+e^{-\alpha})\, d\alpha}$$

$$\alpha e^{\alpha} - e^{\alpha} - \alpha e^{-\alpha} - e^{-\alpha} - \rho e^{\alpha} + \rho e^{-\alpha}$$

$$= \qquad e^{\alpha}(\alpha-1) - e^{-\alpha}(\alpha+1) - \rho(e^{\alpha} - e^{-\alpha})$$

$$= \frac{1}{8i}\left\{ \left[e^{\rho}-e^{-\rho}\right]\left[e^{\alpha}(\alpha-1) - e^{-\alpha}(\alpha+1)\right] - (e^{\alpha} - e^{-\alpha})\left[\rho(e^{\rho}-e^{-\rho}) - (e^{\rho}+e^{-\rho})\right] \right\}$$

$$= \frac{1}{8i}\left\{ \underbrace{(e^{\rho}-e^{-\rho})(e^{\alpha}-e^{-\alpha})}_{e^{\alpha+\rho}+e^{-\alpha-\rho}-e^{\alpha-\rho}-e^{-\alpha+\rho}}(\alpha-\rho) + \underbrace{(e^{\rho}-e^{-\rho})(e^{\alpha}+e^{-\alpha}) + (e^{\rho}+e^{-\rho})(e^{\alpha}-e^{-\alpha})} \right\}$$

$$= \cancel{-e^{\alpha+\rho}} + e^{\alpha-\rho} - e^{-\alpha+\rho} + \cancel{e^{-\alpha-\rho}}$$

$$+ \cancel{e^{\alpha+\rho}} + e^{\alpha-\rho} - e^{-\alpha+\rho} - \cancel{e^{-\alpha-\rho}}$$

$$= \frac{1}{32i}\left\{ 2i\zeta\left[e^{2\mu}+e^{-2\mu}-e^{2i\zeta}-e^{-2i\zeta}\right] + 2(e^{2i\zeta}-e^{-2i\zeta}) \right\}$$

$$= \frac{1}{8}\left\{ \zeta\left[\cosh 2\mu - \cos 2\zeta\right] + \sin 2\zeta \right\}$$

$$\frac{\partial\psi}{\partial\zeta} = \frac{1}{8}\left\{ \cosh 2\mu - \cos 2\zeta + 2\zeta \sin 2\zeta + 2\cos 2\zeta \right\}$$

$$= \frac{1}{8}\left\{ \cosh 2\mu + 2\zeta \sin 2\zeta + \cos 2\zeta \right\}$$

$$\frac{\partial\psi}{\partial\mu} = \frac{1}{8}\cdot 2\zeta \sinh 2\mu$$

$$x + iy = \sin(\zeta + i\mu) = \frac{e^{i\zeta - \mu} - e^{-i\zeta + \mu}}{2} = \frac{e^{-\mu}(\cos\zeta + i\sin\zeta) - e^{\mu}(\cos\zeta - i\sin\zeta)}{2}$$

$x^2+y^2 =$

$$x = \cos\zeta \sinh\mu$$

$$y = \sin\zeta \cosh\mu$$

$$1 = -\sin\zeta \cdot \frac{\partial\mu}{\partial y} \cdot \sinh\mu + \cos\zeta\cosh\mu \frac{\partial\mu}{\partial x} \quad \Big| \sin\zeta\sinh\mu$$

$$0 = \sin\zeta \frac{\partial\mu}{\partial x} \sinh\mu + \cos\zeta\cosh\mu \frac{\partial\mu}{\partial y} \quad \Big| \cos\zeta\cosh\mu$$

$$-\sin\zeta\sinh\mu = \sin^2\zeta\sinh^2\mu + \cos^2\zeta\cosh^2\mu$$

$$\frac{\partial\mu}{\partial y} = -\frac{\sin\zeta\sinh\mu}{\sin^2\zeta\sinh^2\mu + \cos^2\zeta\cosh^2\mu}$$

$$\frac{\partial\mu}{\partial x} = \frac{\cos\zeta\cosh\mu}{}$$

$$\frac{1}{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2} = \sin^2\zeta\sinh^2\mu + \cos^2\zeta\cosh^2\mu = \sinh^2\mu + \cos^2\zeta$$

$$\frac{\partial^2\psi}{\partial\mu^2} + \frac{\partial^2\psi}{\partial\zeta^2} = \zeta\sinh^2\mu + \zeta\cos^2\zeta = \zeta\,\frac{e^{2\mu} + e^{-2\mu} + e^{2i\zeta} + e^{-2i\zeta}}{4}$$

$$= \frac{\alpha-\beta}{8i}\left(e^{\alpha+\beta} + e^{-\alpha-\beta} + e^{\alpha-\beta} + e^{-\alpha+\beta}\right)$$

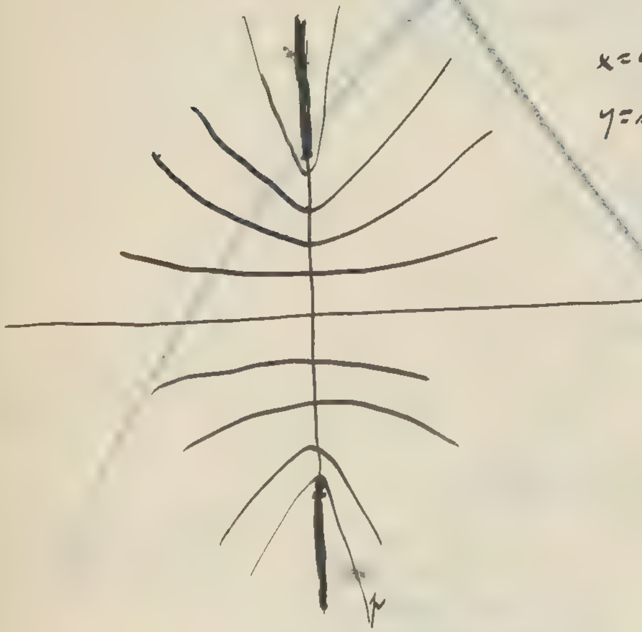
$\alpha+\beta = 2\mu$
$\alpha-\beta = 2i\zeta$

$=$

$$\frac{\partial\psi}{\partial x} = \frac{\cosh^2\xi - \sin^2\mu}{4}\left[2\xi\left(\frac{\sinh\xi\,\cancel{\cosh\xi}}{\cancel{\cosh\xi}\,\cos\mu} + \frac{\cancel{\sin\mu}\,\cos\mu}{\cancel{\sin\mu}\,\sinh\xi}\right) - \frac{\sinh^2\xi + \cos^2\mu}{\cos\mu\,\cosh\xi}\right]$$

$$= \left(\frac{\cosh^2\xi - \sin^2\mu}{4\cos\mu}\right)(\sinh^2\xi + \cos^2\mu)\left[\frac{2\xi}{\cancel{\sinh}\,\xi} - \frac{1}{\cosh\xi}\right]$$

$$= \frac{(\cosh^2\xi - \sin^2\mu)^2}{4\cos\mu}\left[\frac{2\xi}{\sinh\xi} - \frac{1}{\cosh\xi}\right] = v$$



$x = \cos\mu\,\sinh\xi$

$y = \sin\mu\,\cosh\xi$

$$\frac{x^2}{\sinh^2\xi} + \frac{y^2}{\cosh^2\xi} = 1$$

$$-\frac{x^2}{\cos^2\mu} + \frac{y^2}{\sin^2\mu} = 1$$

$$\frac{\partial\psi}{\partial y} = \frac{\sinh^2\xi + \cos^2\mu}{2\sin\mu\,\sinh 2h\xi}\left[2\xi\sinh\xi\cosh\xi + 2\xi\sinh\xi\sin^2\mu - \cosh^3\xi + \sin^2\mu\cosh\xi\right]$$

$$= \frac{\sinh^2\xi + \cos^2\mu}{4\sin\mu}\left[2\xi\left(\cosh\xi + \frac{\sin^2\mu}{\cosh\xi}\right) - \frac{\cosh^2\xi}{\sinh\xi} + \frac{\sin^2\mu}{\sinh\xi}\right]$$

$$\cancel{\frac{\xi\sin^2\mu}{\cosh\xi} + (\cosh^2\xi + \sin^2\mu)\frac{\xi}{\cosh\xi} - \frac{\mu}{\sinh\xi}}$$

$$\cosh^2\xi - \sin^2\mu = \sinh^2\xi + \cos^2\mu$$

$$\cosh^2\xi - \sin^2\mu = \frac{\cosh 2\xi + 1}{2} - \frac{1 - \cos 2\mu}{2} = \frac{\cosh 2\xi + \cos 2\mu}{2}$$

$$\cancel{\cosh^2\xi - \sin^2\mu = \frac{\cosh 2\xi + 1}{2} - \frac{1 - \cos 2\mu}{2}}$$

$$\cancel{\cosh 2\xi + \cos 2\mu = 2\cosh^2\xi - 1 + 1 - 2\sin^2\mu = 2(\cosh^2\xi - \sin^2\mu)}$$

$$\cancel{= 2\sinh^2\xi + 1 + 2\cos^2\mu - 1 = 2(\sinh^2\xi + \cos^2\mu)}$$

$$\cosh^2\xi + \sin^2\mu = \frac{\cosh 2\xi + 1}{2} + \frac{1 - \cos 2\mu}{2} = \cosh 2\xi - \cos 2\mu + 1$$

$$= \frac{\cosh 2\xi + \cos 2\mu}{8\sin\mu}\left[2\xi\,\frac{\cosh 2\xi - \cos 2\mu + 1}{2\cosh\xi} - \frac{\cosh 2\xi + \cos 2\mu}{2\sinh\xi}\right]$$

$$= \frac{\cosh^2\xi - \sin^2\mu}{4\sin\mu}\left[2\xi\,\frac{\cosh^2\xi + \sin^2\mu}{\cosh\xi} - \frac{\cosh^2\xi - \sin^2\mu}{\sinh\xi}\right] = -u$$

$$\psi = \frac{1}{32}\left\{ e^{2\zeta}[2\zeta - 2] + e^{-2\zeta}[2+2\zeta] - 2\zeta\left(e^{2i\varphi} + e^{-2i\varphi}\right)\right\}$$

$$= \frac{1}{16}\left\{ \zeta\left[e^{2\zeta} + e^{-2\zeta} - \left(e^{2i\varphi} + e^{-2i\varphi}\right)\right] - \left(e^{2\zeta} - e^{-2\zeta}\right)\right\}$$

$$= \frac{1}{8}\left\{ \zeta\left[\cosh 2\zeta - \cos 2\varphi\right] - \sinh 2\zeta\right\} +$$

$$\frac{\partial\psi}{\partial y} = \frac{\frac{\partial\psi}{\partial\zeta}}{\frac{\partial\zeta}{\partial y}} + \frac{\frac{\partial\psi}{\partial\varphi}}{\frac{\partial\varphi}{\partial y}}$$

$$\frac{\partial\psi}{\partial\zeta} = \frac{1}{8}\left\{\cosh 2\zeta - \cos 2\varphi + 2\zeta\sinh 2\zeta - 2\cosh 2\zeta\right\}$$

$$= \frac{1}{8}\left\{ 2\zeta\sinh 2\zeta - \underbrace{(\cosh 2\zeta + \cos 2\varphi)}_{=2(\sinh^2\zeta + \cos^2\varphi)}\right\}$$

$$= \frac{1}{4}\left\{2\zeta\sinh\zeta\cosh\zeta - \sinh^2\zeta - \cos^2\varphi\right\}$$

$$\frac{\partial\psi}{\partial\varphi} = \frac{1}{4}\zeta\sin 2\varphi = \frac{1}{2}\zeta\sin\varphi\cos\varphi$$

$$\frac{\partial\psi}{\partial y} = \left[\frac{\sinh^2\zeta + \cos^2\varphi}{4}\right]\left[2\zeta\frac{\sinh\zeta\cosh\zeta - \sinh^2\zeta - \cos^2\varphi}{\sin\varphi\sinh\zeta} + \frac{2\zeta\sin\varphi\cos\varphi}{\cos\varphi\cosh\zeta}\right]$$

$$= \frac{\sinh^2\zeta + \cos^2\varphi}{\sin 2\varphi\sinh 2\zeta}\left[2\zeta\sinh\zeta\cosh\zeta\cos\varphi + 2\zeta\sinh\zeta\sin\varphi\cos\varphi - \sinh\zeta\cosh\zeta\cos\varphi - \cos^3\varphi\cosh\zeta\right]$$

$\xi + i\mu = \alpha$ $\quad$ $\xi = \frac{\alpha+\beta}{2}$ $\quad$ $\mu = \frac{\alpha-\beta}{2i}$

$\xi - i\mu = \beta$

$$\frac{\partial^2\psi}{\partial\mu^2} + \frac{\partial^2\psi}{\partial\xi^2} = \xi\cos^2\mu + \xi\sinh^2\xi = \xi\,\frac{e^{2\xi}+e^{-2\xi}+e^{2i\mu}+e^{-2i\mu}}{4}$$

$$4\,\frac{\partial^2\psi}{\partial\alpha\partial\beta} = \frac{\alpha+\beta}{2}\;\frac{e^{\alpha+\beta}+e^{-\alpha-\beta}+e^{\alpha-\beta}+e^{-\alpha+\beta}}{4}$$

$$\frac{\partial^2\psi}{\partial\alpha\partial\beta} = \frac{1}{32}(\alpha+\beta)\left[e^{\beta}(e^{\alpha}+e^{-\alpha}) + e^{-\beta}(e^{\alpha}+e^{-\alpha})\right]$$

$$\psi = \iint \frac{(\alpha+\beta)}{32}(e^{\alpha}+e^{-\alpha})(e^{\beta}+e^{-\beta})\,d\alpha\,d\beta$$

$$\int x e^{x}\,dx = x e^{x} - e^{x} \qquad \int x e^{-x}\,dx = -x e^{-x} - e^{-x}$$

$$\psi = \frac{1}{32}\int (e^{\beta}+e^{-\beta})\left[\alpha e^{\alpha} - e^{\alpha} - \alpha e^{-\alpha} - e^{-\alpha} + \beta(e^{\alpha}-e^{-\alpha})\right]d\beta$$

$$= \frac{1}{32}\left\{(e^{\beta}-e^{-\beta})\left[e^{\alpha}(\alpha-1) - e^{-\alpha}(\alpha+1)\right] + \left[e^{\alpha}-e^{-\alpha}\right]\left[\beta e^{\beta} - e^{\beta} - \beta e^{-\beta} - e^{-\beta}\right]\right\}$$

$$= \frac{1}{32}\Big\{(e^{\beta}-e^{-\beta})\alpha(e^{\alpha}-e^{-\alpha}) + (e^{\alpha}-e^{-\alpha})(e^{\beta}-e^{-\beta})\beta -$$

$$- e^{\alpha+\beta} + \cancel{e^{\alpha-\beta}} \cancel{- e^{-\alpha+\beta}} + e^{-\alpha-\beta} - e^{\alpha+\beta} \cancel{+ e^{-\alpha+\beta}} \cancel{- e^{\alpha-\beta}} + e^{-\alpha-\beta}\Big\}$$

$$= \frac{1}{32}\Big\{(\alpha+\beta)\underbrace{(e^{\beta}-e^{-\beta})(e^{\alpha}-e^{-\alpha})}_{e^{\alpha+\beta}+e^{-\alpha-\beta}-e^{\alpha-\beta}-e^{-\alpha+\beta}} - 2e^{\alpha+\beta} + 2e^{-\alpha-\beta}\Big\}$$

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$$x + iy = \sinh(\xi + i\mu) = \frac{e^{\xi+i\mu} - e^{-\xi-i\mu}}{2}$$

$$x = \frac{e^{\xi}\cos\mu - e^{-\xi}\cos\mu}{2} = \cos\mu \sinh\xi$$

$$y = \frac{e^{\xi}\sin\mu + e^{-\xi}\sin\mu}{2} = \sin\mu \cosh\xi$$

$\cosh^2\mu = 1 + \sinh^2\mu$

$1 + \cosh 2\mu = 2\cosh^2\mu$

$\cosh 2\mu - 1 = 2\sinh^2\mu$

$$\cancel{\cosh(\xi+i\mu)\cdot\cosh(\xi-i\mu) = \frac{(e^{\xi+i\mu} - e^{-\xi-i\mu})(e^{\xi-i\mu} - e^{-\xi+i\mu})}{4}}$$

$$\cancel{= \frac{e^{2\xi} - e^{-2i\mu} - e^{2i\mu} + e^{-2\xi}}{4}} = \cancel{\frac{\cosh(2\xi) - \cos 2\mu}{2}}$$

$$1 = -\sin\mu\sinh\xi\cdot\frac{\partial\mu}{\partial x} + \cos\mu\cosh\xi\cdot\frac{\partial\mu}{\partial y} \quad \Big| \ \sin\mu\sinh\xi$$

$$0 = -\sin\mu\sinh\xi\,\frac{\partial\mu}{\partial y} - \cos\mu\cosh\xi\cdot\frac{\partial\mu}{\partial x} \quad \Big| \ \cos\mu\cosh\xi$$

$$\sin\mu\sinh\xi = -(\sin^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi)\,\frac{\partial\mu}{\partial x}$$

$$\frac{\partial\mu}{\partial x} = \frac{-\sin\mu\sinh\xi}{\sin^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi} \ ; \quad \frac{\partial\mu}{\partial y} = \frac{\cos\mu\cosh\xi}{\ldots\ldots}$$

$$\frac{1}{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2} = \sin^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi$$

$$= \sin^2\mu\,\frac{\cosh 2\xi - 1}{2} + \cos^2\mu\,\frac{\cosh 2\xi + 1}{2} = \frac{\cosh 2\xi}{2} + \frac{\cos 2\mu}{2}$$

$$= \sinh^2\xi - \cos^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi = \sinh^2\xi + \cos^2\mu$$

$$x + iy = f_0(\xi + i\mu)$$

$$\Delta^2 \psi = \zeta$$

$$\frac{\partial^2\psi}{\partial\mu^2} + \frac{\partial^2\psi}{\partial\xi^2} = \frac{\zeta}{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2}$$

$$x = f_1(\mu, \xi)$$

$$y = f_2(\mu, \xi)$$

$$1 = \left(\frac{\partial\xi}{\partial x} + i\frac{\partial\mu}{\partial x}\right) f_0' = \left(\frac{\partial\mu}{\partial y} + i\frac{\partial\mu}{\partial x}\right) f_0'$$

$$\frac{1}{\frac{\partial\mu}{\partial y} + i\frac{\partial\mu}{\partial x}} = f_0'(\xi + i\mu)$$

$$\frac{1}{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2} = f_0'(\xi + i\mu)\, f_0'(\xi - i\mu)$$

$$\psi = \dots$$

$$\Delta^2\varphi = 0 \qquad \frac{\partial^2\varphi}{\partial\mu^2} + \frac{\partial^2\varphi}{\partial\xi^2} = 0$$

$$\varphi = \Re f_0(\mu \pm i\xi)$$

$$u = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial y} = \frac{\partial\varphi}{\partial\mu}\frac{\partial\mu}{\partial x} + \frac{\partial\varphi}{\partial\xi}\frac{\partial\xi}{\partial x} - \frac{\partial\psi}{\partial\mu}\frac{\partial\mu}{\partial y} - \frac{\partial\psi}{\partial\xi}\frac{\partial\xi}{\partial y}$$

$$= \frac{\partial\mu}{\partial y}\left(\frac{\partial\varphi}{\partial\xi} - \frac{\partial\psi}{\partial\mu}\right) + \frac{\partial\mu}{\partial x}\left(\frac{\partial\varphi}{\partial\mu} + \frac{\partial\psi}{\partial\xi}\right)$$

$$= \frac{\frac{\partial\varphi}{\partial\xi} - \frac{\partial\psi}{\partial\mu}}{\frac{\partial y}{\partial\mu}} + \frac{\frac{\partial\varphi}{\partial\mu} + \frac{\partial\psi}{\partial\xi}}{\frac{\partial x}{\partial\mu}}$$

Rozwiązanie równania: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = f(x,y)$ || Przyczem u oznacza zmienną jakąś bądź

$$x+iy=\alpha \qquad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial \alpha}\frac{\partial \alpha}{\partial x}+\frac{\partial u}{\partial \beta}\frac{\partial \beta}{\partial x} = \frac{\partial u}{\partial \alpha}+\frac{\partial u}{\partial \beta}$$

$$x-iy=\beta \qquad \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \alpha^2}+2\frac{\partial^2 u}{\partial \alpha\,\partial \beta}+\frac{\partial^2 u}{\partial \beta^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \alpha}\frac{\partial \alpha}{\partial y}+\frac{\partial u}{\partial \beta}\frac{\partial \beta}{\partial y} = i\left(\frac{\partial u}{\partial \alpha}-\frac{\partial u}{\partial \beta}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[\frac{\partial^2 u}{\partial \alpha^2}-2\frac{\partial^2 u}{\partial \alpha\,\partial \beta}+\frac{\partial^2 u}{\partial \beta^2}\right]$$

$$\Delta^2 u = 4\frac{\partial^2 u}{\partial \alpha\,\partial \beta} = f\left(\frac{\alpha+\beta}{2},\ \frac{\alpha-\beta}{2i}\right)$$

$$u = \frac{1}{4}\int d\alpha\left[\int f\left(\frac{\alpha+\beta}{2},\ \frac{\alpha-\beta}{2i}\right)d\beta\right]+f_1(\alpha)+f_2(\beta)$$

$$u = \frac{1}{4}\iint f\left(\frac{\alpha+\beta}{2},\ \frac{\alpha-\beta}{2i}\right)d\alpha\,d\beta+\underbrace{f_1(\alpha)+f_2(\beta)}_{R\,f(x+iy)}$$

Transformacya na ρ, ξ zamiast x, y

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial x}+\frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \rho^2}\left(\frac{\partial \rho}{\partial x}\right)^2+\frac{\partial u}{\partial \rho}\frac{\partial^2 \rho}{\partial x^2}+2\frac{\partial^2 u}{\partial \rho\,\partial \xi}\frac{\partial \rho}{\partial x}\frac{\partial \xi}{\partial x}+\frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2+\frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2}$$

$$\Delta^2 u = \frac{\partial^2 u}{\partial \rho^2}\left[\left(\frac{\partial \rho}{\partial x}\right)^2+\left(\frac{\partial \rho}{\partial y}\right)^2\right]+\frac{\partial u}{\partial \rho}\left[\cancel{\frac{\partial^2 \rho}{\partial x^2}+\frac{\partial^2 \rho}{\partial y^2}}\right]+2\frac{\partial^2 u}{\partial \rho\,\partial \xi}\left[\cancel{\frac{\partial \rho}{\partial x}\frac{\partial \xi}{\partial x}+\frac{\partial \rho}{\partial y}\frac{\partial \xi}{\partial y}}\right]+$$

$$+\frac{\partial^2 u}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2+\left(\frac{\partial \xi}{\partial y}\right)^2\right]+\frac{\partial u}{\partial \xi}\left[\cancel{\frac{\partial^2 \xi}{\partial x^2}+\frac{\partial^2 \xi}{\partial y^2}}\right]$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = \left[\frac{\partial^2 u}{\partial \rho^2}+\frac{\partial^2 u}{\partial \xi^2}\right]\left[\left(\frac{\partial \rho}{\partial x}\right)^2+\left(\frac{\partial \rho}{\partial y}\right)^2\right]$$

$$\text{(I).}\quad \boxed{\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ &\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta \end{aligned}}$$

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = -\frac{\partial \zeta}{\partial y} \qquad \text{(II).}$$

$$\frac{\partial p}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\partial \zeta}{\partial x}$$

$$\text{(III).}\quad \Delta^2 p = 0 \qquad \Delta^2 \zeta = 0$$

$$f(x+iy) = g(x,y) + i\,h(x,y)$$

$$f' = \frac{\partial g}{\partial x} + i\,\frac{\partial h}{\partial x} = \frac{1}{i}\left[\frac{\partial g}{\partial y} + i\,\frac{\partial h}{\partial y}\right]$$

$$\left|\ \frac{\partial g}{\partial x} = \frac{\partial h}{\partial y}\ \right|\ \frac{\partial g}{\partial y} = -\frac{\partial h}{\partial x}\ \right|$$

$$g = \zeta \qquad h = p$$

1). $\Delta^2 u = -\frac{\partial \zeta}{\partial y}$

$$u = -J\left(\frac{\partial \zeta}{\partial y}\right) + \varphi$$

$$v = J\left(\frac{\partial \zeta}{\partial x}\right) + \varphi'$$

Wtedy φ i φ' jednak nie są dowolne tylko: muszą jednak spełniać warunki (I). t.j. $\left\{\begin{aligned} &\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi'}{\partial y} = 0 \\ &\frac{\partial \varphi'}{\partial x} - \frac{\partial \varphi}{\partial y} = 0 \end{aligned}\right.$

zatem $\nabla^2 u = 0$.

2). albo lepiej: znaleźć $J(h) = F$ $\quad \Delta^2 F = h$

wtedy: $u = -\frac{\partial F}{\partial y} + \varphi$

$$v = \frac{\partial F}{\partial x} + \varphi'$$

3). Najlepiej $\quad u = -\frac{\partial F}{\partial y} + \frac{\partial A}{\partial x}$

$$v = +\frac{\partial F}{\partial x} + \frac{\partial A}{\partial y}$$

$$\Delta^2 A = 0$$

$$\Delta^2 F = h = p$$

$$\frac{\partial v}{\partial \alpha} + \frac{\partial v}{\partial \beta} + \left[\frac{\partial u}{\partial \alpha} i - \frac{\partial u}{\partial \beta} i\right] \cdot \frac{\alpha - \beta}{i i} \quad \frac{\partial^2 u}{\partial \alpha \partial \beta}$$

$$\frac{\partial}{\partial \alpha}(v + u i) + \frac{\partial}{\partial \beta}\, v - u i =$$

$$\frac{\partial}{\partial \alpha}(u \mp i v) \mp \frac{\partial}{\partial \beta}(u + i v) = 2(\alpha - \beta) \frac{\partial^2 u}{\partial \alpha \partial \beta}$$

$$u = p x \pm f y \qquad \frac{\partial u}{\partial x} = p + x \frac{\partial p}{\partial x} \pm y \frac{\partial f}{\partial y} \qquad \frac{\partial u}{\partial y} = x \frac{\partial p}{\partial y} \pm f \mp y \frac{\partial f}{\partial x}$$

$$v = -p y \pm f x \qquad \frac{\partial v}{\partial y} = -p \mp y \frac{\partial p}{\partial y} \pm x \frac{\partial f}{\partial x} \qquad \frac{\partial v}{\partial x} = -y \frac{\partial p}{\partial x} \pm f \pm x \frac{\partial f}{\partial y}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$\cancel{u = p y \quad f y}$

$\cancel{v =}$

$$u = p x + f \qquad \frac{\partial u}{\partial x} = p + x \frac{\partial p}{\partial x} \qquad \frac{\partial u}{\partial y} = x \frac{\partial p}{\partial y}$$

$$v = \qquad\qquad \frac{\partial v}{\partial y} =$$

$\cancel{u = p y} \quad \cancel{\frac{\partial u}{\partial x} = y \frac{\partial p}{\partial x}}$

$\cancel{v = y}$

1

$$\frac{\partial(u+iv)}{\partial p} + i\,\frac{\partial(u+iv)}{\partial \zeta} = i\zeta\left[\frac{\partial^2(u+iv)}{\partial p^2} + \frac{\partial^2(u+iv)}{\partial \zeta^2}\right]$$

$p + i\zeta = \alpha \qquad u + iv = \omega \qquad p = \frac{\alpha+\beta}{2}$

$p - i\zeta = \beta \qquad\qquad\qquad\qquad \zeta = \frac{\alpha-\beta}{2i}$

$$\cancel{\frac{\partial\omega}{\partial\alpha}} + \frac{\partial\omega}{\partial\beta} + i\left[\cancel{\frac{\partial\omega}{\partial\alpha}}\, i - \frac{\partial\omega}{\partial\beta}\, i\right] = \frac{\alpha-\beta}{2} \cdot 4\,\frac{\partial^2\omega}{\partial\alpha\,\partial\beta}$$

$$2\,\frac{\partial\omega}{\partial\beta} = 2(\alpha-\beta)\,\frac{\partial^2\omega}{\partial\alpha\,\partial\beta}$$

$$\frac{1}{\alpha-\beta} = \frac{\partial}{\partial\alpha}\left(\log\frac{\partial\omega}{\partial\beta}\right)$$

$$\log(\alpha-\beta) + f_1(\beta) = \log\frac{\partial\omega}{\partial\beta}$$

$$\cancel{\omega = f_1(\alpha) + \int f(\beta)\,d\beta + \int \log(\alpha-\beta)\,d\beta}$$

$$\frac{\partial\omega}{\partial\beta} = e^{\log(\alpha-\beta)+f_1(\beta)} = (\alpha-\beta)\cdot F(\beta)$$

$$\omega = f_1(\alpha) + \int(\alpha-\beta)\,F(\beta)\,d\beta$$

$\cancel{u = \tfrac{1}{4}\left[\zeta y + \rho x\right]}$

$\cancel{v = \tfrac{1}{4}\left[-\rho y + \zeta x\right]}$

$$\Delta^2\psi = \zeta\left[\left(\frac{\partial\zeta}{\partial x}\right)^2 + \left(\frac{\partial\zeta}{\partial y}\right)^2\right]\left[\frac{\partial^2\psi}{\partial\rho^2} + \frac{\partial^2\psi}{\partial\zeta^2}\right]$$

$$\frac{\partial^2\psi}{\partial\rho^2} + \frac{\partial^2\psi}{\partial\zeta^2} = \zeta\left[\sin^2 h\rho \cos^2\zeta + \cos^2 h\rho \sin^2\zeta\right]$$

$$\frac{\partial^2\psi}{\partial\alpha\,\partial\beta} = \frac{1}{4}\,\frac{\alpha-\beta}{2i}\left[2\left[e^{2\alpha} + e^{-2\alpha} - 2 + e^{2\beta} - e^{-2\beta} - 2\right]\right]$$

$$e^{\alpha-\beta}\ \cancel{+ e^{\beta-\alpha} - e^{-\alpha-\beta} - e^{\alpha+\beta}}$$

$$\frac{\partial v}{\partial\rho} + \frac{\partial u}{\partial\zeta} = \zeta\left(\frac{\partial^2 u}{\partial\rho^2} + \frac{\partial^2 u}{\partial\zeta^2}\right)$$

$$\frac{\partial v}{\partial\zeta} - \frac{\partial u}{\partial\rho} = \zeta\left(\frac{\partial^2 v}{\partial\rho^2} + \frac{\partial^2 v}{\partial\zeta^2}\right)$$

$$u = \frac{1}{4}\left[\rho x + \zeta y\right] + f_1(\rho+i\zeta) + f_1(\rho-i\zeta)$$

$$v = \frac{1}{4}\left[\zeta x - \rho y\right] + f_2(\rho+i\zeta)$$

$$\frac{\partial v}{\partial\rho} = \frac{1}{4}\left[\zeta\frac{\partial x}{\partial\rho} - y - \rho\frac{\partial y}{\partial\rho}\right] + f_2'(\rho+i\zeta) + f_2'(\rho-i\zeta)$$

$$\frac{\partial u}{\partial\zeta} = \frac{1}{4}\left[\rho\frac{\partial x}{\partial\zeta} + y - \zeta\frac{\partial y}{\partial\zeta}\right] + i\left[f_1'(\rho+i\zeta) - f_1'(\rho-i\zeta)\right]$$

$$\frac{\partial x}{\partial\rho} = -\frac{\partial y}{\partial\zeta}$$

$$\frac{\partial v}{\partial\rho} + \frac{\partial u}{\partial\zeta} = \frac{1}{2}\zeta\frac{\partial x}{\partial\rho} + (f_2' + i f_1')(\rho+i\zeta) + (f_2' - i f_1')(\rho-i\zeta)$$

$$\underbrace{\frac{\partial v}{\partial\rho} + i\frac{\partial v}{\partial\zeta} + \frac{\partial u}{\partial\zeta} - i\frac{\partial u}{\partial\rho}}_{\frac{\partial(u+iv)}{\partial\zeta} + \frac{1}{i}\frac{\partial(u+iv)}{\partial\rho}} = \zeta\left[\frac{\partial^2(u+iv)}{\partial\rho^2} + \frac{\partial^2(u+iv)}{\partial\zeta^2}\right]$$

$$u = \frac{1}{4}\left[px + \xi y\right] + \varphi(p,\xi) \qquad \frac{\partial \xi}{\partial x} = \frac{\partial p}{\partial y}$$

$$v = \frac{1}{4}\left[-py + \xi x\right] + \psi(p,\xi) \qquad \frac{\partial \xi}{\partial y} = -\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{4}\left[p + x\frac{\partial p}{\partial x} + y\frac{\partial \xi}{\partial x}\right] + \frac{\partial \varphi}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial \varphi}{\partial \xi}\frac{\partial p}{\partial y}$$

$$\frac{\partial v}{\partial y} = \frac{1}{4}\left[-p - y\frac{\partial p}{\partial y} + x\frac{\partial \xi}{\partial y}\right] + \frac{\partial \psi}{\partial p}\frac{\partial p}{\partial y} - \frac{\partial \psi}{\partial \xi}\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left.\begin{aligned}\frac{\partial v}{\partial x} &= \frac{1}{4}\left[-y\frac{\partial p}{\partial x} + \xi + x\frac{\partial \xi}{\partial x}\right]\\ \frac{\partial u}{\partial y} &= \frac{1}{4}\left[x\frac{\partial p}{\partial y} + \xi + y\frac{\partial \xi}{\partial y}\right]\end{aligned}\right\} \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\cancel{u+iv=\tfrac{1}{4}} \quad f(p+i\xi) = \cancel{\tfrac{1}{4}\left[p(x-iy) + \xi(x+iy)\right] +}$$

$$\varphi(p,\xi) + i\,\psi(p,\xi)$$

$$f' = \frac{\partial \varphi}{\partial p} + i\frac{\partial \psi}{\partial p} \quad \Big| \frac{\partial p}{\partial x}$$

$$if' = \frac{\partial \varphi}{\partial \xi} + i\frac{\partial \psi}{\partial \xi} \quad \Big| \frac{\partial \xi}{\partial x}$$

$$f'\frac{\partial(p+i\xi)}{\partial x} = \frac{\partial \varphi}{\partial x} + i\frac{\partial \psi}{\partial x}$$

$$z = p + i\xi$$

$$f'(z) = \frac{\partial f}{\partial p} =$$

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$$u = \frac{1}{4}\left[\xi\,\underbrace{\cosh\mu\cos\xi}_{y} - \mu\,\underbrace{\sinh\mu\sin\xi}_{-x}\right]$$

$$u = \frac{1}{4}(\xi y + \mu x)$$

$$\frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\xi^2} = -\sinh\mu\cos\xi$$

$$= -\frac{e^{\mu}-e^{-\mu}}{2}\,\frac{e^{i\xi}+e^{-i\xi}}{2} = -\frac{1}{4}\left[e^{\mu+i\xi} - e^{-(\mu+i\xi)} + e^{\mu-i\xi} - e^{-\mu+i\xi}\right]$$

$$\frac{\partial^2 v}{\partial\alpha\partial\beta} = -\frac{1}{16}\left[e^{\alpha} - e^{-\alpha} + e^{\beta} - e^{-\beta}\right]$$

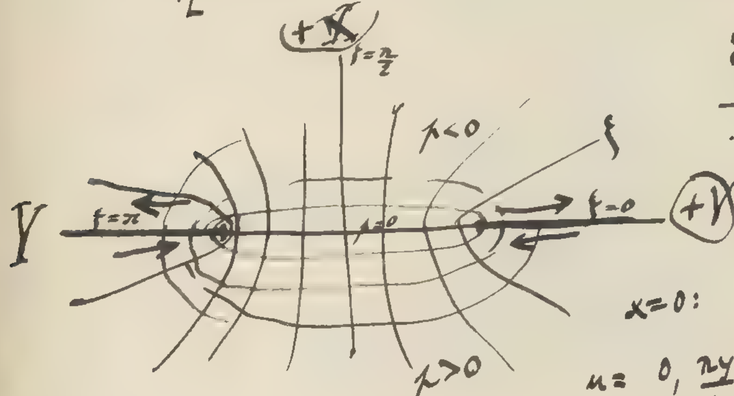
$$v = -\frac{1}{16}\left[\beta\,(e^{\alpha}+e^{-\alpha}) + \alpha\,(e^{\beta}+e^{-\beta})\right]$$

$$= -\frac{1}{16}\left[(\mu-i\xi)(\cosh\mu\cos\xi + i\sinh\mu\sin\xi) + (\mu+i\xi)(\cosh\mu\cos\xi - i\sinh\mu\sin\xi)\right]$$

$$v = -\frac{1}{4}\left[\mu\cosh\mu\cos\xi + \xi\sinh\mu\sin\xi\right]$$

$$v = \frac{1}{4}\left[-\mu y + \xi x\right]$$

	$\tanh\mu = 0$	$\tanh = \frac{y}{x}$ ~~$\tanh 0$~~
$x = 0:$	$\xi = 0$	$u = 0;\ v = -\frac{\mu y}{4}$
$y = 0:$	$\xi = \pm\frac{\pi}{2}$	$u = \frac{\mu x}{4};\ v = \frac{\pi x}{8}$



$x = 0:$

$u = 0, \frac{\pi y}{4};\quad v = -\frac{\mu y}{4},$

$$y - ix = \cosh(\mu + i\zeta) = \frac{e^{\mu+i\zeta} + e^{-\mu-i\zeta}}{2}$$

$$y = \cosh\mu \cos\zeta \qquad \frac{x^2}{\sinh^2\mu} + \frac{y^2}{\cosh^2\mu} = 1$$

$$x = -\sinh\mu \sin\zeta \qquad \frac{y^2}{\cos^2\zeta} - \frac{x^2}{\sin^2\zeta} = 1$$

$$1 = -\sinh\mu \cos\zeta \frac{\partial\zeta}{\partial x} - \cosh\mu \sin\zeta \frac{\partial\mu}{\partial x} \qquad \frac{\partial\zeta}{\partial x} = \frac{\partial\mu}{\partial y}$$

$$0 = \sinh\mu \cos\zeta \frac{\partial\mu}{\partial x} - \cosh\mu \sin\zeta \frac{\partial\zeta}{\partial x} \qquad \frac{\partial\zeta}{\partial x} = \frac{\partial\mu}{\partial y}$$

$$\begin{vmatrix} \sinh\mu\cos\zeta & \cosh\mu\sin\zeta \\ \cosh\mu\sin\zeta & -\sinh\mu\cos\zeta \end{vmatrix}$$

$$\frac{\partial\mu}{\partial y} = -\frac{\sinh\mu\cos\zeta}{\sinh^2\mu\cos^2\zeta + \cosh^2\mu\sin^2\zeta}$$

$$\frac{\partial\mu}{\partial x} = \frac{-\cosh\mu\sin\zeta}{\cosh^2\mu\sin^2\zeta + \sinh^2\mu\cos^2\zeta}$$

$$\frac{\partial^2 u}{\partial\zeta^2} + \frac{\partial^2 u}{\partial\mu^2} = -\cosh\mu\sin\zeta$$

$$= -\frac{e^{\mu}+e^{-\mu}}{2}\,\frac{e^{i\zeta}-e^{-i\zeta}}{2i} = \frac{1}{4i}\left[-e^{\mu+i\zeta} - e^{-\mu+i\zeta} + e^{\mu-i\zeta} + e^{-\mu-i\zeta}\right]$$

$$\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{1}{16i}\left[-e^{\alpha} + e^{-\alpha} + e^{\beta} - e^{-\beta}\right]$$

$$u = \frac{1}{16i}\left[-\beta(e^{\alpha}+e^{-\alpha}) + \alpha(e^{\beta}+e^{-\beta})\right]$$

$$= \frac{1}{8}\left[-(\mu-i\zeta)(\cosh\mu\cos\zeta + i\sinh\mu\sin\zeta) + (\mu+i\zeta)(\cosh\mu\cos\zeta - i\sinh\mu\sin\zeta)\right]$$

$$= \frac{1}{8}\left[\cancel{-\mu\cosh\mu\cos\zeta - \zeta\sinh\mu\sin\zeta}\right.$$

$$v = -\frac{1}{16}\Big[\beta(e^{\alpha}-e^{-\alpha}) - \alpha(e^{\beta}-e^{-\beta})\Big]$$

$$= -\frac{1}{16}\Big[(\mu-i\xi)(e^{\mu+i\xi}-e^{-\mu-i\xi}) - (\mu+i\xi)(e^{\mu-i\xi}-e^{-\mu+i\xi})\Big]$$

$$= -\frac{1}{16}\Big[\mu e^{\mu}[(\cos\xi+i\sin\xi) - (\cos\xi-i\sin\xi)] + \ldots$$

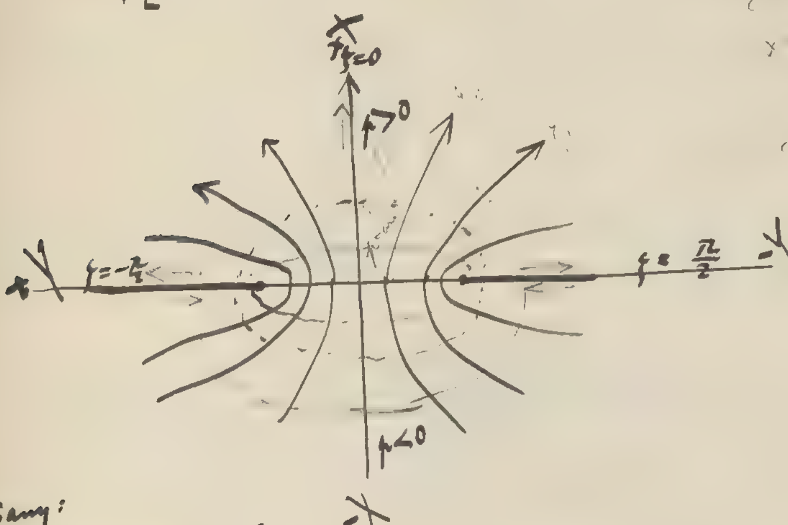
$$+\mu e^{-\mu}[-(\cos\xi-i\sin\xi)+(\cos\xi+i\sin\xi)]$$

$$+i\xi e^{\mu}[-(\cos\xi+\cancel{i\sin\xi})-(\cos\xi-\cancel{i\sin\xi})]$$

$$+i\xi e^{-\mu}[\cos\xi-\cancel{i\sin\xi}+\cos\xi+\cancel{i\sin\xi}]$$

$$v = \frac{1}{4}\left[-\mu\cosh\mu\sin\xi + \xi\sinh\mu\cos\xi\right]$$

$$v = \frac{1}{4}\left[\mu y + \xi x\right] + R\, f(\mu+i\xi)$$



Ściany:

$x=0$: $u = -\frac{1}{4}\frac{\pi}{2}y$

$v = \frac{\mu y}{4}$

$$x - iy = \sinh(\mu + i\zeta)$$

$$x = \sinh\mu\cos\zeta \qquad \frac{x^2}{\sinh^2\mu} + \frac{y^2}{\cosh^2\mu} = 1$$

$$y = -\cosh\mu\sin\zeta \qquad \frac{y^2}{\sin^2\zeta} - \frac{x^2}{\cos^2\zeta} = 1$$

$$\begin{array}{l|c|c} 1 = \cosh\mu\cos\zeta\,\frac{\partial\mu}{\partial x} - \sinh\mu\sin\zeta\,\frac{\partial\zeta}{\partial x} & \cosh\mu\cos\zeta & \sinh\mu\sin\zeta \\ 0 = -\sinh\mu\sin\zeta\,\frac{\partial\mu}{\partial x} - \cosh\mu\cos\zeta\,\frac{\partial\zeta}{\partial x} & -\sinh\mu\sin\zeta & \cosh\mu\cos\zeta \end{array}$$

$$\cosh\mu\cos\zeta = (\cosh^2\mu\cos^2\zeta + \sinh^2\mu\sin^2\zeta)\,\frac{\partial\mu}{\partial x}$$

$$\frac{\partial\mu}{\partial x} = \frac{\cosh\mu\cos\zeta}{\cosh^2\mu\cos^2\zeta + \sinh^2\mu\sin^2\zeta} \qquad \Bigg\| \qquad \frac{\partial\mu}{\partial y} = -\frac{\sinh\mu\sin\zeta}{\sinh^2\mu\sin^2\zeta + \cosh^2\mu\cos^2\zeta}$$

$$\frac{\partial^2 u}{\partial\mu^2} + \frac{\partial^2 u}{\partial\zeta^2} = \cosh\mu\cos\zeta$$

$$= \frac{e^{\mu}+e^{-\mu}}{2}\,\frac{e^{i\zeta}+e^{-i\zeta}}{2} = \frac{1}{4}\left[e^{\mu+i\zeta} + e^{-(\mu+i\zeta)} + e^{-\mu+i\zeta} + e^{\mu-i\zeta}\right]$$

$$u = \frac{1}{4}\left[\mu\sinh\mu\cos\zeta + \zeta\cosh\mu\sin\zeta\right]$$

$$u = \frac{1}{4}\left[\mu x - \zeta y\right] + R\,f.(\mu+i\zeta)$$

$$\frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\zeta^2} = -\sinh\mu\sin\zeta = -\frac{e^{\mu}-e^{-\mu}}{2}\,\frac{e^{i\zeta}-e^{-i\zeta}}{2} = -\frac{1}{4}\left[e^{\mu+i\zeta} + e^{-\mu-i\zeta} - e^{\mu-i\zeta} - e^{-\mu+i\zeta}\right]$$

$$16\,\frac{\partial^2 v}{\partial\alpha\,\partial\beta} = -\left[e^{\alpha} + e^{-\alpha} - e^{\beta} - e^{-\beta}\right]$$

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial x}\left(-\frac{\partial v}{\partial y}\right) + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = -\frac{\partial \zeta}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial x^2} - \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\partial \zeta}{\partial x}$$

$$\left.\begin{aligned} u &= \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} \\ v &= \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x} \end{aligned}\right\} \qquad \begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= \Delta^2 \varphi = 0 \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} &= \Delta^2 \psi = \zeta \end{aligned}$$

$$\Delta^2 u = -\frac{\partial \zeta}{\partial y} = \frac{\partial p}{\partial x}$$

$$\Delta^2 v = \frac{\partial \zeta}{\partial x} = \frac{\partial p}{\partial y}$$

$$\Delta^2 p = 0 \qquad \Delta^2 \zeta = 0$$

$$\Delta^4 \psi = 0$$

$$f(x+iy) = \varphi + i\psi$$

$$\left.\begin{aligned} f' &= \frac{\partial \varphi}{\partial x} + i\frac{\partial \psi}{\partial x} \\ i f' &= \frac{\partial \varphi}{\partial y} + i\frac{\partial \psi}{\partial y} \end{aligned}\right| i \Bigg\} \qquad \begin{aligned} \frac{\partial \varphi}{\partial y} &= -\frac{\partial \psi}{\partial x} \\ \frac{\partial \varphi}{\partial x} &= \frac{\partial \psi}{\partial y} \end{aligned}$$

$$f(x-iy) = \varphi + i\psi$$

$$\left.\begin{aligned} f' &= \frac{\partial \varphi}{\partial x} + i\frac{\partial \psi}{\partial x} \\ -i f' &= \frac{\partial \varphi}{\partial y} + i\frac{\partial \psi}{\partial y} \end{aligned}\right| i \Bigg\} \qquad \begin{aligned} \frac{\partial \varphi}{\partial x} &= -\frac{\partial \psi}{\partial y} \\ \frac{\partial \varphi}{\partial y} &= \frac{\partial \psi}{\partial x} \end{aligned}$$

$$\boxed{f(x-iy) = p + i\zeta}$$

$$\frac{\partial p}{\partial x} = -\frac{\partial \zeta}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\partial \zeta}{\partial x}$$

$f = Az$

$p = x$

$q = y = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$

$u = -\dfrac{a\frac{y^2}{2} + b\frac{x^2}{2}}{a+b}$ $\qquad -\frac{x^2}{2} + \frac{\partial\varphi}{\partial x}$

$\psi = \frac{y^3}{6}, \quad \frac{x^2 y}{2}$

$v = \dfrac{bxy}{a+b}$ $\qquad xy + \frac{\partial\varphi}{\partial y}$

$\psi = \dfrac{a\frac{y^3}{6} + b\frac{x^2 y}{2}}{a+b}$

$\left.\begin{aligned} \frac{\partial u}{\partial x} &= -x \\ \frac{\partial v}{\partial y} &= +x \end{aligned}\right\} = 0 \qquad \left.\begin{aligned} \frac{\partial v}{\partial x} &= y \\ -\frac{\partial u}{\partial y} &= 0 \end{aligned}\right\} = q$

$1 = -1$

$0 = 0$

$u = -\frac{p^2}{2} \qquad \frac{\partial u}{\partial p} = -p \qquad \frac{\partial^2 u}{\partial p^2} = -1 \qquad \frac{\partial^2 u}{\partial q^2} = 0$

$v = pq$

$\partial^2 u = -1 = \frac{1}{1}$

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$$-\frac{e^{p}-e^{-p}}{2}\sin\xi = \left[\left(\frac{e^{p}-e^{-p}}{2}\sin\xi\right)^2 + \left(\frac{e^{p}+e^{-p}}{2}\cos\xi\right)^2\right]\frac{\partial p}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{-\sinh p \sin\xi}{\cos^2\xi + \sinh^2 p}$$

$\Delta u = \cosh p \cos\xi$ $\qquad$ $\Delta v = -\sinh p \sin\xi$

$$u = \frac{1}{4}\left[p \sinh p \cos\xi - \xi \cosh p \sin\xi\right] + \Phi(p+i\xi) + \Psi(p-i\xi)$$

$$v = \frac{1}{4}\left[p \cosh p \sin\xi + \xi \sinh p \cos\xi\right]$$

$$\frac{\partial u}{\partial \xi} = \frac{1}{4}\left[-p \sinh p \sin\xi - \cosh p \sin\xi - \xi \cosh p \cos\xi\right]$$

$$\frac{\partial v}{\partial p} = \frac{1}{4}\left[+\cosh p \sin\xi + p \sinh p \sin\xi + \xi \cosh p \cos\xi\right]$$

$$\frac{\partial v}{\partial p} = \xi \cosh p \cos\xi -$$

$u = \frac{1}{2} p x$

$\frac{\partial u}{\partial x} = \frac{1}{2}\left[p + x\frac{\partial p}{\partial x}\right]$ $\qquad$ $\frac{\partial u}{\partial y} = \frac{1}{2} x \frac{\partial p}{\partial y}$

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\left[2\frac{\partial p}{\partial x} + x\frac{\partial^2 p}{\partial x^2}\right]$ $\qquad$ $\frac{\partial^2 u}{\partial y^2} = \frac{1}{2} x \frac{\partial^2 p}{\partial y^2}$

$$\Delta^2 u = \frac{\partial p}{\partial x} + x\cancel{\Delta p}$$

$u = -\frac{1}{2}\xi y$

$\frac{\partial u}{\partial x} = -\frac{1}{2} y \frac{\partial \xi}{\partial x} = -\frac{1}{2} y \frac{\partial p}{\partial y}$ $\Big|$ $\frac{\partial u}{\partial y} = \frac{1}{2}\left(\xi - y\frac{\partial p}{\partial x}\right)$

$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{2} y \frac{\partial^2 p}{\partial x \partial y}$ $\Big\|$ $\frac{\partial^2 u}{\partial y^2} = \frac{1}{2}\left(-2\frac{\partial p}{\partial x} + y\frac{\partial^2 p}{\partial x \partial y}\right)$

$$\Delta^2 u = \frac{\partial p}{\partial x}$$

$v = \frac{1}{2} p y$

$\frac{\partial v}{\partial x} = \frac{1}{2} y \frac{\partial p}{\partial x}$ $\qquad$ $\frac{\partial v}{\partial y} = \frac{1}{2}\left(p + y\frac{\partial p}{\partial y}\right)$

$\frac{\partial^2 v}{\partial x^2} = \frac{1}{2} y \frac{\partial^2 p}{\partial x^2}$ $\qquad$ $\frac{\partial^2 v}{\partial y^2} = \frac{1}{2}\left(2\frac{\partial p}{\partial y} + y\frac{\partial^2 p}{\partial y^2}\right)$

$$\Delta^2 v = \frac{\partial p}{\partial y}$$

$v = \frac{1}{2}\xi x$

$\frac{\partial v}{\partial x} = \frac{1}{2}\left(\xi + x\frac{\partial p}{\partial y}\right)$ $\Big\|$ $\frac{\partial v}{\partial y} = -\frac{1}{2} x \frac{\partial p}{\partial x}$

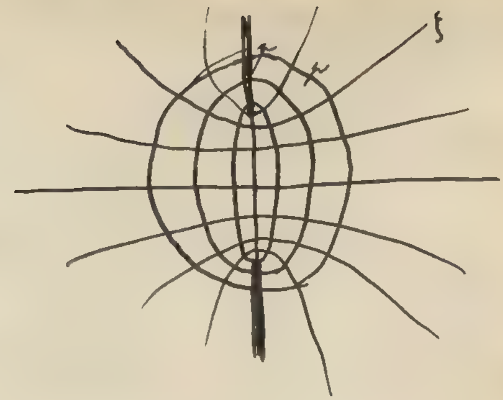
$\frac{\partial^2 v}{\partial x^2} = \frac{1}{2}\left(2\frac{\partial p}{\partial y} + x\frac{\partial^2 p}{\partial x \partial y}\right)$ $\Big\|$ $\frac{\partial^2 v}{\partial y^2} = -\frac{1}{2} x \frac{\partial^2 p}{\partial x \partial y}$

$$\Delta^2 v = \frac{\partial p}{\partial y}$$

$$x+iy = \sinh(\mu+i\xi)$$

$$x = \sinh\mu \cdot \cos\xi$$

$$y = \cosh\mu \cdot \sin\xi$$



$$\frac{x^2}{\sinh^2\mu} + \frac{y^2}{\cosh^2\mu} = 1 \qquad \frac{y^2}{\sin^2\xi} - \frac{x^2}{\cos^2\xi} = 1$$

$y=0$: $\xi = 0$

$x=0$: $y<1$: $\mu = 0$

$y>1$: $\xi = -\frac{\pi}{2}, +\frac{\pi}{2}$

$$1 = \cosh(\mu+i\xi)\cdot\left(\frac{\partial\mu}{\partial x} + i\frac{\partial\xi}{\partial x}\right) \qquad \Bigg| \quad i = \cosh(\mu+i\xi)\cdot\left(\frac{\partial\mu}{\partial y} + i\frac{\partial\xi}{\partial y}\right)$$

$$1 = \left[\frac{e^{\mu}+e^{-\mu}}{2}\cos\xi + \frac{e^{\mu}-e^{-\mu}}{2}\, i\sin\xi\right]\left[\frac{\partial\mu}{\partial x} + i\frac{\partial\xi}{\partial x}\right]$$

[illegible]

$$1 = \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi \cdot \frac{\partial\mu}{\partial x} - \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi \cdot \frac{\partial\mu}{\partial y} \qquad \Bigg\| \quad 0 = \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi\,\frac{\partial\mu}{\partial y} + \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi\,\frac{\partial\mu}{\partial x}$$

$$0 = \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi\,\frac{\partial\mu}{\partial x} + \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi \cdot \frac{\partial\mu}{\partial y}$$

$$\frac{e^{\mu}+e^{-\mu}}{2}\cos\xi = \left[\left(\frac{e^{\mu}+e^{-\mu}}{2}\cos\xi\right)^2 + \left(\frac{e^{\mu}-e^{-\mu}}{2}\sin\xi\right)^2\right]\frac{\partial\mu}{\partial x}$$

$$\left[\cosh^2\mu\,\cos^2\xi + \sinh^2\mu - \sinh^2\mu\,\cos^2\xi\right] = \left[\cancel{\cos^2\xi + \sinh^2\mu}\right]$$

$$\frac{\partial\mu}{\partial x} = \frac{\cosh\mu\,\cos\xi}{\cos^2\xi + \sinh^2\mu}$$

[illegible]

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial \xi}\underbrace{\frac{\partial \xi}{\partial x}}_{=\frac{\partial p}{\partial y}} \\ \frac{\partial u}{\partial y} &= \frac{\partial u}{\partial p}\frac{\partial p}{\partial y} + \frac{\partial u}{\partial \xi}\underbrace{\frac{\partial \xi}{\partial y}}_{=-\frac{\partial p}{\partial x}} \end{aligned}\right\} \quad \frac{\partial p}{\partial x}\left(\frac{\partial u}{\partial p} - \frac{\partial v}{\partial \xi}\right) + \frac{\partial p}{\partial y}\left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial p}\right) = 0$$

$$\frac{\partial v}{\partial x} = \xi + \frac{\partial u}{\partial y} = \xi + \frac{\partial u}{\partial p}\frac{\partial p}{\partial y} - \frac{\partial u}{\partial \xi}\frac{\partial p}{\partial x} = \frac{\partial v}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial v}{\partial \xi}\frac{\partial p}{\partial y} \quad \Bigg| \quad \frac{\partial p}{\partial x} \quad \Bigg| \quad \frac{\partial p}{\partial y}$$

$$\frac{\partial v}{\partial y} = -\left(\frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial p}{\partial y}\right) = \frac{\partial v}{\partial p}\frac{\partial p}{\partial y} - \frac{\partial v}{\partial \xi}\frac{\partial p}{\partial x} \quad \Bigg| \quad \frac{\partial p}{\partial y} \quad \Bigg| \quad -\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial p} = \frac{1}{4}\left[\cosh p \sin\xi + p \sinh p \sin\xi - \cosh p\, \xi \cos\xi\right] \sinh p \sin\xi \quad \Big|$$

$$\frac{\partial u}{\partial \xi} = \frac{1}{4}\left[p \cosh p \cos\xi - \sinh p \cos\xi + \sinh p\, \xi \sin\xi\right] \cosh p \cos\xi \quad \Big| \; -$$

$$\frac{\partial v}{\partial y} = \frac{1}{4}\Big[\cosh p \sinh p + p\underbrace{(\sin^2 h p \sin^2\xi - \cos^2 h p \cos^2\xi)}_{-\cos^2 h p + \cos^2 h p \sin^2} - \xi \sinh p \cosh p \sin\xi\cos\xi\Big]$$

$$\xi\frac{\partial p}{\partial x} - \frac{\partial u}{\partial \xi}\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right] = \frac{\partial v}{\partial p}\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right]$$

$$\xi\frac{\partial p}{\partial y} + \frac{\partial u}{\partial p}\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right] = \frac{\partial v}{\partial \xi}\left[\left(\frac{\partial p}{\partial x}\right)^2 + \ldots\right]$$

$$\frac{\partial v}{\partial p} = \xi\,\frac{\frac{\partial p}{\partial x}}{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2} - \frac{\partial u}{\partial \xi} = +\xi\,(\Delta)u - \frac{\partial u}{\partial \xi}$$

$$\frac{\partial v}{\partial \xi} = \xi\,\frac{\frac{\partial p}{\partial y}}{(\quad)} + \frac{\partial u}{\partial p} = \xi\,(\Delta)^2 v + \frac{\partial u}{\partial p}$$

$$u = \frac{1}{2}\left[\,p \cosh p \sin\xi - \sinh p \cdot \xi \cos\xi\,\right]$$

$$u = \frac{1}{2}\left[\,p x - \xi y\,\right] \quad + ax + by +$$

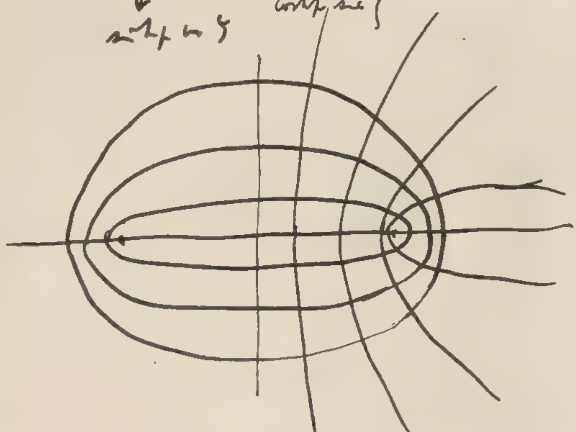
$$v = +\frac{1}{2}\left[\,p y + \xi x\,\right]$$

$\sinh p \cos\xi$ $\qquad$ $\cosh p \sin\xi$

$x = 0 \qquad \xi = 0 \qquad u = 0 \qquad v = \frac{-1}{4} p y$

$y = 0 \quad x > c : \xi = \frac{\pi}{2} \qquad u = \frac{1}{4} p x \;\Big|\; v = -\frac{\pi x}{8}$

$x < c : p = 0 \qquad u = 0 \qquad v = -\frac{1}{4}\xi x$



$$\frac{\partial p}{\partial x} = -\frac{\partial \xi}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\partial \xi}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{4}\left[\,p + x\frac{\partial p}{\partial x} - y\frac{\partial \xi}{\partial y}\,\right]$$

$$\frac{\partial v}{\partial y} = +\frac{1}{4}\left[\,p + y\frac{\partial p}{\partial y} + x\frac{\partial \xi}{\partial x}\,\right] \qquad \Bigg\| \; z = \frac{\pi}{2}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = +\frac{1}{4}\left[\,\cancel{p} + y\frac{\partial p}{\partial x} + \cancel{\xi} + x\frac{\partial \xi}{\partial y} \pm x\frac{\partial p}{\partial y} + \cancel{\xi} + y\frac{\partial \xi}{\partial x}\,\right] \quad -\frac{\xi}{2}$$

$$\frac{\partial v}{\partial p} = +\,\xi \sinh p \sin\xi - \frac{1}{2}\left[\,p\cosh p \cos\xi - \sinh p \cos\xi + \sinh p \cdot \xi \sin\xi\,\right]$$

$$= \qquad -\frac{1}{2}\left[\,p\cosh p \cos\xi - \sinh p \cos\xi - \xi \sinh p \sin\xi\,\right]$$

1.1.1

$$\cancel{x = \frac{e^{\xi}-e^{-\xi}}{2}\sin\mu}$$

$$\cancel{y = \frac{e^{\xi}+e^{-\xi}}{2}\cos\mu}$$

$\cancel{\eta = 0 \quad x=0}$ $\qquad \cos i + \sin i = e^{i}$

$\cos - i\sin = e^{-i}$

$$\frac{e^{i}-e^{-i}}{2i}$$

$\cancel{y=0 \quad \mu = \pm\frac{\pi}{2}}$

By machine:

$$x+iy = c\left[\frac{e^{\mu}+e^{-\mu}}{2}\,\frac{e^{i\xi}-e^{-i\xi}}{2i} + i\,\frac{e^{\mu}-e^{-\mu}}{2}\,\frac{e^{i\xi}+e^{-i\xi}}{2}\right]$$

$$= \frac{c}{4i}\left[\cancel{e^{\mu+i\xi}} + \cancel{e^{-\mu+i\xi}} - e^{\mu-i\xi} - \cancel{e^{-\mu-i\xi}}\right.$$

$$\left. - \cancel{e^{\mu+i\xi}} + e^{-\mu+i\xi} - e^{\mu-i\xi} + \cancel{e^{-\mu-i\xi}}\right]$$

$$= \frac{c}{2i}\left[-e^{+\mu-i\xi} + e^{-(\mu-i\xi)}\right]$$

$$x = c\cosh\mu\sin\xi$$

$$y = -c\sinh\mu\cos\xi$$

$$\boxed{\frac{x^2}{\cosh^2\mu} + \frac{y^2}{\sinh^2\mu} = c^2}$$

$$\boxed{\frac{x^2}{\sin^2\xi} - \frac{y^2}{\cos^2\xi} = c^2}$$

$\psi = \frac{\pi}{2} - \xi \ldots$

$$\left.\begin{array}{ll} x=0 & \xi=0 \\ y=0 & \xi=\frac{\pi}{2} \end{array}\right\}$$

$$\frac{\partial\mu}{\partial x} = \frac{\sinh\mu\sin\xi}{\sinh^2\mu+\cos^2\xi} \qquad \frac{\partial\mu}{\partial y} = -\frac{\cosh\mu\cos\xi}{\sinh^2\mu+\cos^2\xi}$$

$$\frac{\partial^2 u}{\partial\mu^2} + \frac{\partial^2 u}{\partial\xi^2} \qquad \frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\xi^2}$$

$$8\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{1}{2}\left[e^{\frac{\alpha+\beta}{2}} - e^{-\frac{\alpha+\beta}{2}}\right]\left[\frac{e^{\frac{\alpha-\beta}{2}} - e^{-\frac{\alpha-\beta}{2}}}{i}\right] = \left(e^{\alpha} - e^{-\beta} - e^{\beta} + e^{-\alpha}\right) = \frac{\cosh\alpha - \cosh\beta}{i}$$

$$\frac{\partial}{\partial\alpha}\left[\beta\sinh\alpha - \alpha\sinh\beta\right] = \beta\cosh\alpha - \sinh\beta$$

$$= \cosh\alpha - \cosh\beta$$

$$\frac{\partial}{\partial\beta}\left[\quad\right]$$

$$v = (\mu-i\xi)\frac{(e^{\mu+i\xi} - e^{-\mu-i\xi})}{i} - (\mu+i\xi)\frac{(e^{\mu-i\xi} - e^{-\mu+i\xi})}{i} = e^{\mu}\left[(\mu-i\xi)(\cos\xi+i\sin\xi) - (\mu+i\xi)(\cos\xi - i\sin\xi)\right]$$

$$+ e^{-\mu}\left[-(\mu-i\xi)(\cos\xi - i\sin\xi) + (\mu+i\xi)(\cos\xi + i\sin\xi)\right]$$

$$= \cancel{2}\,[\mu\sin\xi - \xi\cos\xi]\,e^{\mu} + (\mu\sin\xi + \xi\cos\xi)\,e^{-\mu} = \mu\sin\xi\cosh\mu - \xi\cos\xi\sinh\mu$$

$$\frac{\partial^2 v}{\partial p^2} + \frac{\partial^2 v}{\partial \zeta^2} = -\left(\frac{e^p + e^{-p}}{2}\right) \sin\zeta \quad \cancel{+ \alpha x + \beta y}$$

$$\cancel{\quad} \; p \sin\zeta \left(\frac{e^p - e^{-p}}{2}\right) \mp \zeta\cos\zeta \, \frac{e^p + e^{-p}}{2} \qquad \Big| \quad p\cos\zeta \, \frac{e^p - e^{-p}}{2} \pm \cos\zeta \, \frac{e^p + e^{-p}}{2} \mp \zeta \sin\zeta \, \frac{\cdot}{\cdot}$$

$$\sin\zeta \, \frac{e^p - e^{-p}}{2} + p \sin\zeta \, \frac{e^p + e^{-p}}{2} \mp \zeta\cos\zeta \, \frac{e^p - e^{-p}}{2}$$

$$\sin\zeta \, \frac{e^p + e^{-p}}{2} + \sin\zeta \, \frac{e^p + e^{-p}}{2} + p \sin\zeta \, \cancel{\frac{e^p - e^{-p}}{2}} \; \Big\| \; \pm \zeta \cos\zeta \, \cancel{\frac{e^p + e^{-p}}{2}}$$

$$\pm \sin\zeta \, \frac{e^p + e^{-p}}{2} \; \pm \sin\zeta \, \frac{e^p + e^{-p}}{2} - p \cancel{\sin\zeta \, \frac{e^p - e^{-p}}{2}} \; \pm \cancel{\zeta\cos\zeta \, \frac{e^p + e^{-p}}{2}}$$

$$v = \pm \frac{1}{4}\left[p \underbrace{\sin\zeta \sinh p}_{y} - \zeta \underbrace{\cos\zeta \cosh p}_{x}\right] +$$

$$u = \frac{1}{4}\left[p x + y \zeta\right] + Ax + By$$

$$v = \frac{1}{4}\left[p y - x\zeta\right]$$

$y = 0 : x < c : \; p = 0 \quad \| \; u = 0 \qquad v = -\frac{x\zeta}{4}$

$x > c : \; \zeta = 0, \pi \; \| \; u = \frac{px}{4} \; ; \; v = 0 \; ; \; -\frac{x\pi}{4}$

$x = 0 : \qquad \zeta = \frac{\pi}{2} \quad \| \; u = \frac{y\pi}{8} \quad ; v = \frac{py}{4}$

$$p\left(e^p + e^{-p}\right) \sin\zeta \mp \zeta \cos\zeta \left(e^p - e^{-p}\right)$$

$$\left(e^p + e^{-p}\right)\sin\zeta + p\left(e^p - e^{-p}\right)\sin\zeta \pm \zeta\cos\zeta\left(e^p + e^{-p}\right) \; \Big| \; p\left(e^p + e^{-p}\right)\cos\zeta \pm \cos\zeta\left(e^p - e^{-p}\right) \mp \zeta\sin\zeta\left(e^p - e^{-p}\right).$$

$$2\left(e^p - e^{-p}\right)\sin\zeta + \cancel{p\left(e^p + e^{-p}\right)\sin\zeta} \cancel{\pm \zeta\cos\zeta\left(e^p - e^{-p}\right)}$$

$$\mp 2\left(e^p - e^{-p}\right)\sin\zeta - \cancel{p\left(e^p + e^{-p}\right)\sin\zeta} \cancel{\mp \zeta\cos\zeta\left(e^p - e^{-p}\right)}$$

Sth

$$= \frac{(\mu+i\xi)}{16}\left[e^{\mu}(\cos\xi - i\sin\xi) + e^{-\mu}(\cos\xi + i\sin\xi)\right] + (\mu - i\xi)\left[e^{\mu}(\cos\xi + i\sin\xi) + e^{-\mu}(\cos\xi - i\sin\xi)\right]$$

$$= \frac{e^{\mu}}{16}\left[2(\mu\cos\xi + \xi\sin\xi) - i\,\cancel{(\mu\sin\xi - \xi\cos\xi)}\right]$$

$$+ e^{-\mu}\left[2(\mu\cos\xi - \xi\sin\xi)\right]$$

$$u = \frac{1}{4}\left[\cos\xi\cdot\mu\cdot\cancel{\cosh}\mu + \sin\xi\cdot\xi\cdot\cancel{\sinh}\mu\right]$$

$$-\frac{1}{4}\left[\mu x \;\cancel{\quad}\; + y\,\xi \;\cancel{\quad}\;\right]$$

Da $y=0$: also $\mu = 0$ $\quad x<c$

also $\xi = 0, \pi, \boxed{2\pi \ldots\ldots}$ $\quad x>c$

$$u = 4\mu x \,\mathrm{tgh}\,\mu$$

Da $x=0$: $\xi = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$$u = 4y\frac{\pi}{2}\cdot\mathrm{ctgh}\,\mu$$

$$\frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\xi^2} = \frac{\frac{\partial\mu}{\partial\xi}}{(\,\,)^2} = -\cosh\mu\sin\xi$$

$$8\,\frac{\partial^2 v}{\partial\alpha\partial\beta} = -\frac{1}{2}\left(e^{\frac{\alpha+\beta}{2}} + e^{-\frac{\alpha+\beta}{2}}\right)\left(e^{\frac{\alpha-\beta}{2}} - e^{-\frac{\alpha-\beta}{2}}\right)$$

$$= -\frac{1}{2}\left(e^{\alpha} + e^{-\beta} - e^{\beta} - e^{-\alpha}\right) = -\sinh\alpha + \sinh\beta$$

$$\frac{\partial}{\partial\alpha}\left(-\beta\cosh\alpha + \alpha\cosh\beta\right) = -\beta\sinh\alpha + \cosh\beta$$

$$\frac{\partial^2}{\partial\alpha\partial\beta} \qquad = -\sinh\alpha + \sinh\beta$$

$$v = -\frac{1}{8}\left[\beta\cosh\alpha - \alpha\cosh\beta\right] = -\frac{1}{8}\left[(\mu - i\xi)\left(e^{\mu+i\xi} + e^{-\mu-i\xi}\right) - (\mu+i\xi)\left(e^{\mu-i\xi} + e^{-\mu+i\xi}\right)\right]$$

$$= e^{\mu}\left[(\mu-i\xi)(\cos\xi + i\sin\xi) - (\mu+i\xi)(\cos\xi - i\sin\xi)\right] + e^{-\mu}\left[(\mu-i\xi)(\cos\xi - i\sin\xi) - (\mu+i\xi)(\cos\xi + i\sin\xi)\right]$$

$$= i\,e^{\mu}\,2(\mu\sin\xi - \xi\cos\xi) \qquad - 2e^{-\mu}\,i\,(\mu\sin\xi + \xi\cos\xi)$$

$$\frac{\partial \rho}{\partial x} = \frac{\sinh\rho \cos\zeta}{\sinh^2\rho + \sin^2\zeta} \qquad \frac{\partial \rho}{\partial y} = -\frac{\cosh\rho \sin\zeta}{\sinh^2\rho + \sin^2\zeta}$$

$$\frac{\frac{\partial \rho}{\partial x}}{\left(\frac{\partial \rho}{\partial x}\right)^2 + \left(\frac{\partial \rho}{\partial y}\right)^2} = \frac{\sinh\rho \cos\zeta \, . (\sinh^2\rho + \sin^2\zeta)}{\underbrace{\sinh^2\rho \cos^2\zeta + \cosh^2\rho \sin^2\zeta}_{= \sinh^2\rho + \sin^2\zeta}} = \sinh\rho \cos\zeta$$

$$\sinh\rho \cos\zeta = \frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \zeta^2} = \frac{e^{\rho} - e^{-\rho}}{2} \cos\zeta$$

$$\rho + i\zeta = \alpha \qquad \rho = \frac{\alpha+\beta}{2} \qquad \frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \zeta^2} = 4 \frac{\partial^2 u}{\partial \alpha \partial \beta}$$

$$\rho - i\zeta = \beta \qquad \zeta = \frac{\alpha - \beta}{2i}$$

$$8 \frac{\partial^2 u}{\partial \alpha \partial \beta} = \left[e^{\frac{\alpha+\beta}{2}} - e^{-\frac{\alpha+\beta}{2}} \right] \cos \frac{\alpha - \beta}{2i} = \frac{1}{2} \left(e^{\frac{\alpha+\beta}{2}} - e^{-\frac{\alpha+\beta}{2}} \right) \left(e^{\frac{\alpha-\beta}{2}} + e^{-\frac{\alpha-\beta}{2}} \right)$$

$$= \frac{1}{2} \left(e^{\alpha} - e^{-\beta} + e^{\beta} - e^{-\alpha} \right) = \sinh\alpha + \sinh\beta$$

$$\frac{\partial}{\partial \alpha} \left[\alpha \cosh\beta + \beta \cosh\alpha \right] = \cosh\beta + \beta \sinh\alpha$$

$$\frac{\partial}{\partial \alpha \partial \beta} \qquad = \sinh\beta + \sinh\alpha$$

$$u = \frac{1}{8} \left[\alpha \cosh\beta + \beta \cosh\alpha \right]$$

$$= \frac{1}{16} \left[(\rho + i\zeta) \left(e^{\rho - i\zeta} + e^{-\rho + i\zeta} \right) + (\rho - i\zeta) \left(e^{\rho + i\zeta} + e^{-\rho - i\zeta} \right) \right]$$

$$\frac{\sinh\mu\cos\xi}{\cosh^2\mu-\cos^2\xi} = \sqrt{\frac{(\cosh^2\mu-1)\cos^2\xi}{2\left[\left(\frac{1+x^2+y^2}{2}\right)^2-x^2\right]}} = \sqrt{\frac{x^2-\frac{1+y^2+x^2}{2}+\sqrt{\left(\frac{1+x^2+y^2}{2}\right)^2-x^2}}{2\left[\frac{1+\ldots}{2}-x^2\right]}}$$

$\cancel{x+i\xi = e}$

$$x+iy = e^{\mu+i\xi} + \mu + i\xi$$

$\cancel{x = e^{\mu}\cos\xi + \mu}$ $\cancel{\sqrt{x^2+y^2} = e^{\mu}}$ $\cancel{\operatorname{tg}\xi = \frac{y}{x}}$

$\cancel{y = e^{\mu}\sin\xi + \xi}$ $\cancel{\mu = \lg r}$ $\cancel{\xi = \operatorname{arctg}\frac{y}{x} = 0}$

$$1 = \left[e^{\mu+i\xi} + 1\right]\left(\frac{\partial\mu}{\partial x} + i\frac{\partial\xi}{\partial x}\right)$$

$$1 = (e^{\mu}\cos\xi+1)\frac{\partial\mu}{\partial x} - e^{\mu}\sin\xi\frac{\partial\xi}{\partial x} \qquad \begin{vmatrix} e^{\mu}\cos\xi+1 & -e^{\mu}\sin\xi \\ e^{\mu}\sin\xi & e^{\mu}\cos\xi+1 \end{vmatrix}$$

$$0 = e^{\mu}\sin\xi\frac{\partial\mu}{\partial x} + (e^{\mu}\cos\xi+1)\frac{\partial\xi}{\partial x}$$

$$(e^{\mu}\cos\xi+1) = \left[(e^{\mu}\cos\xi+1)^2 + e^{2\mu}\sin^2\xi\right]\frac{\partial\mu}{\partial x}$$

$$\frac{\partial\mu}{\partial x} = \frac{1+e^{\mu}\cos\xi}{1+e^{2\mu}+2e^{\mu}\cos\xi}$$

$$\frac{\partial\mu}{\partial y} = \frac{-e^{\mu}\sin\xi}{1+e^{2\mu}+2e^{\mu}\cos\xi}$$

$\mu e^{\mu}\cos\xi$

$e^{\mu}\cos\xi + \mu e^{\mu}\cos\xi$

$2e^{\mu}\cos\xi + \cancel{\mu e^{\mu}\cos\xi}$

$$\frac{-\frac{\partial\mu}{\partial x}}{\left(\frac{\partial\mu}{\partial x}\right)^2+\left(\frac{\partial\mu}{\partial y}\right)^2} = -\frac{(1+e^{\mu}\cos\xi)\left(1+e^{2\mu}+2e^{\mu}\cos\xi\right)}{\cancel{1+e^{2\mu}+2e^{\mu}\cos\xi}+}$$

$u = \mu e^{\mu}\cos\xi$

$u = -\left(\frac{\mu^2+\mu e^{\mu}\cos\xi}{2}\right) = -\frac{\mu x}{2}$

$+\xi$

$$= -(1+e^{\mu}\cos\xi) = \frac{\partial^2 u}{\partial\mu^2} + \frac{\partial^2 u}{\partial\xi^2}$$

$$\frac{\partial^2 \mu}{\partial \mu^2} + \frac{\partial^2 u}{\partial \xi^2} = \frac{\sin h\mu \cos \xi \,(\sin^4 \xi - \sin^4 h\mu)}{\sin^2 h\mu \cos^2 \xi \left[-\sin^2 h\mu + \sin^2 \xi\right]^2 + \cos^2 h\mu \sin^2 \xi \left[\sin^2 h\mu + \sin^2 \xi\right]^2}$$

$$= \frac{\sin h\mu \cos \xi \left[\sin^4 \xi - \sin^4 h\mu\right]}{\sin^4 \xi \left[\sin^2 \xi + \sin^2 h\mu \,(1 + 2\cos^2 h\mu)\right] + \sin^4 h\mu \left[\sin^2 h\mu + \sin^2 \xi \,(1 - 2\cos^2 \xi)\right]}$$

$$\frac{x^2}{\cos^2 h\mu} + \frac{y^2}{\cos^2 h\mu - 1} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - 1} = 1$$

$a^2 =$

$$x^2 (\cos^2 h\mu - 1) + y^2 (\cos^2 h\mu) = \cos^4 h\mu - \cos^2 h\mu$$

$$\cos^4 h\mu - \cos^2 h\mu \left[1 + x^2 + y^2\right] = -x^2$$

$$\cos^2 h\mu = \frac{1 + x^2 + y^2}{2} \pm \sqrt{\left(\frac{1 + x^2 + y^2}{2}\right)^2 - x^2}$$

$1 \pm a^2$

$a^2 - 1$

$$\frac{x^2}{\cos^2 \xi} - \frac{y^2}{1 - \cos^2 \xi} = 1$$

$$-x^2 (1 - \cos^2 \xi) + y^2 \cos^2 \xi = -\cos^2 \xi + \cos^4 \xi$$

$$\cos^4 \xi - \cos^2 \xi \left[1 + y^2 + x^2\right] = -x^2$$

$$\cos^2 \xi = \frac{1 + y^2 + x^2}{2} \pm \sqrt{}$$

$\frac{1 + a^2}{2} - \frac{1 - a^2}{2}$

$$\cos^2 h\mu - \cos^2 \xi = 2\sqrt{}$$

$$x+iy = \cosh(\mu + i\xi) = \frac{e^{\mu - i\xi} + e^{-\mu + i\xi}}{2}$$

$$x = \cosh\mu \, \cos\xi = \frac{e^{\mu} + e^{-\mu}}{2}\cos\xi$$

$$y = \sinh\mu \, \sin\xi = \frac{e^{\mu} - e^{-\mu}}{2}\sin\xi$$

$$\frac{x^2}{\cosh^2\mu} + \frac{y^2}{\sinh^2\mu} = 1$$

$$\sinh^2\mu - \cosh^2\mu = -1$$

$$\cosh^2\mu = 1 + \sinh^2\mu$$

$$\sinh^2\mu = y^2 + x^2\,\mathrm{tgh}^2\mu$$

$$2\sinh\cosh\mu\,\frac{\partial\mu}{\partial x} = 2x\,\mathrm{tgh}^2\mu + 2x^2\frac{\frac{\partial\mu}{\partial x}}{\cosh^2\mu}\,\mathrm{tgh}\,\mu$$

$$\frac{\partial\mu}{\partial x}\left[\sinh 2\mu - 2\,\mathrm{tgh}\,\mu\cos^2\xi\right] = 2\cosh\mu\cos\xi\,\mathrm{tgh}^2\mu$$

$$\frac{\partial\mu}{\partial x} = \frac{\sinh\mu\,\mathrm{tgh}\,\mu\,\cos\xi}{\sinh\mu\cosh\mu - \cos^2\xi\,\mathrm{tgh}\,\mu} = \frac{\sinh\mu\,\cos\xi}{\cosh^2\mu - \cos^2\xi}$$

$$x^2 + y^2\,\mathrm{ctg}^2h\mu = \cosh^2\mu$$

$$2y\,\mathrm{ctg}^2h\mu - 2y^2\frac{\mathrm{ctgh}\,\mu}{\sinh^2\mu}\frac{\partial\mu}{\partial y} = 2\cosh\mu\sinh\mu\frac{\partial\mu}{\partial y}$$

$$\frac{\partial\mu}{\partial y} = \frac{\sin\xi\,\mathrm{ctg}^2h\mu\,\sinh\mu}{-\sinh\mu\cosh\mu + \sin^2\xi\,\mathrm{ctgh}\,\mu} = \frac{-\cosh\mu\sin\xi}{-\sinh^2\mu + \sin^2\xi}$$

$$\frac{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2}{\frac{\partial\mu}{\partial x}} = \frac{\sinh^2\mu\cos^2\xi\left[-\sinh^2\mu + \sin^2\xi\right]^2 + \cosh^2\mu\sin^2\xi\left[\cosh^2\mu - \cos^2\xi\right]^2}{\sinh\mu\cos\xi\cdot(-\sinh^2\mu + \sin^2\xi)^2(\cosh^2\mu - \cos^2\xi)}$$

$$= \cos^2\xi\sinh^6\mu + \sin^2\xi\cosh^6\mu - 2\sin^2\xi\cos^2\xi\left[\ldots\right] + \cos^2\xi\sin^2\xi\left[\sinh^2\mu\sin^2\xi + \cosh\mu\cos^2\xi\right]$$

$\varphi = \theta = \operatorname{arctg} \frac{y}{x}$ $\qquad x + iy = e^{\xi}(\cos\varphi + i\sin\varphi) = e^{\xi + i\varphi}$

$\xi = \lg r \qquad r = e^{\xi}$

$u = \frac{1}{4}\left(y \lg r^{2u} - x\theta\right) \quad = \frac{1}{4}\left[\xi e^{\xi}\sin\varphi - \varphi e^{\xi}\varphi\cos\varphi\right]$

$v = \frac{1}{4}\left(-x\lg r + y\left(\frac{\pi}{2} - \theta\right)\right)$

$\frac{\partial u}{\partial \varphi} = \frac{1}{4}\left[\xi e^{\xi}\cos\varphi - e^{\xi}\cos\varphi + e^{\xi}\varphi\sin\varphi\right]$

$\frac{\partial^2 u}{\partial \varphi^2} = \frac{1}{4}\left[-\xi e^{\xi}\sin\varphi + 2e^{\xi}\sin\varphi + \cancel{e^{\xi}\ldots} + e^{\xi}\varphi\cos\varphi\right]$

$\frac{\partial u}{\partial \xi} = \frac{1}{4}\left[e^{\xi}\sin\varphi + \xi e^{\xi}\sin\varphi - e^{\xi}\varphi\cos\varphi\right]$

$\frac{\partial^2 u}{\partial \xi^2} = \frac{1}{4}\left[\xi e^{\xi}\sin\varphi + 2e^{\xi}\sin\varphi - e^{\xi}\varphi\cos\varphi\right]$

$\Delta^2 u = e^{\xi}\sin\varphi = y$

$\frac{\partial\varphi}{\partial x} = -\frac{y}{x^2+y^2} \qquad \frac{\partial\varphi}{\partial y} = \frac{x}{x^2+y^2} \qquad \dfrac{\frac{\partial\varphi}{\partial x}}{\left(\frac{\partial\varphi}{\partial x}\right)^2 + (\ )^2} = \dfrac{-\frac{y}{x^2+y^2}}{\frac{y^2+x^2}{(x^2+y^2)^2}} = -y$

Zawsze można stawić $\mu = 1$

$\cos^2\xi\sin^6 h\varphi + \sin^2\xi\cos^6 h\varphi + \cos^2\xi\sin^2\xi\left[\sin^2\xi\,(\cos^2 h\varphi - 1) + \cos^2 h\varphi\sin^2\xi\right]$

$\cos^2 h\varphi - \sin^2\xi \qquad - 2\sin^2 h\varphi$

$- 2\cos^2 h\varphi$

$\cos^2\xi\sin^2\xi\left[-\xi\sin^2 h\varphi\,\cancel{+ \ldots\cos^2 h\varphi}\right.$

$\left.\cancel{+\sin^2 h\varphi} + \cos^2\xi\,(\underbrace{\cos^2 h\varphi - \sin^2 h\varphi}_{=1})\right] - \cos^2 h\varphi - 2\sin^2 h\varphi - \sin^2\xi$

Streamlines:

$$\frac{v}{u} = \frac{dy}{dx} \qquad u\,dy - v\,dx = 0$$

$$\frac{\partial\varphi}{\partial x}dy - \frac{\partial\varphi}{\partial y}dx - \underbrace{\left(\frac{\partial\psi}{\partial y}dy + \frac{\partial\psi}{\partial x}dx\right)}_{d\psi} = 0$$

$$x = f(\mu, \xi) \qquad y = f_1(\mu, \xi)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \mu}\frac{\partial \mu}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \mu\partial x}\frac{\partial \mu}{\partial x} + \frac{\partial u}{\partial \mu}\frac{\partial^2 \mu}{\partial x^2} + \frac{\partial^2 u}{\partial \xi\partial x}\frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2}$$

$$= \frac{\partial^2 u}{\partial \mu^2}\left(\frac{\partial \mu}{\partial x}\right)^2 + 2\frac{\partial^2 u}{\partial\xi\partial\mu}\left(\frac{\partial \mu}{\partial x}\right)\left(\frac{\partial \xi}{\partial x}\right) + \frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial u}{\partial \mu}\frac{\partial^2 \mu}{\partial x^2} + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2}$$

$$\Delta^2 u = \frac{\partial^2 u}{\partial \mu^2}\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right] + 2\frac{\partial^2 u}{\partial\mu\partial\xi}\left(\frac{\partial \mu}{\partial x}\frac{\partial \xi}{\partial x} + \frac{\partial \mu}{\partial y}\frac{\partial \xi}{\partial y}\right) + \frac{\partial^2 u}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right]$$

$$+ \frac{\partial u}{\partial \mu}\cancel{\left[\frac{\partial^2 \mu}{\partial x^2} + \frac{\partial^2 \mu}{\partial y^2}\right]} + \frac{\partial u}{\partial \xi}\cancel{\left[\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right]}$$

$$= \underbrace{\cancel{\frac{\partial^2 u}{\partial \mu^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right]\left[\mu^2 + 1\right]}} + 2\cancel{\frac{\partial^2 u}{\partial\mu\partial\xi}\left[-\mu\frac{\partial \xi}{\partial y}\frac{\partial \xi}{\partial x} + \mu\frac{\partial \xi}{\partial x}\frac{\partial \xi}{\partial y}\right]}$$

$$= \left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right]\left[\mu^2\frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial \xi^2}\right] = \left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right]\left[\frac{\partial^2 u}{\partial \mu^2} + \frac{1}{\mu^2}\frac{\partial^2 u}{\partial \xi^2}\right] = -\frac{\partial \xi}{\partial y}$$

$$\frac{\partial^2 u}{\partial(\mu\xi)^2} + \frac{\partial^2 u}{\partial \xi^2} = \frac{-\frac{\partial \xi}{\partial y}}{\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2}$$

$$\frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial(\mu\xi)^2} = \frac{+\frac{\partial \mu}{\partial x}}{\mu\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right]} = \frac{\mu}{\mu^2}$$

$$x^2 - y^2 = \mu$$
$$2xy = \xi$$

$$x^2 - \left(\frac{\xi}{2x}\right)^2 = \mu$$

$$\cancel{x^4} - \mu x^2 = \frac{\xi^2}{4}$$

$$x = \sqrt{\frac{\mu}{2} + \sqrt{\frac{\xi^2}{4} + \frac{\mu^2}{4}}}$$

$$y = \sqrt{-\frac{\mu}{2} + \sqrt{\frac{\xi^2}{4} + \frac{\mu^2}{4}}}$$

$$\cancel{\frac{\partial \mu}{\partial x} = 2x}; \quad \frac{\partial \mu}{\partial y} = -2y;$$

$$\frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial(\mu\xi)^2} = 4(x^2+y^2) = 2\sqrt{\mu^2+\xi^2}$$

$$u = -\frac{r}{2}\left[\theta\sin\theta + \frac{\cos\theta}{2}\right] + r\,\frac{\left(\frac{\pi}{2}-1\right)\cos\theta + \left(\frac{\pi}{2}+1\right)\sin\theta}{4\pi}\left(\frac{\pi}{2}+1\right)$$

$$v = \frac{r}{2}\left[\theta\cos\theta - \frac{\sin\theta}{2}\right] + r\,\frac{\left(\frac{\pi}{2}-1\right)\cos\theta + \left(\frac{\pi}{2}+1\right)\sin\theta}{4\pi}\left(\frac{\pi}{2}-1\right)$$

$p = p \log r$

$$u = r\cos\theta\left[\frac{\frac{\pi^2}{4}-1}{4\pi} - \frac{1}{4}\right] + r\sin\theta\left[\frac{\left(\frac{\pi}{2}+1\right)^2}{4\pi} - \frac{\theta}{2}\right]$$

$$v = r\cos\theta\left[\frac{\left(\frac{\pi}{2}-1\right)^2}{4\pi} + \frac{\theta}{2}\right] + r\sin\theta\left[\frac{\frac{\pi^2}{4}-1}{4\pi} - \frac{1}{4}\right]$$

$\theta = 0: \quad u = r\,\frac{\frac{\pi^2}{4}-1-\pi}{4\pi} \qquad v = r\,\frac{\left(\frac{\pi}{2}-1\right)^2}{4\pi}$

$\theta = \frac{\pi}{4} \qquad u = 0, \quad v = 0$

$\cancel{\theta = \frac{3\pi}{4} \quad u = -\frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}-1-\pi}{4\pi} + \frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}+\pi+1 - \frac{3\pi^2}{2}}{4\pi} = \frac{\sqrt{2}}{2}\ 2\pi+2 =}$

$\theta = -\frac{\pi}{4} \qquad u = +\frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}-1-\pi}{4\pi} - \frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}+\pi+1+\frac{\pi^2}{2}}{4\pi} = -\frac{\sqrt{2}}{4\pi}\left(\frac{\pi^2}{4}+\pi+1\right)$

$\qquad\qquad v = \frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}-\pi+1+\frac{\pi^2}{2}}{4\pi} - \frac{\sqrt{2}}{2}\left[\frac{\frac{\pi^2}{4}-1-\pi}{4\pi}\right]$

Trhliny v podloží [illegible] superponovat takové [illegible] síly tak ale

$\theta = -\frac{\pi}{4}$ [illegible]

$v = \frac{x}{2}\,\mathrm{arctg}\,\frac{y}{x}$ $\qquad = \frac{\pi}{2}\,\theta\cos\theta$

$u = -\frac{y}{2}\,\mathrm{arctg}\,\frac{y}{x} + \frac{\pi}{4}y$ $\qquad = \frac{\pi}{2}\left[\frac{\pi}{2}\sin\theta - \theta\sin\theta\right] = \frac{\pi}{2}\sin\theta\left[\frac{\pi}{2}-\theta\right]$

$x = u\log 2$

$\theta = 0$: $v=0$, $u=0$ $\qquad$ $\theta=\frac{\pi}{2}$: $v=0$, $u=0$

$$\frac{v}{u} = \frac{\theta}{\frac{\pi}{2}-\theta}\,\mathrm{ctg}\,\theta$$

$$v = \frac{\pi}{2}\left[\theta(\cos\theta+\sin\theta) - \frac{(\sin\theta-\cos\theta)}{2}\right]$$

$$u = \frac{\pi}{2}\left[-\theta(\cos\theta+\sin\theta) + \frac{(\cos\theta-\sin\theta)}{2}\right]$$

$\theta=0$: $\quad v=\frac{\pi}{4}$ $\quad u=-\frac{\pi}{4}$

$\theta=\frac{\pi}{2}$: $\quad v=\frac{\pi(\pi-1)}{4}$ $\quad u=-\frac{\pi(\pi-1)}{4}$

$\cancel{\left(\frac{\pi}{2}+1\right)^2 - \left(\frac{\pi}{2}-1\right)^2} = \frac{1}{2}$

$\cancel{y=0}$

$x=y \qquad \theta=\frac{\pi}{4}$:

$$u = \frac{1}{4}\left[y(\log y+\log 2) - y\frac{\pi}{4}\right] \qquad u=-v$$

$$v = \frac{1}{4}\left[-y(\log y+\log 2) + y\frac{\pi}{4}\right]$$

$\cancel{u = -\frac{\sqrt2}{8}\left(\frac{\pi}{2}-1\right)(x+y)\frac{\sqrt2}{2}}$

$\cancel{u = \frac{\sqrt2}{8}\left(\frac{\pi}{2}-1\right)}$

$$\mathrm{tg}\,\varphi = \frac{\frac{\pi}{2}-1}{\frac{\pi}{2}+1}$$

$$\cos\varphi = \frac{\frac{\pi}{2}+1}{\sqrt{\frac{\pi^2}{4}+1-\pi+\frac{\pi^2}{4}+1+\pi}} = \frac{\frac{\pi}{2}+1}{\sqrt2\sqrt{\frac{\pi^2}{4}+1}}$$

$$\sin\varphi = \frac{\frac{\pi}{2}-1}{\sqrt2\sqrt{\frac{\pi^2}{4}+1}}$$

$$u = \frac{x\sin\varphi+y\sin\varphi}{\cos\left(\frac{\pi}{4}-\varphi\right)\pi} = \frac{x\cos\varphi+y\sin\varphi}{(\cos\varphi+\sin\varphi)\frac{\sqrt2}{2}} = \frac{x\left(\frac{\pi}{2}+1\right)+y\left(\frac{\pi}{2}+1\right)}{\pi\,\frac{\sqrt2}{2}}$$

$$v = -\frac{x\left(\frac{\pi}{2}+1\right)+y\left(\frac{\pi}{2}+1\right)}{4\pi}\left(\frac{\pi}{2}-1\right)$$

$$u = \frac{x\left(\frac{\pi}{2}+1\right)+y\left(\frac{\pi}{2}+1\right)}{4\pi}\left(\frac{\pi}{2}+1\right)$$

$f = \lg z$

$p = \lg z$

$\xi = \theta = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$

$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{1}{r}\frac{\partial\psi}{\partial r}$

$\psi = \frac{r^2\theta}{4}$

$\psi = \frac{(x^2+y^2)}{4} \operatorname{arctg}\frac{y}{x}$

$v = \frac{\partial\psi}{\partial x} = \frac{x}{2}\operatorname{arctg}\frac{y}{x} - \frac{y}{x^2}\frac{x^2+y^2}{4}\frac{1}{1+\frac{y^2}{x^2}} = \frac{r}{2}\left[\theta\cos\theta - \frac{\sin\theta}{2}\right]$

$+u = -\frac{\partial\psi}{\partial y} = -\frac{y}{2}\operatorname{arctg}\frac{y}{x} \pm \frac{x^2+y^2}{4x}\frac{1}{1+\frac{y^2}{x^2}} = -\frac{r}{2}\left[\theta\sin\theta + \frac{\cos\theta}{2}\right]$

$\theta = 0 \qquad v = 0 \qquad +u = -\frac{r}{4}$

$\theta = \frac{\pi}{2}: \quad v = -\frac{r}{4} \qquad +u = -r\frac{\pi}{4}$

$\theta = \frac{\pi}{4}: \quad v = \frac{r}{4}\frac{\sqrt{2}}{2}\left(\frac{\pi}{2} - 1\right) \qquad u = -\frac{r}{4}\frac{\sqrt{2}}{2}\left(\frac{\pi}{2} + 1\right)$

$p = \theta$

$\xi = \lg z = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$

$\psi = \frac{1}{2}\int dx \int \lg(x^2+y^2)\, dx + \int dy \int$

$-\frac{\partial\xi}{\partial y} = \frac{y}{x^2+y^2} = \Delta^2 u$

$+\frac{\partial\xi}{\partial x} = \frac{-x}{x^2+y^2} = \Delta^2 v$

$u = -\frac{1}{4}\left\{y \lg\frac{(x^2+y^2)}{2} - x \operatorname{arctg}\frac{y}{x}\right\} +$

$v = \frac{1}{4}\left\{-x \lg\frac{x^2+y^2}{2} + y \operatorname{arctg}\frac{x}{y}\right\} - \frac{\pi}{8} y$

$x^2+y^2 = 1: \qquad \left.\begin{array}{l} u = -\frac{1}{4}\theta\cos\theta \\ v = -\frac{1}{4}(\theta)\sin\theta \end{array}\right\} V = -\frac{1}{4}\theta$

$$\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = \frac{A}{4}\left\{ \frac{6y}{x^2+y^2} - \cancel{4}2y\frac{\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{2}}} \right\} = \frac{Ay}{x^2+y^2} \quad !$$

$$u = \frac{\partial \Phi_1}{\partial x} - \frac{\partial \Phi_2}{\partial y} + \frac{A}{4}\left\{ \log \frac{x^2+y^2}{2} + \frac{y^2-x^2}{y^2+x^2} \right\} \quad \Bigg\| \quad \frac{A}{4}\left[\log r - \cos 2\theta\right]$$

$$v = \frac{\partial \phi_1}{\partial y} + \frac{\partial \phi_2}{\partial x} - \frac{A}{4}\left\{ \frac{2xy}{x^2+y^2} - \operatorname{arctg}\frac{y}{x} \right\} \quad \Bigg\| \quad -\frac{A}{4}\left[\sin 2\theta - \theta\right]$$

$$1 - \frac{2x^2}{y^2+x^2}$$

$$\frac{-y^2}{y^2+x^2} - 1$$

$\theta = 0$ $\quad u = \log r - 1$ $\quad \theta = \frac{\pi}{4}$ $\quad u = \log r - \cancel{0}$

$\quad v = 0$ $\quad\quad v = 1 - \frac{\pi}{4}$

$\theta = \frac{\pi}{2}$ $\quad u = \log r + 1$

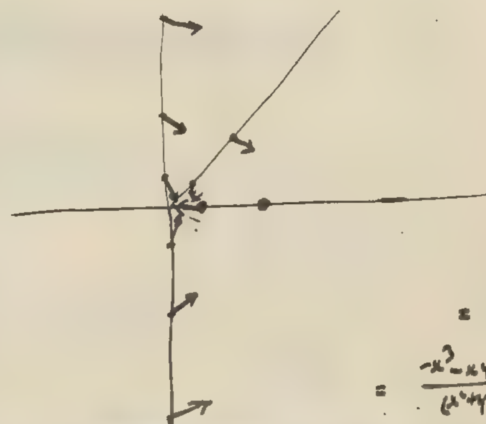
$\quad v = \frac{\pi}{2}$

$\cancel{\theta = \pi}$ $\quad \cancel{u = \log r - 1}$

$\quad \cancel{v = -\pi}$

$\theta = -\frac{\pi}{2}$ $\quad u = \log r + 1$

$\quad v = +\frac{\pi}{2}$



$$= x^3 - 3xy^2$$

$$= \frac{-x^3 - xy^2 - 2xy^2 + 2x^3}{(x^2+y^2)^2}$$

$$\frac{\partial v}{\partial y} = -\frac{2x}{x^2+y^2} + \frac{4xy^2}{(x^2+y^2)^2} + \quad \Bigg| \quad \frac{\partial u}{\partial x} = \frac{-x}{x^2+y^2} - \frac{2x(y^2-x^2)}{(x^2+y^2)^2}$$

$$= 4$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$$\frac{\partial v}{\partial x} = -\frac{2y}{x^2+y^2} + \frac{4x^2y}{(x^2+y^2)^2} \quad\quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = + \frac{4x^2y + 2y^3 - 2x^2y - 6x^2y - 6y^3}{(y^2+x^2)^2}$$

$$\frac{\partial u}{\partial y} = \frac{y}{x^2+y^2} + \frac{2y}{y^2+x^2} - \frac{2y(y^2-x^2)}{(y^2+x^2)^2} \quad\quad = \cancel{-4y^3 - 4x^2y + 4x^2y + 2y^3 - 2x^2y}$$

$$3y^3 + 3yx^2 - 2y^3 + 2yx^2 = y^3 + yx^2 = \frac{y}{x^2+y^2}$$

$$= \frac{1}{4}\,\frac{4y}{y^2+x^2}$$

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} =$$

$$+\frac{A}{2i}\left[\frac{1}{u}-\frac{1}{v}\right] = 4\frac{\partial^2\psi}{\partial u\partial v}$$

$$\psi = u\log v - v\log u$$

$$\frac{\partial\psi}{\partial u} = \log v - \frac{v}{u} \qquad \frac{\partial\psi}{\partial v} = \frac{u}{v} - \log u$$

$$\frac{\partial^2\psi}{\partial u\partial v} = \frac{1}{v} - \frac{1}{u} \qquad \frac{\partial^2\psi}{\partial u\partial v} = \frac{1}{v} - \frac{1}{u}$$

$$-\frac{A}{8i}\left[\frac{1}{v}-\frac{1}{u}\right] = \frac{\partial^2\psi}{\partial u\partial v}$$

$$\psi = -\frac{A}{8i}(u\log v - v\log u) + f_1(u) - f_2(v)$$

$$= -\frac{A}{8}\left[\frac{(x+iy)\log(x-iy) - (x-iy)\log(x+iy)}{i}\right]$$

$$x+iy = r e^{i\theta} \qquad \log = \log r + i\theta$$

$$x-iy = r e^{-i\theta} \qquad \log = \log r - i\theta$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$\psi = -\frac{A}{8}\left\{\frac{[\log r - i\theta]\, r e^{i\theta} - [\log r + i\theta]\, r e^{-i\theta}}{i}\right\}$$

$$= -\frac{A}{8}\left\{r\log r\,\frac{e^{i\theta}-e^{-i\theta}}{i} - \theta\, r(e^{i\theta}+e^{-i\theta})\right\}$$

$$= -\frac{A}{4}\left\{r\log r\,\sin\theta - r\theta\cos\theta\right\}$$

$$\psi = -\frac{A}{4}\left\{+y\,\frac{\log(x^2+y^2)}{2} - x\,\mathrm{arctg}\,\frac{y}{x}\right\}$$

Prüfung: $\frac{\partial\psi}{\partial x} = -\frac{A}{4}\left\{\cancel{\frac{2xy}{x^2+y^2}} - \mathrm{arctg}\,\frac{y}{x} + \cancel{\frac{\frac{y}{x}}{1+\frac{y^2}{x^2}}}\right\}$ $\quad \frac{2xy}{x^2+y^2} - \mathrm{arctg}\,\frac{y}{x} + \frac{\frac{y}{x}}{1+y^2}$

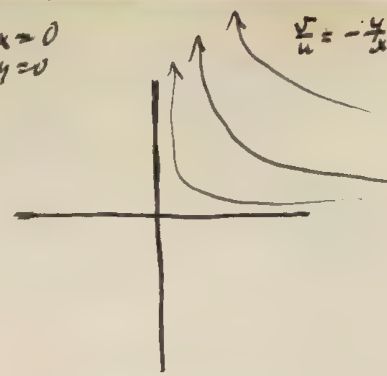
$$\frac{\partial^2\psi}{\partial x^2} = -\frac{A}{4}\left\{\cancel{\frac{2y}{x^2+y^2}} \cancel{\frac{4x^2y}{(x^2+y^2)^2}} + \cancel{\frac{\frac{y}{x^2}}{1+\frac{y^2}{x^2}}}\right\}$$

$$\frac{\partial\psi}{\partial y} = -\frac{A}{4}\left\{+\log\frac{x^2+y^2}{2} + \frac{y^2}{x^2+y^2} - \frac{x^2}{x^2+y^2}\right\}$$

$$\frac{\partial^2\psi}{\partial y^2} = -\frac{A}{4}\left\{+\frac{y}{x^2+y^2} \cdots + \frac{2x^2y}{(x^2+y^2)^2}\right\}$$

u i v znikają równocześnie dla $x=0$, $y=0$

$$\frac{v}{u} = -\frac{y}{x}\,\frac{y^2+3x^2}{3y^2+x^2}$$



$$u = -\frac{A}{6}\left(3xy^2+x^3\right)$$

$$v = \frac{A}{6}\left(y^3+3x^2y\right)$$

$$p = p_0 + \mu A\left(x^2-y^2\right)$$

$$p = 2Axy$$

$$\zeta = A\left(x^2-y^2\right) = \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} \qquad \psi = \frac{A}{12}\left(x^4-y^4\right) + \phi$$

$$u = \frac{Ay^3}{3} +$$

$$v = \frac{Ax^3}{3} -$$

$$p^2+\zeta^2 = \frac{1}{(x^2+y^2)}$$

$$x^2+y^2 = \frac{1}{\sqrt{p^2+\zeta^2}}$$

$$\int \log(z^2)\,dz$$

$$f = \frac{A}{z} = \frac{A}{r}\left(\cos\theta - i\sin\theta\right)$$

$$\frac{\partial\psi}{\partial y} = -\frac{2yx}{(x^2+y^2)^2} = \frac{2p\zeta}{\cancel{\;}}$$

$$\frac{\partial\psi}{\partial x} = \frac{1}{x^2+y^2} - \frac{2x^2}{(\;)^2} = \frac{y^2-x^2}{(x^2+y^2)^2} = (\zeta^2-p^2)$$

$$2x\,2y\,z^2 \qquad 2y\,3z^2$$

$$p = A\frac{\cos\theta}{r} = \frac{Ax}{x^2+y^2}$$

$$\zeta = -A\frac{\sin\theta}{r} = -\frac{Ay}{x^2+y^2} = \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} = +\frac{A}{2i}\left[\frac{+1}{x+iy} - \frac{-1}{x-iy}\right]$$

$$x+iy = u \qquad x = \frac{u+v}{2}$$

$$x-iy = v \qquad y = \frac{u-v}{2}$$

$$\frac{\partial\psi}{\partial x} = \left(\frac{\partial\psi}{\partial u}+\frac{\partial\psi}{\partial v}\right) \qquad \frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi}{\partial u^2} + 2\frac{\partial^2\psi}{\partial u\partial v} + \frac{\partial^2\psi}{\partial v^2}$$

$$\frac{\partial\psi}{\partial y} = i\left(\frac{\partial\psi}{\partial u}-\frac{\partial\psi}{\partial v}\right) \qquad \frac{\partial^2\psi}{\partial y^2} = -\left(\frac{\partial^2\psi}{\partial u^2} - 2\frac{\partial^2\psi}{\partial u\partial v} + \frac{\partial^2\psi}{\partial v^2}\right)$$

$$d\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy = \frac{\partial\psi}{\partial u}du + \frac{\partial\psi}{\partial v}dv$$

$$= \left(\frac{\partial\psi}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial\psi}{\partial v}\frac{\partial v}{\partial x}\right)dx + \cdots$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} = 4\frac{\partial^2\psi}{\partial u\partial v}$$

Exempl. visq. slow

$f(x+iy) = p + i\zeta$

$\cos\varphi = \frac{x}{\cosh\psi}$ $\sin\varphi = \frac{y}{\cosh\psi}$

$\cancel{\frac{x^2}{\cosh^2\psi} + \frac{y^2}{\sinh^2\psi} = c^2}$

$\cosh = e^{\psi} + e^{-\psi}$

$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = -\mu \frac{\partial \zeta}{\partial y} \\ \frac{\partial p}{\partial y} &= \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = +\mu \frac{\partial \zeta}{\partial x} \end{aligned}\right\} \quad \begin{aligned} \Delta^2 p &= 0 \\ \Delta^2 \zeta &= 0 \end{aligned}$$

$p = \text{const}$
$\zeta = \text{const}$ $\perp$

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 = \Delta^2 \varphi = 0$$

$$v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x} \qquad -\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \zeta = \Delta^2 \psi$$

$\int (u\,dy - v\,dx) =$

$\cancel{\Delta^2 u = -\frac{\partial \zeta}{\partial y}}$
$\cancel{\Delta^2 v = \frac{\partial \zeta}{\partial x}}$

Ex.: $f = A z^2$

$p = A r^2 \cos 2\theta \quad = A(x^2 - y^2)$

$\zeta = A r^2 \sin 2\theta \quad = 2Axy$

$$2Axy = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$$

$$\psi = \frac{1}{6} A (x y^3 + x^3 y) + R\Phi(x+iy)$$

$$\boxed{\begin{aligned} \frac{\partial \psi}{\partial x} &= \frac{1}{6}(A y^3 + A 3 x^2 y) \\ \frac{\partial^2 \psi}{\partial x^2} &= A x y \end{aligned}}$$

$$u = \frac{\partial \Phi_1}{\partial x} - \frac{\partial \Phi_2}{\partial y} - \frac{A}{6}(3xy^2 + x^3)$$

$$v = \frac{\partial \Phi_1}{\partial y} + \frac{\partial \Phi_2}{\partial x} + \frac{A}{6}(y^3 + 3x^2 y)$$

$\cancel{+\frac{\partial^2 \Phi_1}{\partial x \partial y} + \frac{\partial^2 \Phi_2}{\partial y^2}}$

$\cancel{\frac{\partial v}{\partial x} = A x y}$ $\cancel{\frac{\partial u}{\partial y} = -Axy}$

$\cancel{\zeta = 2Axy}$

$-\frac{A}{2}(y^2 + x^2)$

$-\frac{1}{2} y^2 x - \frac{1}{2} x^2$

$x \frac{1}{2} y^2$

Supozycya : ~~6~~ ~~Stałe~~

$$\cancel{V_i = \frac{4\pi(a^{n}+\delta)^2 - a^2}{r} 6 + c}$$

$$\frac{4}{3}\pi a^3(\rho-\rho')g = 6\pi\mu a c$$

$$c = \frac{2}{9}\frac{a^2(\rho-\rho')g}{\mu}$$

$\rho-\rho'=1$

$\mu = 0{\cdot}018$

$a = 1\mu = 10^{-4}$

$\delta = 1\mu = 10^{-4}$

$$c = \frac{2}{9}\frac{10^{-8}.980}{4.0{\cdot}018} = \frac{2.98}{4.9.1{\cdot}8} .10^{-4}\frac{cm}{m} = 0{\cdot}3 . 10^{-4}\ cm$$

$$= \frac{1{\cdot}2}{4}.86400 . 10^{-4}\frac{cm}{d} = \frac{1}{4}\frac{10 cm}{dzień}$$

Kolophonium – H_2O ; $\rho = 1{\cdot}07$ $\quad c = \frac{0{\cdot}7 cm}{dzień}$

$2r_1 = 10^{-7}$ cm (für Lochen)

$2r_2 = 10^{-4}$ cm

$$\frac{m_1}{m_2} = 10^{-9} \qquad \frac{R_1}{R_2} = 10^9$$

$h_1 = 10$ km $= 10^6$ cm

$h_2 = 10^{-3}$ cm

aieby to było widoczne, musiałoby być

$2r_2 = 10^{-5}$ cm $= 0{\cdot}1\mu$ $\quad h_2 = 1$ cm

albo jeśli $\rho = 1{\cdot}07$

$h_2 = 13$ cm

---

aż zmalało 10^{6} cm !

$$\text{Capp.}\ \overbrace{W_1 + W_2 - W_{12}}^{\Delta_C W} = \alpha\, 4\pi\left(2a^2 - a_{12}^{\ 2}\right) = 4\pi a^2 \alpha\left(2 - \sqrt[3]{4}\right)$$

$\alpha_{water-air} = 80$ (CGS) $\qquad \varphi_0 - \varphi_i = 1$ Volt $= \frac{1}{300}$ $\qquad \delta = 10^{-7}$ cm

$$\frac{\Delta_E W}{\Delta_C W} = \frac{\frac{(\varphi_0-\varphi_i)^2}{\delta}}{4\pi\alpha} = \frac{\left(\frac{1}{300}\right)^2 . 10^7}{4\pi . 80} = \frac{10}{9.4.\pi.0{\cdot}8} = \frac{1}{10}$$

~~[illegible]~~ ale łatwo mogą być oba wpływy równe jeśli α_{12} zmniejsza

Oliwa – H_2O : 22

$V_a = \varphi_a$ supozycja: $\varphi_a - \varphi_i$ = stałe

$V_i = \varphi_i$ (myślałby takie supozycje że 6 stałe)

$$\varphi_a = \cancel{\ldots} = \varphi\left(\frac{1}{a} - \frac{1}{a+\delta}\right) + c \qquad \varphi_a - \varphi_i = \varphi\left(\frac{1}{a} - \frac{1}{a+\delta}\right)$$

$$\cancel{\varphi = \ldots} \quad \varphi_i = \varphi\left(\frac{1}{r} - \frac{1}{r}\right) + c$$

$$\varphi = \frac{(\varphi_a - \varphi_i)}{\frac{1}{a} - \frac{1}{a+\delta}} = (\varphi_a - \varphi_i)\,\frac{a(a+\delta)}{\delta}$$

$$\sigma_a = (\varphi_a - \varphi_i)\,\frac{\left(1 + \frac{\delta}{a}\right)}{4\pi\delta}$$

$$\sigma_i = (\varphi_a - \varphi_i)\,\frac{\left(1 + \frac{\delta}{a}\right)^{-1}}{4\pi\delta}$$

$$W = (V_a - V_i)\,\varphi = (\varphi_a - \varphi_i)^2\,\frac{a(a+\delta)}{\delta}$$

$$a_{12}^3 = 2a^3 \qquad a_{12} = a\sqrt[3]{2}$$

$$W_1 + W_2 - W_{12} = \frac{(\varphi_a - \varphi_i)^2}{\delta}\left[2(a^2 + a\delta) - \left(a^2\sqrt[3]{4} + a\delta\sqrt[3]{2}\right)\right]$$

$$\Delta_E W = (\varphi_a - \varphi_i)^2\,\frac{\left[a^2(2 - \sqrt[3]{4}) + a\delta(2 - \sqrt[3]{2})\right]}{\delta}$$

dodatnie w kierunku powiększania $\underline{a}$

$$\cancel{\Delta^2\varphi = \varepsilon = \ldots} \qquad \varphi = f(r) \qquad \frac{\partial\varphi}{\partial x} = f'\frac{x}{r} \qquad \frac{\partial^2\varphi}{\partial x^2} = \frac{f'}{r} - \frac{x^2}{r^3}f' + f''\frac{x^2}{r^2}$$

$$\Delta^2\varphi = \frac{2f'}{r} + f''$$

$$\varepsilon = -\frac{1}{4\pi}\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{2}{r}\frac{\partial\varphi}{\partial r}\right)$$

$$\int 4\pi(a+\xi)^2\,\varepsilon\,d\xi = -\int \underbrace{\left[r^2\frac{\partial^2\varphi}{\partial r^2} + 2r\frac{\partial\varphi}{\partial r}\right]}_{\frac{d}{dr}\left(r^2\frac{\partial\varphi}{\partial r}\right)} dr = r^2\frac{\partial\varphi}{\partial r}\Big|_{-\delta}^{+\delta} = 0$$

$$\frac{\partial p}{\partial x} = -\frac{\mu}{\rho}\frac{\partial \zeta}{\partial y} \qquad \Delta^2 p = 0$$

$$\frac{\partial p}{\partial y} = +\frac{\mu}{\rho}\frac{\partial \zeta}{\partial x} \qquad \Delta^2 \zeta = 0$$

$$f(x+iy) = \varphi + i\psi$$

$$\Delta^2\varphi = 0 \qquad \Delta^2\psi = 0$$

$$-\frac{\frac{\partial\psi}{\partial x}}{\frac{\partial\varphi}{\partial y}} = \frac{\frac{\partial\varphi}{\partial y}}{\frac{\partial\psi}{\partial x}}$$

$$\cancel{\frac{1}{\rho}\frac{\partial p}{\partial x}\frac{\partial \zeta}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial y}\frac{\partial \zeta}{\partial y}}$$

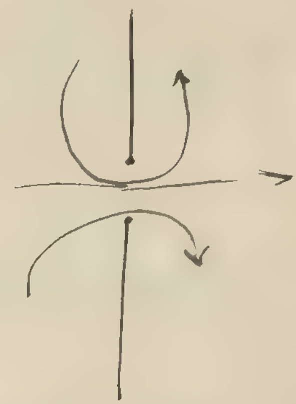
$$\frac{\partial p}{\partial x}\frac{\partial \zeta}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial \zeta}{\partial y} = 0$$

$$f(x+iy) = p + i\zeta$$

$$\zeta = \psi(x,y) = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$$

$$\frac{\partial \zeta}{\partial x} = \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial x^2} = -\Delta^2 v$$

$$\frac{\partial \zeta}{\partial y} = \qquad \Delta^2 u$$



→ zatem przy ściankach $\nabla^2\zeta = 0$, $\zeta^2 = -\nabla^2\Omega$

$p, \zeta = f(x, y^2)$ $\quad p_\infty = \frac{c_1}{c_2}$ ----

$p(0, y^2) = \text{const}$

$\zeta = f(x^2, y)$ $\quad \zeta(x, 0) = 0$

$\bar{\zeta}$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial p}{\partial x}=u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}+v\overbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}^{\zeta}$$

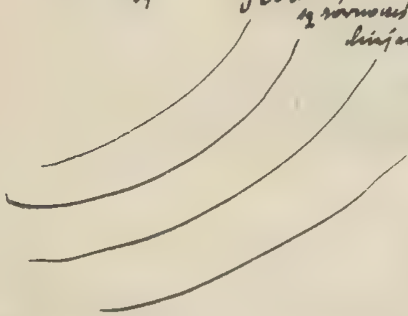
$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial p}{\partial y}=u\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+u\frac{\partial u}{\partial y}+v\frac{\partial v}{\partial y}$$

$$\left.\begin{aligned} v\zeta &= \frac{\partial}{\partial x}\left(\frac{p}{\rho}-\frac{u^2+v^2}{2}\right)\\ u\zeta &= -\frac{\partial}{\partial y}\underbrace{\left(\frac{p}{\rho}-\frac{u^2+v^2}{2}\right)}_{\Omega}\end{aligned}\right\}\quad V\zeta=\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}\right)(\Omega)$$

$$\Omega=\text{const}$$

$$\frac{v}{u}=-\frac{\frac{\partial}{\partial x}}{\frac{\partial}{\partial y}}$$

$\Omega=\text{const}$ są równoległe liniami prądu a spadek $\nabla\Omega$ jest miarą wirów [illegible] ∇/ζ



$$\frac{\partial\Omega}{\partial x}dx+\frac{\partial\Omega}{\partial y}dy=0$$

$$\frac{dy}{dx}=-\frac{\frac{\partial\Omega}{\partial x}}{\frac{\partial\Omega}{\partial y}}=\frac{v}{u}$$

$$\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=\mu\left(-\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}\right)=\mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)$$

$$\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=\mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

$$v\zeta=\frac{\partial(\Omega)}{\partial x}+\mu\frac{\partial\zeta}{\partial y}$$

$$u\zeta=-\left[\frac{\partial\Omega}{\partial y}+\mu\frac{\partial\zeta}{\partial x}\right]$$

$$v\frac{\partial\zeta}{\partial y}+u\frac{\partial\zeta}{\partial x}=\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\zeta$$

$$v\frac{\partial\zeta}{\partial x}-u\frac{\partial\zeta}{\partial y}-\zeta^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Omega$$

$$\frac{v}{u}=\frac{\frac{\partial\Omega}{\partial x}+\mu\frac{\partial\zeta}{\partial y}}{-\frac{\partial\Omega}{\partial y}+\mu\frac{\partial\zeta}{\partial x}}$$

$$(\sigma\nabla)\zeta=\mu\nabla^2\zeta$$

$$\Delta X - \frac{\partial F}{\partial x} = \overbrace{4\pi A^2_\mu \lambda}^{a} \frac{\partial X}{\partial t} + \beta \frac{\partial^2 X}{\partial t^2})$$

$$X = P\frac{x}{r}\ ; \quad Y = P\frac{y}{r}\ ; \dots$$

$$F = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 3\frac{P}{r} - \frac{P(x^2+y^2+z^2)}{r^3} + \frac{dP}{dr}\left(\frac{x^2}{r^2} + \dots\right)$$

$$= \frac{2P}{r} + \frac{dP}{dr}$$

$$\frac{\partial X}{\partial x} = \frac{P}{r} - \frac{Px^2}{r^3} + \frac{dP}{dr}\frac{x^2}{r^2} \qquad + \frac{d^2P}{dr^2}\frac{x^3}{r^3}$$

$$\frac{\partial^2 X}{\partial x^2} = -\frac{3Px}{r^3} + \frac{3Px^3}{r^5} + \frac{dP}{dr}\left(-\frac{3x^2}{r^3}\right)\frac{x}{r} + \frac{dP}{dr}\frac{2x}{r^2}$$

$$\frac{\partial X}{\partial y} = -P\frac{xy}{r^3} + \frac{dP}{dr}\frac{xy}{r^2}$$

$$\frac{\partial^2 X}{\partial y^2} = -\frac{Px}{r^3} + 3\frac{Pxy^2}{r^5} - \frac{dP}{dr}\frac{xy^2}{r^4} + \frac{dP}{dr}\frac{x}{r^2} - 2\frac{dP}{dr}\frac{xy^2}{r^4} + \frac{d^2P}{dr^2}\frac{xy^2}{r^3}$$

$$\Delta^2 X = P\left\{-\frac{5x}{r^3} + \frac{3x^3 + 3xy^2 + 3xz^2}{r^5}\right\}$$

$$+ \frac{dP}{dr}\left\{\frac{x}{r^2} - \frac{3x^3}{r^4} + \frac{4x}{r^2} - \frac{3xy^2}{r^4} - \frac{3xz^2}{r^4}\right\} + \frac{d^2P}{dr^2}\left[\frac{x^3}{r^3} + \frac{x(y^2+z^2)}{r^3}\right]$$

$$= P\ -\frac{2x}{r^3} + \frac{dP}{dr}\left\{\cancel{\frac{1}{r} - \frac{3x^2}{r^3} + \dots + \frac{3x^3}{r^4}}\right\} + \frac{x}{r}\frac{d^2P}{dr^2}$$

$$+ \frac{2x}{r^2}\frac{dP}{dr}$$

$$-\cancel{\frac{2x}{r^3}P} + \cancel{\frac{2x}{r^2}\frac{dP}{dr}} + \cancel{\frac{x}{r}\frac{d^2P}{dr^2}} - \cancel{2\frac{x}{r^2}\frac{dP}{dr}} + \cancel{2P\frac{x}{r^3}} - \cancel{\frac{d^2P}{dr^2}\frac{x}{r}} = a\frac{x}{r}\frac{\partial P}{\partial t} + \beta\frac{x}{r}\frac{\partial^2 P}{\partial t^2} = 0$$

$$P = \cancel{1} + P_0 e^{-\frac{4\pi\lambda}{\varepsilon}t}$$

$$P = \varphi(r) + \psi(r)\, e^{-\frac{4\pi\lambda}{\varepsilon}t}$$

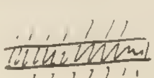
$t=0:\ P=0 \qquad \varphi(r)+\psi(r)=0$

$t=\infty:\ P=0 \qquad \varphi(r) = 0$

$\Big\}\ P=0$

1). Stat. $f6^e$ || 1). Plattencondens. 2). ${}^5C_y$

Schutzring condens.

3).  2: Medien

4). $f6^e$ $\angle x$;

$\Delta^2 V = 0$ $\quad$ $\Delta^2\theta = 0$ $\quad$ $\Delta^2\varrho = 0$

$c\varrho \frac{\partial\theta}{\partial t} = \kappa \Delta^2\theta$ $\quad$ $\frac{\partial\varrho}{\partial t} = k \frac{\partial^2\varrho}{\partial x^2}$

właściwie powinno się to nazwać przewodzeniem potencyału

$t = 86400 . 365$
259200
51840
4332
$31,547.600$

~~[illegible]~~

$\frac{x}{2\sqrt{kt}} = 1$

$x = 2\sqrt{kt}$

$= 2\sqrt{3.10^{10}} = 3{,}4.10^5 \text{ cm} = 3{,}4 \text{ km}$

$T = 100.10^6 \text{ lat} = 3.10^{15} \text{ sec}$

$A = 2a \sqrt{\frac{kt}{\pi}} = 2a\sqrt{10^{10}} = 2a.10^5$

Gdyby przed 10^8 lat pow. morza wszystka woda pokryta została, i gdyby woda bezustannie ohładzana (ale bez prądów) toby dotychczas tylko na 2 km głębokości została absolutnie wystudzona

W 1 dniu: $A = 2\sqrt{\frac{\cancel{[illegible]}}{\pi}} = 1 \text{ cm}$

Z tego pochodzi powolność wzrostu roślin, wogóle organizmów i długość życia.

Rozmieszanie atmosfery i powietrze nad morzem nie jest absolutnie wystudzone.

Meteoryty (stałe w których niema prądów) powstają uniwersyjnie niezmiennie i metale obok siebie leżące nie to się zmieniają.

Takie prawo Newtona daje się pewnikiem zastosow. do θ i do ϱ

$\frac{k}{c\varrho} = 0{,}015$ | $\sqrt{kt} = \sqrt{4{,}5.10^{13}} = 7.10^6$

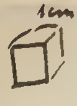
$k \text{ diff.} = 2{,}67.10^{-5}$

$\int^{0{,}477} = \frac{1}{2}$

spadnie na połowę gdy

$\frac{z}{2\sqrt{kt}} = 0{,}477$

[illegible]



Ayrton & Perry Wied. I 574, Pr. R. S. 27 p. 219 (1878) mult. $\times 10^4/2$ = Hg = 1

Paraffin: 46° Widerstand = 34.000 . 10^{13} Ω

50° 1000 . 10^{12}

70° 10 . 10^{12}

77.8 1.35 . 10^{12}

Guttapercha: 24° 83 . 10^{12}

44° 8.9 . 10^{12}

83° 0.5 . 10^{12}

Ebonit: 36° 61.000 . 10^{12}

96.8° 9700 . 10^{12}

Kohlrausch H_2O 21.5° $\lambda = 0.71$ Hg = 10^{10}

Pogg. Erg. B. 8 p. 1 (1876)

Kohlr. 24 p. 48 (1885) ? Vacuum dest. $\lambda = 0.25$ Hg = 10^{10}

Dewar Heat Wied. 20 p. 280 (1893)

Sr. sehr schlechte Isolator

d. Äther Pfeiffer Wied. Ann. 26 $\ll 0.003 \cdot 10^{-10}$ (Hg = 1)

Heydweiller Wied. Ann. 69 p. 531 Glassorte: 2.10^{-4} CGS ext.

$= 2.10^{-26}$ CGS. em.

Luft > 5 mm Hg. ... < 5 CGS. ext. $\lambda < \lambda_{Glas}$

5 – 0.1 ...... =

...... $\lambda = 10^{-1}$ ext.

10^{-23} em.

$$V \frac{2\pi\lambda\, l \cdot 10^{-18}}{1} = 1\%\, V \frac{2\pi}{l} \cdot 60$$

$$l^2 = 6.10^{17} \frac{1}{2} = 3.10^{17}$$

$$l = 10^8 \sqrt{30} = 5.5 \cdot 10^8$$

$= 5.5 \cdot 10^2$ km

= 5500 km

$$l^2 = \frac{1}{2 \cdot 0.00046} = 10^3$$

$l = 31$ m = 31 cm

Analogja i różnice statystyk prądów elektrycznych, cieplnych i dyfuzji.

$\Delta^2\varphi = 0$ .. nietylko się bardzo często w fizyce, ogólnie wszędzie pewna analogja

ale czasem tylko przypadkowa [illegible] n.p. fale akustyczne, pole elektrostatyczne i ruch cieczy płynnej.

Czy istnieją analogie rzeczywiste i pozorne ?

Ściśle biorąc, tylko różnice jakościowe, bo właściwie wszystko w fizyce teoret. jest tylko analogją; ale gdy tylko mało cech wspólnych to żadnej korzyści, i odwrotnie przeciwnie

Bardzo wielka analogja istnieje między owemi przypadkami [illegible] jeśli n.p. prąd w elektrolitach.

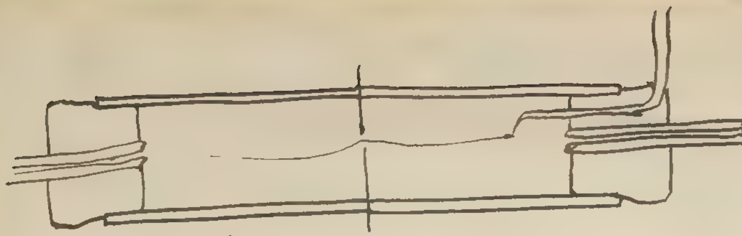
Różnice zakresu wielkości współczynników.

	Elektr.
Ag	60
Cu	56
Hg	1
conc. NaCl	~~[illegible]~~ 2.10^{-5}
H_2O	$\frac{1}{4}.10^{-14}$
Paraffin 40°:	3.10^{-22}
powietrze Hydr.	2.10^{-17}
vacuum	
Eter	10^{-18}

	Ciepło
Ag	1·096 (Weber)
Cu	1·1 – 0·63
Hg	0·02 (Angot)
~~[illegible]~~	0·001 – 0·0023
H_2~~[illegible]~~	0·0013 (Winkelmann)
Paraffin	0·0002 Weber
Lutt [illegible]	0·00046 Lees
~~[illegible]~~ Kork	0·00013 "
Kiel. Kautschuk, Watta, ~~[illegible]~~	0·00009 (?) Forbes
powietrze	0·00006

	Dyf.
Caramel	$5·4.10^{-7}$
~~[illegible]~~	16·7 "
HCl	267.10^{-7}
NaCl	123.10^{-7}
$CuSO_4$	24.10^{-7}
H – O	0·722
$O - CO_2$	0·180
Au – Pb	500° : 3·19
	100° : 0·00002
	$NaCl - H_2O = 1$
$CO_2 - H_2O$	16.10^{-5}
Kautschuk – CO_2	$0·54.10^{-5}$

981



	η 20°
Glycerin	8·0
Vaselinöl	0·91
H_2O	0·0099
Methylenchlorid	0·0041

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad \Big| \ \frac{\partial}{\partial y}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad \Big| \ \frac{\partial}{\partial x}$$

$$-u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y} = v^2 \frac{\partial\left(\frac{u}{v}\right)}{\partial y} =$$

$$+u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}$$

$$\left.\begin{array}{l} \cancel{-\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}} + \cancel{\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}} - u\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^2 u}{\partial y^2} \\ -\cancel{\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}} - \cancel{\frac{\partial v}{\partial x}\frac{\partial u}{\partial x}}\left[+u\frac{\partial^2 v}{\partial x^2} - v\frac{\partial^2 u}{\partial x^2}\right] \end{array}\right\} = \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$-u\,\Delta^2 v + v\,\Delta^2 u = \frac{\mu}{\rho}\Delta^2\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$\frac{\partial c}{\partial t} = \mathrm{div}$$

[illegible]

Kula w cieczy ściśliwej:

Quellen & Senken analog. do płynów

Double sources w optyce światła

$$\frac{\partial\rho}{\partial t} = -\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \dots\right)$$

$$= -\rho\,\Delta^2\varphi$$

gęstość jest substancyą:

~~[illegible]~~ Phys. Zs. 4 (1903)

Przykłady Schuster etc.

ale mimo to tylko przedstawienia

podobnie jak Seillmeier-Optik etc. Plattner-wody

Więc może rozróżnimy analogię formalną i rzeczywistą?

Jednak ~~[illegible]~~ jest ta różnica tylko stopniowa; do pewnego punktu wspólność, po za tem nie | gdyby zupełna to zupełna nie potrafilibyśmy ich odróżnić tylko

A zresztą a priori nie można rozróżnić n.p. Lichtäther elektr. – elast. ; jak Wasser Therm.-elektr. Ath. zawsze formalnie!

Ostatecznie w ogóle w fizyce każde wytłomaczenie = tylko analogia, Gastheorie = analogia z mechanizmem etc.

Nazwy „analogii" wtedy tylko używamy do analogii niezupełnych uproszczenie przez rozmyślanie

Więc porzućmy stosunkowo daleko idące; ~~[illegible]~~ wspólność: substancyalizacja

może nawet kiedyś dalej jeszcze idące np. elektr.-dyf. = ~~[illegible]~~ elektrolytach, a elektrony? Wiedemann-Franz

Wielka we fizyce która analogii ciepło wartość przypisujemy ciepło substancya, wtedy ruch?

tor Elektryczne masy czy istnieją? Magnetyzm?

$$\frac{1}{x} = \frac{\cosh 2b + \cos 2a}{\sin 2a} = r \cos\theta$$

$$\frac{\cosh 2b + \cos 2a}{\sinh 2b} = r \sin\theta$$

$$r^2 = (\cosh 2b + \cos 2a)^2 \; \frac{\sin^2 2a + \sinh^2 2b}{\sin^2 2a \sinh^2 2b}$$

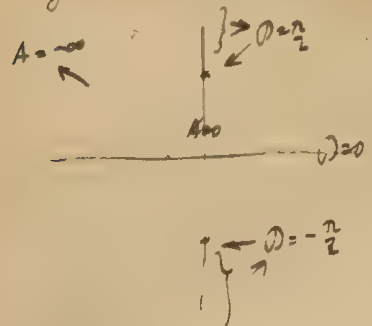
$$\cosh 2\delta = \frac{1+\delta+\frac{\delta^2}{2} + 1-\delta+\frac{\delta^2}{2}}{2} \qquad \cosh 2b = 1 + \frac{\delta^2}{2}$$

$$= 1+\frac{\delta^2}{2} \qquad e^{2b} - \frac{\delta}{2} \cdot \delta^4$$

$$b = \frac{\delta^2}{2} \qquad = \ldots$$

$$\lim_{r=\infty} r b = \boxed{\text{finite const.}}$$

$$\lg(z+\sqrt{1+z^2}) = \operatorname{arcsinh} z = A + i\Theta$$



$$e^a \cos b = x + \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2}$$

$$e^a \sin b = y + \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2}$$

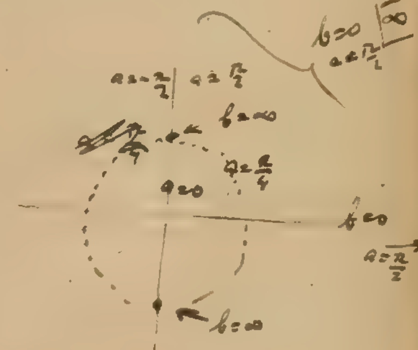
$$A = \lg\sqrt{r^2 + r r_2 + 2r\sqrt{r_1 r_2}\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}$$

$$\Theta = \arcsin\left\{\frac{r\sin\theta + \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2}}{\sqrt{\dots}}\right\}$$

$$\operatorname{arctg} z = a + ib$$

$$x = \frac{\sin 2a}{\cosh 2b + \cos 2a}\,; \qquad y = \frac{\sinh 2b}{\cosh 2b + \cos 2a}\,;$$

$$a = \frac{1}{2}\operatorname{arctg}\frac{2x}{1-r^2} \qquad b = \frac{1}{2}\operatorname{arctgh}\frac{2y}{1+r^2}$$



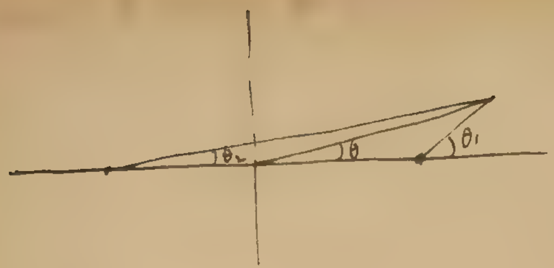
$$\frac{\nu-\mu}{2} = \frac{xy}{r^4}$$

$$\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right) = \dots = \frac{1}{2}(\nu-\mu) = -\frac{\sin\theta\cos\theta}{r^2}$$

$$\frac{r}{\sqrt{r_1 r_2}} = 1 + \frac{\cos 2\theta}{2r^2}$$

$y=0$: $\quad b=0$:

$$x = \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha} = \operatorname{tg}\alpha$$



$$\sin\theta = \theta - \frac{\theta^3}{3!} = \frac{\eta}{r} = \frac{\eta}{1+\xi}$$

$$\cos\theta = 1 - \frac{\theta^2}{2} = 1 - \frac{\eta^2}{2}$$

$$\frac{\theta}{2} = \frac{\eta}{2+\xi}$$

$$\cos\frac{\theta}{2} = 1 - \frac{\eta^2}{32}$$

linge $\theta = \theta_2 = 0$

$$u = -\frac{2y}{\sqrt{2r_1}} \sin\frac{\theta_1}{2}$$

$$v = +\frac{2x}{\sqrt{2r_1}} \sin\frac{\theta_1}{2} - \frac{2}{\sqrt{2r_1}} \sin\frac{\theta_1}{2} = \frac{2\xi}{\sqrt{2r}} \sin\frac{\theta}{2}$$

$$\sin\theta \cos\frac{\theta_1+\theta_2}{2} - \cos\theta \sin\frac{\theta_1+\theta_2}{2} =$$

$$\sin\theta\left[\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} - \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right] - \cos\theta\left[\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2} + \cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right] =$$

$$= \eta(1-\xi)\left[\cos\frac{\theta_1}{2}\left(1-\frac{\eta^2}{32}\right) - \sin\frac{\theta_1}{2}\,\frac{\eta}{4}\left(1-\frac{\xi}{4}\right)\right] - \left(1-\frac{\eta^2}{2}\right)\left[\sin\frac{\theta_1}{2}\left(1-\frac{\eta^2}{32}\right) + \cos\frac{\theta}{2}\frac{\eta}{4}\left(1-\frac{\xi}{4}\right)\right]$$

$$= \cos\frac{\theta_1}{2}\left[\eta(1-\xi)\left(1-\frac{\eta^2}{32}\right) - \left(1-\frac{\eta^2}{2}\right)\frac{\eta}{4}\left(1-\frac{\xi}{4}\right)\right]$$

$$- \sin\frac{\theta_1}{2}\left[\left(1-\frac{\eta^2}{2}\right)\left(1-\frac{\eta^2}{32}\right) + \frac{\eta^2}{4}(1-\xi)\left(1-\frac{\xi}{4}\right)\right]$$

$$\cancel{32 - 16 + 1 + 8} \quad \frac{25}{32}$$

$$8 - 1(- 1$$

$$= \cos\frac{\theta_1}{2}\left[\eta - \eta\xi - \frac{\eta}{4} + \frac{\eta\xi}{16}\right] - \sin\frac{\theta_1}{2}\underbrace{\left[1 - \frac{\eta^2}{2} - \frac{\eta^2}{32} + \frac{\eta^2}{4}\right]}_{1 - \frac{9\eta^2}{32}}$$

$$u = -\frac{2r\sin\theta}{\sqrt{2r}}\left(1+\frac{3\xi}{4}\right)\sin\frac{\theta}{2} = -\sqrt{2r}\sin\theta\sin\frac{\theta}{2}$$

$$= -\sqrt{\frac{2}{r}}\,\eta\,\sin\frac{\theta}{2}$$

$$\frac{r}{\sqrt{r_1 r_2}} = \frac{1+\xi}{\sqrt{r_1(2+\xi)}} = \frac{1}{\sqrt{2r_1}}\,(1+\xi)\underbrace{\left(1+\frac{\xi}{2}\right)^{-\frac{1}{2}}}_{1-\frac{\xi}{4}}$$

$$\frac{1}{\sqrt{r_1 r_2}} = \frac{1}{\sqrt{2r_1}}\left(1+\frac{\xi}{2}\right)^{-\frac{1}{2}}$$

$$\frac{x r}{\sqrt{r_1 r_2}} = \frac{1}{\sqrt{2r_1}}\,\underbrace{(1+\xi)^2\left(1-\frac{\xi}{4}\right)}_{1+\frac{7}{4}\xi}$$

$$v = \sin\frac{\theta_1}{2}\left\{+\frac{2}{\sqrt{2r_1}}\left(1+\frac{7}{4}\xi\right)\left(1-\frac{9\eta^2}{32}\right) - \frac{2}{\sqrt{2r_1}}\left(1+\frac{\xi}{4}\right)\left(1-\frac{\eta^2}{32}\right)\right]$$

$$+\cos\frac{\theta_1}{2}\left\{-\frac{2}{\sqrt{2r_1}}\left(1+\frac{7}{4}\xi\right)\frac{3\eta}{4} - \frac{2}{\sqrt{2r_1}}\left(1-\frac{\xi}{4}\right)\frac{\eta}{4}\right]$$

$$= \frac{\sqrt{2}}{\sqrt{r_1}}\left[\sin\frac{\theta_1}{2}\left(1+\frac{7}{4}\xi-1+\frac{\xi}{4}\right) - \cos\frac{\theta_1}{2}\left(\frac{3\eta}{4}+\frac{\eta}{4}\right)\right]$$

$$= \sqrt{\frac{2}{r_1}}\left[\sin\frac{\theta_1}{2}\cdot 2\xi - \cos\frac{\theta_1}{2}\cdot\eta\right]$$

$$= \sqrt{2r_1}\left[2\sin\frac{\theta_1}{2}\cos\theta_1 - \cos\frac{\theta_1}{2}\sin\theta_1\right]$$

$$= \underbrace{\sin\frac{\theta_1}{2}\cos\theta_1 - \cos\frac{\theta_1}{2}\sin\theta_1}_{-\sin\frac{\theta}{2}} + \sin\frac{\theta}{2}\cos\theta_1$$

$$= \sin\frac{\theta}{2}\left[\cos\theta - 1\right]$$

$$v = \sqrt{2r}\,\sin\frac{\theta}{2}(\cos\theta-1) = \sqrt{\frac{2}{r}}\,\sin\frac{\theta}{2}(\xi - r)$$

Obliczenie poprawki od skutków bezwładności

$u = u_0 + \frac{\rho}{\mu} u_1$

$p = p_0 + \frac{\rho}{\mu} p_1$

I). $\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} = \frac{1}{\mu}\frac{\partial p_0}{\partial x}$

II). $\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} = \frac{1}{\mu}\frac{\partial p_1}{\partial x} + \frac{\rho}{\mu}\left(u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y}\right)$

$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0$

$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} = \frac{1}{\mu}\frac{\partial p_1}{\partial y} + \frac{\rho}{\mu}\left(u_0 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial v_0}{\partial y}\right)$

$$\frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} = -\rho\left[\frac{\partial}{\partial x}\underbrace{\left(u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right)}_{u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial v_0}{\partial x} - v_0\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)} + \frac{\partial}{\partial y}\underbrace{\left(u_0\frac{\partial v_0}{\partial x} + v_0\frac{\partial v_0}{\partial y}\right)}_{u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} + u\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}\right]$$

$$= -\rho\left[\frac{\partial}{\partial y}(u\zeta) - \frac{\partial}{\partial x}(v\zeta) + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{u^2+v^2}{2}\right)\right]$$

$$= -\rho\left[u\frac{\partial \zeta}{\partial y} - v\frac{\partial \zeta}{\partial x} - \zeta^2 + \qquad\right]$$

$F = \int (H^2 - S'') z\, dz$

$u = -\frac{\zeta y}{2}$

$v = \frac{\zeta x}{2} + 2 J F$

$= u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} - v\zeta = \frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right) - v\zeta$

$\frac{\partial u}{\partial x} = -\frac{y}{2}\frac{\partial \zeta}{\partial x}$

$\frac{\partial u}{\partial y} = -\frac{\zeta}{2} - \frac{y}{2}\frac{\partial \zeta}{\partial y}$

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\zeta y^2}{4}\frac{\partial \zeta}{\partial x} - \frac{yx}{4}\zeta\frac{\partial \zeta}{\partial y} - \frac{\zeta^2 x}{4}$

$- \left(\zeta + y\frac{\partial \zeta}{\partial y}\right) J F$

Najpierw dla prostszego przykładu: przepływ na około krawędzi:

$$u = -\frac{\sqrt{2}\,y}{\sqrt{r}}\sin\frac{\theta}{2} \qquad\qquad \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{r}}\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)$$

$$v = \frac{\sqrt{2}\,x}{\sqrt{r}}\sin\frac{\theta}{2} - \sqrt{2}\sqrt{r}\sin\frac{\theta}{2} \qquad\qquad \frac{1}{\sqrt{\beta}} = \frac{1}{\sqrt{r}}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$$

$$\frac{1}{\sqrt{r}}\sin\frac{\theta}{2} = \frac{1}{2i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right)$$

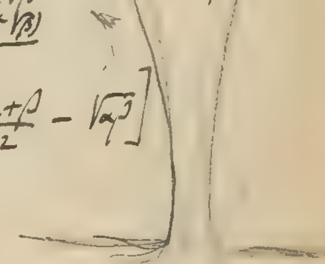
$$y = \frac{\alpha-\beta}{2i} \qquad\qquad \sqrt{r}\sin\frac{\theta}{2} = \frac{1}{2i}\left(\sqrt{\alpha}-\sqrt{\beta}\right)$$

$$x = \frac{\alpha+\beta}{2} \qquad\qquad \left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) = \frac{\sqrt{\alpha}-\sqrt{\beta}}{\sqrt{\alpha\beta}}$$

$$u = -\frac{\alpha-\beta}{2i}\,\frac{1}{2i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) = \frac{\alpha-\beta}{4}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) = \frac{(\sqrt{\alpha}-\sqrt{\beta})^2(\sqrt{\alpha}+\sqrt{\beta})}{4\sqrt{\alpha\beta}}$$

$$v = \frac{\alpha+\beta}{2}\,\frac{1}{2i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) - \frac{1}{2i}\left(\sqrt{\alpha}-\sqrt{\beta}\right) = \left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right)\frac{1}{2i}\left[\frac{\alpha+\beta}{2} - \sqrt{\alpha\beta}\right]$$

$$= \frac{1}{4i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right)\left(\sqrt{\alpha}-\sqrt{\beta}\right)^2 = \frac{1}{4i}\,\frac{(\sqrt{\alpha}-\sqrt{\beta})^3}{\sqrt{\alpha\beta}}$$



$$u = -y\sqrt{\frac{1-\cos\theta}{r}} = -y\,\frac{\sqrt{r-x}}{r} \qquad\qquad u^2+v^2 = (r-x)\left(\frac{y^2}{r^2} + \frac{x^2}{r^2} - \frac{2x}{r} + 1\right)$$

$$v = \frac{x\sqrt{r-x}}{r} - \sqrt{r-x} = \left(\frac{x}{r}-1\right)\sqrt{r-x} \qquad\qquad = 2(r-x)\left(1-\frac{x}{r}\right)$$

$$= \frac{2(r-x)^2}{r}$$

$$\zeta = \frac{2\sqrt{2}}{\sqrt{r}}\sin\frac{\theta}{2} = \frac{2\sqrt{r-x}}{r}$$ (control

$$u\zeta = -\frac{2y(r-x)}{r^2} \qquad\qquad v\zeta = 2\left(\frac{x}{r}-1\right)\frac{r-x}{r} = -2\left(\frac{r-x}{r}\right)^2 = -2\left(1-\frac{x}{r}\right)^2$$

$$\frac{\partial}{\partial y}(\;) = -\frac{2(r-x)}{r^2} + \frac{4y^2(r-x)}{r^4} - \frac{2y^2}{r^3} \qquad\qquad \frac{\partial}{\partial x}(\;) = +4\left(1-\frac{x}{r}\right)\left(\frac{1}{r} - \frac{x^2}{r^3}\right)$$

$$= 4\,\frac{r-x}{r}\,\frac{y^2}{r^3}$$

$$-\frac{2(r-x)}{r^2} + \frac{4y^2(r-x)}{r^4} - \frac{2y^2}{r^3} - 4\frac{y^2}{r^3} + \frac{4xy^2}{r^4} = \cancel{-\frac{6(r-x)}{r^2} - \frac{2y^2}{r^3} - \frac{8xy^2}{r^4}}$$

$$= \cancel{-\frac{6}{r} + \frac{6x}{r^2} - \frac{2y^2}{r^3} - \frac{8xy^2}{r^4}} = -\frac{2}{r} + \frac{2x}{r^2} - \frac{2y^2}{r^3}$$

$$u^2+v^2 = \frac{2(r-x)^2}{r} = 2\left(r - 2x + \frac{x^2}{r}\right)$$

$$\frac{\partial}{\partial x} = 2\left\{\frac{x}{r} - 2 + \frac{2x}{r} - \frac{x^3}{r^3}\right\} = 2\left[-2 + 3\frac{x}{r} - \frac{x^3}{r^3}\right]$$

$$\frac{\partial^2}{\partial x^2} = 2\left[\frac{3}{r} - \frac{3x^2}{r^3} - \frac{3x^2}{r^3} + 3\frac{x^4}{r^5}\right]$$

$$\frac{\partial}{\partial y} = 2\left[\frac{y}{r} - \frac{x^2 y}{r^3}\right]$$

$$\frac{\partial^2}{\partial y^2} = 2\left[\cancel{\frac{1}{r} - \frac{y^2}{r^3}} - \cancel{\frac{x^2}{r^3}} + \frac{3x^2y^2}{r^5}\right]$$

$$\nabla^2 = 2\left[\frac{3}{r} - \frac{6x^2}{r^3} + \frac{3x^2}{r^3}\right] = 2\left[\frac{3}{r} - \frac{3x^2}{r^3}\right]$$

$$\cancel{-\frac{6}{r}} + \cancel{\frac{8x}{r^2}} - \cancel{\frac{2y^2}{r^3}} - \cancel{\frac{8xy^2}{r^4}} + \cancel{\frac{4}{r}} - \frac{3x^2}{r^3} = \quad + \frac{4}{r} - \frac{4x}{r^2} - \frac{3}{r} + \frac{3x^2}{r^3}$$

$$\cancel{\frac{6x}{r^2}} - \cancel{\frac{6x^2}{r^3}} - \cancel{\frac{2}{r}} + \frac{2x^2}{r^3} - \cancel{\frac{8x}{r^2}} + \cancel{\frac{8x^3}{r^4}} = -\frac{2}{r} - \cancel{\frac{2x}{r^2}} - \cancel{\frac{4x^2}{r^3}} + \frac{8x^3}{r^4}$$

$$+ \frac{4(r-x)}{r^2} +$$

$$\frac{3}{r} - \frac{3x^2}{r^3} - \frac{2}{r} + \frac{2x}{r^2} - \frac{2}{r^2} + \frac{2x^2}{r^3}$$

$$= -\frac{1}{r} + \frac{2x}{r^2} - \frac{x^2}{r^3} \qquad (\text{r}\ \text{right})$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{(r-x)^2 x}{r^3} \qquad \Big| \qquad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{(r-x)^2 y}{r^3}$$

$$\frac{\partial}{\partial x}(\qquad) + \frac{\partial}{\partial y}(\qquad) = -\frac{(r-x)^2}{r^3} = -\frac{1}{r}\left(1 - \frac{x}{r}\right)^2$$

$$= -\frac{1}{r}(1-\cos\theta)^2 = -\frac{4\sin^4\frac{\theta}{2}}{r}$$

$$\frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} = \frac{1}{r}\left(1 - \frac{x}{r}\right)^2 = \frac{4\sin^4\frac{\theta}{2}}{r}$$

$$x = \frac{\alpha+\beta}{2} \qquad y = \frac{\alpha-\beta}{2i}$$

$$r^2 = \frac{\alpha^2 + 2\alpha\beta + \beta^2}{4} - \frac{\alpha^2 - 2\alpha\beta + \beta^2}{4}$$

$$= \frac{4\alpha\beta}{4} = \alpha\beta$$

$$\alpha^2 + \beta^2 = 2(x^2 - y^2)$$

$$4\frac{\partial^2 p}{\partial\alpha\,\partial\beta} = \frac{\left[\sqrt{\alpha\beta} - \frac{\alpha+\beta}{2}\right]^2}{(\alpha\beta)^{3/2}}$$

$$= \frac{1}{4}\frac{\left[-\alpha + 2\sqrt{\alpha\beta} - \beta\right]^2}{\sqrt{\alpha\beta}^3} = \frac{1}{4}\frac{\left[\sqrt{\alpha} - \sqrt{\beta}\right]^4}{\sqrt{\alpha\beta}^3} = \frac{1}{2}\left[\frac{\sqrt{\alpha}}{\sqrt{\alpha\beta^3}} - \frac{2}{\sqrt{\alpha\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha^3\beta}}\right]$$

$$\cancel{\int\frac{1}{\sqrt{\alpha\beta^3}}\,d\alpha\,d\beta = -4\sqrt{\frac{\alpha}{\beta}}} \qquad \cancel{\int\frac{d\alpha\,d\beta}{\alpha\beta} = \log\alpha\,\log\beta} \qquad \frac{(2i\sqrt{r}\sin\frac{\theta}{2})^4}{r^3} = \frac{\sin^4\frac{\theta}{2}}{r}$$

$$\int\frac{d\alpha\,d\beta}{\sqrt{\alpha\beta}} = 4\sqrt{\alpha\beta} \qquad \int\sqrt{\frac{\alpha}{\beta^3}}\,d\alpha\,d\beta = -\frac{1}{3}\sqrt{\frac{\alpha^3}{\beta}}$$

$$p = 8\left[-\frac{1}{3}\sqrt{\frac{\alpha^3}{\beta}} - 8\sqrt{\alpha\beta} - \frac{1}{3}\sqrt{\frac{\beta^3}{\alpha}}\right] = \frac{-8}{\sqrt{\alpha\beta}}\left[\frac{1}{3}\sqrt{\alpha^4} + 8(\sqrt{\alpha\beta})^2 + \frac{1}{3}\sqrt{\beta^4}\right]$$

$$= \frac{-8}{\sqrt{\alpha\beta}}\left[\frac{\alpha^2}{3} + 8\alpha\beta + \frac{\beta^2}{3}\right] = -\frac{8}{r}\left[\frac{2(x^2-y^2)}{3} + 8r^2\right]$$

Prüfen $\quad 8r + \frac{2}{3}\left(\frac{x^2}{r} - \frac{y^2}{r}\right)$

$$8\frac{x}{r} + \frac{2}{3}\left[\frac{2x}{r} - \frac{x^3}{r^3} + \frac{y^2x}{r^3}\right] \qquad \Big|\; 8\frac{y}{r} + \frac{2}{3}\left(-\frac{x^2y}{r} - \frac{2y}{r} + \frac{y^3}{r^3}\right)$$

$$8\frac{1}{r} - 8\frac{x^2}{r^3} + \frac{2}{3}\left[\frac{2}{r} - \frac{2x^2}{r^3} - \frac{3x^2}{r^3} + \frac{3x^4}{r^5} + \frac{y^2}{r^3} - \frac{3y^2x^2}{r^5}\right]$$

$$y^2 = r^2 - x^2$$
$$y^4 = r^4 - 2r^2x^2 + x^4$$

$$8\frac{1}{r} - 8\frac{y^2}{r^3} + \frac{2}{3}\left[-\frac{2}{r} + \frac{2y^2}{r^3} + \frac{3y^2}{r^3} - \frac{3y^4}{r^5} - \frac{x^2}{r^3} + \frac{3x^2y^2}{r^5}\right]$$

$$\frac{8}{r} + \frac{2}{3}\left[6\frac{y^2}{r^3} - \frac{x^2}{r^3} + \frac{3x^4}{r^5} - \frac{3y^4}{r^5}\right] = \frac{8}{r} + \frac{2}{3}\left[\underbrace{6\left(\frac{1}{r} - \frac{2x^2}{r^3}\right) + \frac{3x^4}{r^5} - \frac{1}{r} + \frac{2x^2}{r^3} - \frac{x^4}{r^5}}_{3\left(\frac{1}{r} - 2\frac{x^2}{r^3}\right)}\right]$$

x^2-y^2

$$x=\int \frac{1}{16}\frac{(\sqrt{\alpha}-\sqrt{\beta})^4}{\sqrt{\alpha\beta}^3}\,d\alpha\,d\beta \qquad \begin{matrix}\sqrt{\alpha}=u & \alpha=u^2\\ \sqrt{\beta}=v & \beta=v^2\end{matrix}$$

$$=\frac{4}{16}\int\frac{(u-v)^4}{u^3v^3}\,uv\,du\,dv=\frac{1}{4}\int\frac{(u-v)^4}{u^2v^2}\,du\,dv=$$

$$=\frac{1}{4}\int\frac{u^4-4u^3v+6u^2v^2-4uv^3+v^4}{u^2v^2}\,du\,dv=$$

$$=\frac{1}{4}\int\left(\frac{u^2}{v^2}-4\frac{u}{v}+6-4\frac{v}{u}+\frac{v^2}{u^2}\right)du\,dv=$$

$$=\frac{1}{4}\left[-\frac{u^3}{3v}-2u^2\lg v+6uv-2v^2\lg u-\frac{v^3}{3u}\right]$$

$$=\frac{-1}{4}\left[\frac{u^4+v^4}{3uv}-6uv+2(u^2\lg v+v^2\lg u)\right]$$

$$=-\frac{1}{4}\left[\frac{\alpha^2+\beta^2}{3\sqrt{\alpha\beta}}-6\sqrt{\alpha\beta}+(\alpha\lg\beta+\beta\lg\alpha)\right]$$

$$=-\frac{1}{4}\left[\frac{2(x^2-y^2)}{3r}-6r+\left[(x+iy)(\lg r-i\theta)+(x-iy)\lg(r+i\theta)\right]\right]$$

$$2(x\lg r+y\theta)\Big]$$

Tito: $\frac{2x}{3r}-\frac{x^3-xy^2}{3r^3}-6\frac{x}{r}+2\left(\lg r+\frac{x^2}{r^2}-\frac{y^2}{r^2}\right)$

$\cancel{\frac{2}{3r}}\;\cancel{\frac{x^2}{3r^3}}\;-\frac{2y}{3r}-\frac{x^2y-y^3}{3r^3}-6\frac{y}{r}+2\left(\frac{xy}{r^2}+\theta+y\frac{x}{r^2}\right)$

$\cancel{\frac{2}{3r}}-\frac{2x^2}{3r^3}-\frac{3x^2-y^2}{3r^3}+\frac{x^4-x^2y^2}{r^5}+\cancel{\frac{6}{r}}-\frac{6x^2}{r^3}+2\left(\frac{x}{r^2}+\frac{2x}{r^2}-\frac{2x^3}{r^4}+\frac{2xy^2}{r^4}\right)$

$-\cancel{\frac{2}{3r}}+\frac{2y^2}{3r^3}-\frac{x^2-3y^2}{3r^3}+\frac{x^2y^2-y^4}{r^5}-\frac{6}{r}-\frac{6y^2}{r^3}+2\left(\frac{x}{r^2}-\frac{2xy^2}{r^4}+\frac{x}{r^2}+\frac{x}{r^2}\right)$

$$= \frac{2(y^2-x^2) - 4x^2+4y^2+3x^2-3y^2}{3r^3} + \frac{6}{r} - + 8\frac{x}{r^2}$$

$$= \frac{3y^2-3x^2}{3r^3} - \quad = \frac{y^2-x^2}{r^3} \dots \quad = \frac{2}{r} - \frac{4x^2}{r^3} - \frac{6}{r} + \frac{8x}{r^2}$$

$$= \cancel{-\frac{4}{r} + \frac{8x}{r^2} - \frac{2x^2}{r^3}}$$

$$-\frac{4}{r} + \frac{8x}{r^2} - \frac{4x^2}{r^3}$$

$$= \frac{(y-ix)^2}{r^3} \qquad (\text{[illegible]})!$$

$$p = -\frac{1}{4}\left[\frac{2}{3}\,\frac{r^2(\cos^2-\sin^2)}{r} - 6r + 2(x\lg r + y\theta)\right]$$

$$= -\frac{1}{4}\left[\frac{2}{3}\cos 2\theta - 6 + 2\left[\cos\theta\lg r + \theta\sin\theta\right]\right]$$

Dodać można bob odjąć dowolne funkcje zadość czyniące $\Delta^2 f = 0$

$$z \lg z = x\lg r - y\theta$$

Ale mimo wszystko nie da się usunąć p od przybierania nieskończenie wielkich wartości w ∞ odległości. Jest to naturalnem zupełnie ponieważ prędkość w ∞ się staje ∞ a zatem też ciśnienie hydrodynamiczne.

~~[illegible]~~ Gdyby odrzucić część $\Delta^2(\ldots)$:

$$\Delta^2 p = -\frac{2}{r} + \frac{2x}{r^2} - \frac{2y^2}{r^3} = -\frac{4}{r} + \frac{2x}{r^2} + \frac{2x^2}{r^3}$$

$$= -\frac{4}{\sqrt{\alpha\beta}} + \frac{\alpha+\beta}{\alpha\beta} + \frac{(\alpha+\beta)^2}{2\sqrt{\alpha\beta}^3} = -\frac{4}{\sqrt{\alpha\beta}} + \frac{1}{\beta} + \frac{1}{\alpha} + \left(\frac{1}{2}\sqrt{\frac{\alpha}{\beta^3}} + \frac{1}{\sqrt{\alpha\beta}} + \frac{1}{2}\sqrt{\frac{\beta}{\alpha^3}}\right)$$

$$u = -2\frac{y^2 x}{r^4} \qquad \frac{\partial u}{\partial x} = -\frac{2y^2}{r^4} + 8\frac{y^2x^2}{r^6} \qquad \frac{\partial u}{\partial y} = -\frac{4yx}{r^4} + 8\frac{y^3x}{r^6}$$

$$v = -2\frac{y^3}{r^4} \qquad \frac{\partial v}{\partial x} = \cancel{\cdot} \cdot 8\frac{y^3x}{r^6} \qquad \frac{\partial v}{\partial y} = -6\frac{y^2}{r^4} + 8\frac{y^4}{r^6}$$

$$\frac{4y^4x}{r^8} - \frac{16y^4x^3}{r^{10}} + \frac{8y^4x}{r^8} - \frac{16y^6x}{r^{10}} = -\frac{4y^4x}{r^8} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$-16\frac{y^5x^2}{r^{10}} + 12\frac{y^5}{r^8} - 16\frac{y^7}{r^{10}} = -\frac{4y^5}{r^8} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$-4\left[\frac{y^4}{r^8} - 8\frac{y^4x^2}{r^{10}} + \frac{5y^4}{r^8} - \frac{8y^6}{r^{10}}\right] =$$

$$-4\left[-2\frac{y^4}{r^8}\right] = 8\frac{y^4}{r^8} = -\Delta^2 p$$

$$y = \frac{\alpha - \beta}{2i} \qquad r = \sqrt{\alpha\beta}$$

$$-4\frac{\partial^2 p}{\partial\alpha\partial\beta} = \frac{1}{2}\frac{(\alpha-\beta)^4}{(\alpha\beta)^4} = \frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 = \frac{1}{2}\left(\frac{1}{\alpha^4} - \frac{4}{\alpha^3\beta} + \frac{6}{\alpha^2\beta^2} - \frac{4}{\alpha\beta^3} + \frac{1}{\beta^4}\right)$$

$$-8p = -\frac{\beta}{3\alpha^3} + \frac{2}{\alpha^2}\lg\beta + \frac{6}{\alpha\beta} + \frac{2}{\beta^2}\lg\alpha - \frac{\alpha}{3\beta^3}$$

$$= \cancel{-\frac{1}{3r^3}\ldots + \frac{6}{r^2} \ldots}$$

$$+ \frac{2}{r^2}[i\theta(\cos 2\theta - i\sin 2\theta) + i\theta(\cos \ldots]$$

$$- 2\sin 2\theta$$

$$= -\frac{1}{3r^2}\left[e^{-i\theta - 3i\theta} + e^{i\theta + 3i\theta}\right] + \ldots$$

$$= -\frac{2}{3r^2}\cos 4\theta + \frac{6}{r^2} + \frac{2}{r^2}\left[\lg r (\cos 2\theta - i\sin 2\theta) + (\cos 2\theta + i\sin 2\theta)\ldots\right]$$

$$+ 2\cancel{\ldots}\cos 2\theta$$

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$$p = -\frac{1}{4r^2}\left\{3 - \frac{\cos 4\theta}{3} + 2\log r \cdot \cos 2\theta - 2\,\theta \sin 2\theta\right\}$$

$$\left.\begin{aligned}\cos 2(\tfrac{\pi}{2}-\varphi) &= \cos(\pi - 2\varphi) = -\cos 2\varphi \\ \cos 2(\tfrac{\pi}{2}+\varphi) &= \cos(\pi+2\varphi) = -\cos 2\varphi\end{aligned}\right\}$$

$$\frac{1}{\alpha^2}\log\alpha = \frac{1}{r^2}(\cos 2\theta - i\sin 2\theta)(\log r + i\,\theta)$$

$$= \frac{1}{r^2}\left[\cos 2\theta \cdot \log r + \theta \sin 2\theta\right]$$

$$\cos 4(\tfrac{\pi}{2}-\varphi) = \cos(2\pi - 4\varphi) = \cos 4\varphi = \cos 4(\tfrac{\pi}{2}+\varphi)$$

symetrycznie względem Y

$$\theta \sin 2\theta \quad \left|\begin{aligned}&(\tfrac{\pi}{2}-\varphi)\sin(\pi - 2\varphi) = +(\tfrac{\pi}{2}-\varphi)\sin 2\varphi \\ &(\tfrac{\pi}{2}+\varphi)\sin(\pi+2\varphi) = -(\tfrac{\pi}{2}+\varphi)\sin 2\varphi\end{aligned}\right.$$

asymetrycznie, zatem to trzeba wyrugować dodając

$$p = -\frac{1}{4r^2}\left\{3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta \cdot \log r\right\}$$

$$= \frac{-1}{4\alpha\beta}\left\{3 - \frac{1}{6}\left(\frac{\beta^2}{\alpha^2} + \frac{\alpha^2}{\beta^2}\right) + 2\left(\frac{\beta}{\alpha} + \frac{\alpha}{\beta}\right)(\log\alpha + \log\beta)\right\}$$

$$= \frac{-1}{4}\left\{\frac{3}{\alpha\beta} - \frac{1}{6}\left[\frac{\beta}{\alpha^3} + \frac{\alpha}{\beta^3}\right] + 2\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)(\log\alpha + \log\beta)\right\}$$

$\cos 4\theta =$ ~~...~~

~~$= \mathfrak{R}(\cos 2\theta + i\sin 2\theta)(\ldots)$~~

~~$= \mathfrak{R}\ldots$~~

$$\frac{\beta^2}{\alpha^2} + \frac{\alpha^2}{\beta^2} = e^{-4i\theta} + e^{+4i\theta} = 2\cos 4\theta$$

$$\cos 2\theta = \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$$

$$\operatorname{Pot} p = \frac{1}{16}\left[3\log\alpha \cdot \log\beta + \frac{1}{4}\frac{\beta^2}{\alpha^2} + \frac{1}{4}\frac{\alpha^2}{\beta^2} \underbrace{- \frac{2\beta}{\alpha}(\log\alpha + \cancel{1}) - \frac{2\alpha}{\beta}(\log\beta + \cancel{1}) - \frac{2\alpha}{\beta}(\log\alpha - \cancel{1}) - \frac{2\beta}{\alpha}(\log\beta - \cancel{1})}_{= -2\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)\log\alpha\beta}\right]$$

$$\int \frac{\log x}{x^2}\,dx = -\frac{\log x}{x} + \int \frac{1}{x^2}\,dx = -\frac{1}{x}(\log x + 1)$$

$$\int \log x\,dx = x(\log x - 1)$$

$$\int \frac{\log x}{x^3}\,dx = -\frac{1}{2}\frac{\log x}{x^2} + \frac{1}{2}\int\frac{dx}{x^3} = -\frac{1}{4x^2} - \frac{1}{2}\frac{\log x}{x^2}$$

~~$\frac{1}{x^2}(\log x + 1) = \ldots$~~

$$+\cancel{\frac{1}{2x^3}} \cancel{-\frac{1}{2x^3}} + \frac{\log x}{x^3}$$

$$\text{Pot } p = \frac{1}{16}\left[3 \log\alpha \log\beta + \frac{1}{4}\left(\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}\right) - 2\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)\log\alpha\beta\right]$$

$$= \frac{1}{16}\left[3(\log r + i\theta)(\log r - i\theta) + \frac{1}{4}\cos 4\theta - 4\cos 2\theta \log r\right]$$

$$= \frac{1}{16}\left\{\left[3(\log r)^2 + 3\theta^2\right] + \frac{\cos 4\theta}{4} - 4\cos 2\theta \log r\right\}$$

$$R(\log\alpha)^2 = (\log r)^2 - \theta^2$$

$$\text{Pot } p = \frac{1}{16}\left\{6(\log r)^2 + \frac{\cos 4\theta}{4} - 4\cos 2\theta \log r\right\}$$

$$u = \frac{\partial \text{ Pot } p}{\partial x} \qquad\qquad p = \frac{1}{4r^2}\left\{3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta . \log r\right\}$$

$$\frac{\partial p}{\partial x} = -\frac{x}{2r^4}\left\{3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta . \log r\right\}$$

$$+ \frac{1}{4r^2}\left[\left\{-\frac{4\sin 4\theta}{3} + 8\sin 2\theta \log r\right\}\frac{\sin\theta}{r} + 4\cos 2\theta . \frac{x}{r^2}\right]$$

$$= \frac{1}{2r^3}\left\{-\left[3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta . \log r\right]\cos\theta + 2\cos 2\theta . \cos\theta + \left[-\frac{2\sin 4\theta}{3} + 4\sin 2\theta \log r\right]\sin\theta\right\}$$

$$-\cos 2\theta \cos\theta + \sin 2\theta \sin\theta = -\cos 3\theta$$

$$\cos\theta \cos 4\theta - 2\sin\theta \sin 4\theta =$$

$$\frac{\partial p}{\partial \alpha} = \frac{\partial p}{\partial \alpha} + \frac{\partial p}{\partial \beta}$$

$$\frac{\partial p}{\partial y} = i\left(\frac{\partial p}{\partial \alpha} - \frac{\partial p}{\partial \beta}\right)$$

$x = \frac{\alpha+\beta}{2}$

$$= \frac{1}{4}\left[-\frac{3}{r^2} + \frac{x^4-6x^2y^2+y^4}{3r^6} - 4\frac{(x^2-y^2)}{r^4}\log r\right]$$

$$p = -\frac{1}{4}\left[\frac{3}{\alpha\beta} - \frac{1}{6}\left[\frac{\beta}{\alpha^3}+\frac{\alpha}{\beta^3}\right] + \left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)(\log\alpha + \log\beta)\right]$$

$$\frac{\partial p}{\partial \alpha} = -\frac{1}{4}\left[\frac{-3}{\alpha^2\beta} - \frac{3}{\alpha\beta^2} + \frac{3\beta}{6\alpha^4} - \frac{1}{6\alpha^3} - \frac{1}{6\beta^3} + \frac{3\alpha}{6\beta^4} + \left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) - 2\left(\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right)(\log\alpha + \log\beta)\right]$$

$$= -\frac{1}{4}\left[-\frac{2}{\alpha^2\beta} - \frac{2}{\alpha\beta^2} + \frac{\beta}{2\alpha^4} + \frac{\alpha}{2\beta^4} + \frac{5}{6\alpha^3} + \frac{5}{6\beta^3} + \cancel{\frac{2}{\alpha^3}} + \frac{2}{\beta^3} + \frac{2}{\alpha\beta^2} + \cancel{\frac{2}{\alpha^2\beta}} - 2\left(\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right)\log(\alpha\beta)\right]$$

$$u\frac{\partial u}{\partial \alpha} + v\frac{\partial u}{\partial y} = -\frac{4\left(\frac{\alpha-\beta}{2i}\right)^4\left(\frac{\alpha+\beta}{2}\right)}{(\alpha\beta)^4} = -\frac{1}{8}\frac{\left[\alpha^3-3\alpha^2\beta+3\alpha\beta^2-\beta^3\right]\left[\alpha^2-\beta^2\right]}{\alpha^4\beta^4} =$$

$$= -\frac{1}{8}\,\frac{\alpha^5-3\alpha^4\beta+3\alpha^3\beta^2-\alpha^2\beta^3-\alpha^3\beta^2+3\alpha^2\beta^3-3\alpha\beta^4+\beta^5}{\alpha^4\beta^4} =$$

$3/4 - \frac{1}{8} = \frac{5}{8}$

$$= -\frac{1}{8}\left[\frac{\alpha}{\beta^4} + \frac{\beta}{\alpha^4} - \frac{3}{\beta^3} - \frac{3}{\alpha^3} + \frac{2}{\alpha\beta^2} + \frac{2}{\alpha^2\beta}\right]$$

$-\frac{5}{3}+3 = \frac{4}{3}$

$-\frac{1}{4}+\frac{3}{8} = \frac{5}{8}$

$-\frac{5}{2}+2 = \frac{9}{2}$

$$+2\left(\frac{1}{\alpha\beta^2}+\frac{1}{\alpha^2\beta}\right)$$

$$\Delta^2 u = \frac{1}{8}\left[\cancel{-7\left(\frac{1}{\alpha\beta^2}+\frac{1}{\alpha^2\beta}\right)} - 2\left(\frac{\beta}{\alpha^4}+\frac{\alpha}{\beta^4}\right) + \frac{4}{3}\left(\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right) + 4\left(\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right)\log(\alpha\beta)\right]$$

$$u = \frac{1}{8}\left[\cancel{4\left(\frac{1}{\beta}\log\alpha + \frac{1}{\alpha}\log\beta\right)} - \frac{5}{6}\left(\frac{\beta^2}{\alpha^3}+\frac{\alpha^2}{\beta^3}\right) - \frac{5}{2}\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right) + 2\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right)\right.$$

$$\left.\cancel{+4\left(\frac{\beta\log\alpha}{\alpha^2}+\frac{\alpha\log\beta}{\beta^2}\right)} + 4\left(\frac{\beta\log\beta}{\alpha^2} - \frac{\beta}{\alpha^2}\right) + 4\left(\frac{\alpha\log\alpha}{\beta^2} - \frac{\alpha}{\beta^2}\right)\right]$$

$$= \frac{1}{8}\left[-\frac{9}{2}\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right) - \frac{5}{6}\left(\frac{\beta^2}{\alpha^3}+\frac{\alpha^2}{\beta^3}\right) + 4\left(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}\right) + 4\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right)\log\alpha\beta\right]$$

$$\cancel{u = \frac{1}{8}\left[\frac{1}{r}\cos 3\theta + \frac{5}{3r}\cos 5\theta - 8\,\frac{x\log r - y\theta}{r^2} - \frac{8}{r}\cos 3\theta \log r\right]}$$

$$\Delta^2 u = 4\frac{\partial^2 u}{\partial\alpha\,\partial\beta} = \frac{1}{4}\left[\left(\frac{1}{\beta^2}+\frac{1}{\alpha^2}\right) - \left(\frac{\beta}{\alpha^4}+\frac{\alpha}{\beta^4}\right) + \frac{2}{3}\left(\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right) + 2\left(\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right)\log\alpha\beta\right]$$

$$(u) = \frac{1}{16}\left[-\left(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}\right) + \frac{1}{6}\left(\frac{\beta^2}{\alpha^3}+\frac{\alpha^2}{\beta^3}\right) - \frac{1}{3}\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right) + 2\left[-\frac{1}{4}\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right) - \right.\right.$$

$$\left.\left. -\frac{1}{2}\left(\frac{\beta\log\alpha}{\alpha^2}+\frac{\alpha\log\beta}{\beta^2}\right) - \frac{1}{2}\left(\frac{\alpha\log\alpha}{\beta^2}+\frac{\beta\log\beta}{\alpha^2}\right) + \frac{1}{2}\left(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\right)\right]\right]$$

$$= \frac{1}{16}\left[+\frac{1}{6}\left(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\right) + \frac{1}{6}\left(\frac{\alpha^2}{\beta^3}+\frac{\beta^2}{\alpha^3}\right) - \left(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right)\log\alpha\beta\right]$$

$$(u) = \frac{1}{8}\left[\frac{1}{6}\,\frac{\cos 3\theta}{r} + \frac{1}{6}\,\frac{\cos 5\theta}{r} - \frac{x\log r - y\theta}{r^2} - 2\,\frac{\cos 3\theta}{r}\log r\right]$$

$$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} = -\frac{1}{4}\left[\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)(1+2\log 2) - \left(\frac{\log\alpha}{\beta^2}+\frac{\log\beta}{\alpha^2}\right)\right]$$

$$\cos 3\left(\tfrac{\pi}{2}-\varphi\right) = \cos\left(\tfrac{3\pi}{2}-3\varphi\right) = -\sin 3\varphi$$

$$\cos 3\left(\tfrac{\pi}{2}+\varphi\right) = \cos\left(\tfrac{3\pi}{2}+3\varphi\right) = \sin 3\varphi$$

$$\frac{1}{z} = \frac{\cos\theta - i\sin\theta}{r}$$

$$\left(\tfrac{\pi}{2}-\varphi\right)\cos\varphi$$

$$\left(\tfrac{\pi}{2}+\varphi\right)\cos\varphi \qquad \boxed{-\frac{\frac{\pi}{2}\sin\theta}{r}}$$

$$= -\frac{1}{4}\left[\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{\log\alpha}{\alpha^2}+\frac{\log\beta}{\beta^2}\right] + \cancel{\frac{\log\beta}{\alpha}+\frac{\log\alpha}{\beta^2}} - \cancel{\left(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha^2}\right)}$$

$$= -\frac{1}{4}\left[R\left(\frac{1}{\alpha^2}\right) + R\left(\frac{\log\alpha}{\alpha^2}\right)\right] = -\frac{1}{2}\left[\frac{\cos 2\theta}{r^2} + \frac{\log r}{r^2}\right]$$

$$\left(\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial\beta}\right)\left[f(\alpha)+f(\beta)\right] + \left(\frac{\partial}{\partial\alpha}-\frac{\partial}{\partial\beta}\right)\left[\varphi(\alpha)-\varphi(\beta)\right]$$

$$= f'(\alpha)+f'(\beta)+\varphi'(\alpha)+\varphi'(\beta)$$

$$= -\frac{1}{4}\,R\left(\frac{\log\alpha+1}{\alpha^2}\right)$$

$$f = -\frac{\log x}{x} \qquad \varphi = -\frac{1}{x}$$

$$\phi = -\frac{1}{4}\left\{ \frac{3}{\alpha\beta} - \frac{1}{6}\left[\frac{\beta}{\alpha^3} + \frac{\alpha}{\beta^3}\right] + \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right) \lg \alpha\beta \right\} \qquad \left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)(\lg\alpha + \lg\beta)$$

$$+\frac{\partial\psi}{\partial y} = -\frac{i}{4}\left\{ -\frac{3}{\alpha^2\beta} + \frac{3}{\alpha\beta^2} + \cancel{\frac{\beta}{2\alpha^4}} + \frac{1}{6\alpha^3} - \frac{1}{6\beta^3} - \cancel{\frac{\alpha}{2\beta^4}} + \frac{-2}{\alpha^3}(\lg\alpha + \lg\beta) + \frac{1}{\alpha^3} + \frac{1}{\alpha\beta^2} \right.$$

$$\left. + \frac{2}{\beta^3}(\lg\alpha + \lg\beta) - \frac{1}{\alpha^2\beta} - \frac{1}{\beta^3} \right\}$$

$$= -\frac{i}{4}\left\{ 4\left(\frac{1}{\alpha\beta^2} - \frac{1}{\alpha^2\beta}\right) + \frac{7}{6}\left(\frac{1}{\alpha^3} - \frac{1}{\beta^3}\right) + \frac{1}{2}\left(\frac{\beta}{\alpha^4} - \frac{\alpha}{\beta^4}\right) + 2\left(\frac{1}{\beta^3} - \frac{1}{\alpha^3}\right)\lg\alpha\beta \right\}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{4}{(\alpha\beta)^4}\left(\frac{\alpha-\beta}{2i}\right)^5 = i\,\frac{(\alpha-\beta)^5}{8(\alpha\beta)^4} = \frac{i}{8}\left\{ \frac{\alpha}{\beta^4} - \frac{5}{\beta^3} + \frac{10}{\alpha\beta^2} - \frac{10}{\alpha^2\beta} + \frac{5}{\alpha^3} - \frac{\beta}{\alpha^4} \right\}$$

$-\frac{7}{3}+5 \qquad -\frac{13}{3}+5$

$$\Delta^2 v = \frac{i}{8}\left\{ \cancel{20\left(\frac{1}{\alpha\beta^2} - \frac{1}{\alpha^2\beta}\right)} + \frac{8}{3}\left(\frac{1}{\alpha^3} - \frac{1}{\beta^3}\right) - 2\left(\frac{\beta}{\alpha^4} - \frac{\alpha}{\beta^4}\right) + 4\left(\frac{1}{\beta^3} - \frac{1}{\alpha^3}\right)\lg\alpha\beta \right\}$$

$$2\left(\frac{1}{\alpha\beta^2} - \frac{1}{\alpha^2\beta}\right)$$

Probe:

$$\Delta^2\frac{\partial v}{\partial y} = -\Delta^2\frac{\partial u}{\partial x} = 2\left(\frac{2}{\alpha^2\beta^2} + \frac{2}{\alpha\beta^3} + \frac{2}{\alpha^3\beta}\right) + 2\left(\frac{1}{\alpha^4} + \frac{1}{\beta^4} - \frac{4\beta}{\alpha^5} - \frac{4\alpha}{\beta^5}\right) + 4\left(\frac{1}{\alpha\beta^3} + \frac{1}{\beta\alpha^3}\right)$$
$$-12\left(\frac{1}{\alpha^4} + \frac{1}{\beta^4}\right)\lg\alpha\beta$$

$$\Delta^2\psi = \frac{i}{4}\left[\left(\frac{1}{\alpha\beta^2} - \frac{1}{\alpha^2\beta}\right) - \left(\frac{\beta}{\alpha^4} - \frac{\alpha}{\beta^4}\right) + \frac{4}{3}\left(\frac{1}{\alpha^3} - \frac{1}{\beta^3}\right) + 2\left(\frac{1}{\alpha^3} - \frac{1}{\beta^3}\right)\lg\alpha\beta\right]$$

$$(\psi) = \frac{i}{16}\left[-\frac{\lg\alpha}{\beta} + \frac{\lg\beta}{\alpha} + \frac{1}{6}\left(\frac{\beta^2}{\alpha^3} - \frac{\alpha^2}{\beta^3}\right) - \frac{2}{3}\left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right) + 2\left\{ -\frac{1}{4}\left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right) - \right.\right.$$

$$\left.\left. -\frac{1}{2}\left(\frac{\beta\lg\alpha}{\alpha^2} - \frac{\alpha\lg\beta}{\beta^2}\right) - \frac{1}{2}\left(\frac{\beta\lg\beta}{\alpha^2} - \frac{\alpha\lg\alpha}{\beta^2}\right) + \frac{1}{2}\left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right)\right\}\right]$$

$$= \frac{i}{16}\left[-\frac{1}{6}\left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right) + \frac{1}{6}\left(\frac{\beta^2}{\alpha^3} - \frac{\alpha^2}{\beta^3}\right) - \left(\frac{\lg\alpha}{\beta} - \frac{\lg\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right)\lg\alpha\beta \right]$$

$$= \frac{1}{8}\left[-\frac{\sin 3\theta}{6r} + \frac{\sin 5\theta}{6r} + \frac{yr + x\theta}{r^2} - 2\frac{\sin 3\theta}{r}\lg r \right]$$

$$\delta\mu = \frac{\alpha^4+\beta^4}{3(\alpha\beta)^3} + \frac{6}{\alpha\beta} + 2 - -$$

$$= \frac{2(x^4-6x^2y^2+y^4)}{3r^6} + \frac{6}{r^2} + 2\,\frac{[(x^2-y^2)\log r - 2xy\theta]}{r^4} + \cancel{\frac{2xy}{r}}$$

$$\frac{\delta\partial\mu}{\partial x} = \frac{8x^3-24xy^2}{3r^6} - \frac{12(x^5-6x^3y^2+xy^4)}{3r^8} - \frac{12x}{r^4} + 2\,\frac{2x\log r + x - 2y\theta +}{r^4}$$

$$- 8\,\frac{[(x^2-y^2)\log r - 2xy\theta]\,x}{r^6}$$

$$= x\left[\frac{8x^2-24y^2}{3r^6} - \frac{4(x^4-6x^2y^2+y^4)}{r^8} - \frac{12}{r^4} + \frac{24}{r^4}\right] + \log r\left[\frac{4x}{r^4} - \frac{8(x^2-y^2)x}{r^6}\right]$$

$$+\left(\frac{16x^2y}{r^6} - \frac{4y}{r^4}\right)\theta$$

$$= 4\log r\;\frac{r^2-2x^2+2y^2}{r^6}\,x + \frac{4\theta}{r^6}\,y\,[4x^2-x^2-y^2] + x[\;\cdot\;\cdot\quad]$$

$$= \frac{4}{r^6}\left[x(3y^2-x^2)\log r + 4y(3x^2-y^2)\,\theta\right] + x\left[\frac{8x^2+24x^2}{3r^6} - \frac{8}{r^4} - \frac{12}{r^4} + \frac{2}{r^4}\right.$$

$$\left. - \frac{4(x^4+6x^4+x^4)}{r^8} + \frac{32x^2}{r^6} - \frac{4}{r^4}\right]$$

$$= -\frac{22}{r^4} + \frac{32\cdot 4x^2}{3r^6} - \frac{32x^4}{r^8}$$

$$~~(\upsilon) = \frac{1}{48}\left[\frac{-\sin 3\theta + \sin 5\theta}{r}\right] + \frac{\theta\sin\theta + 2\cos\theta - 2\sin 3\theta\log r}{8r}~~$$

$$4\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{\partial p}{\partial\alpha} + \frac{\partial p}{\partial\beta} + F$$
$$4\frac{\partial^2 v}{\partial\alpha\partial\beta} = i\left(\frac{\partial p}{\partial\alpha} - \frac{\partial p}{\partial\beta}\right) + G$$

$$4\frac{\partial^2 p}{\partial\alpha\partial\beta} = -\left[\frac{\partial F}{\partial\alpha} + \frac{\partial F}{\partial\beta} + i\frac{\partial G}{\partial\alpha} - i\frac{\partial G}{\partial\beta}\right]$$
$$= -\left[\frac{\partial}{\partial\alpha}(F+iG) + \frac{\partial}{\partial\beta}(F-iG)\right]$$
$$4p = -\int(F+iG)\,d\beta - \int(F-iG)\,d\alpha + \cdots$$

$$4\left(\frac{\partial p}{\partial\alpha} + \frac{\partial p}{\partial\beta}\right) = -\cancel{(F-iG)} - \cancel{(F+iG)} - \int\frac{\partial(F+iG)}{\partial\alpha}\,d\beta - \int\frac{\partial(F-iG)}{\partial\beta}\,d\alpha$$

$$4\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{F}{2} - \frac{1}{4}\int\frac{\partial}{\partial\alpha}(F+iG)\,d\beta - \frac{1}{4}\int\frac{\partial(F-iG)}{\partial\beta}\,d\alpha$$
$$u = \frac{1}{8}\int F\,d\alpha\,d\beta - \frac{1}{16}\iint(F+iG)\,d\beta\,d\beta - \frac{1}{16}\iint(F-iG)\,d\alpha\,d\alpha +$$

Poprawione u, v, tak aby spełniały warunek div = 0 :

$$((u)) = \frac{1}{16}\left[\frac{1}{6}\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + \frac{1}{6}\left(\frac{\alpha^2}{\beta^3} + \frac{\beta^2}{\alpha^3}\right) - \underbrace{\left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right)\log\alpha\beta}_{\cancel{\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right)\log\alpha\beta}} - 2\left(\frac{\log\alpha}{\alpha} + \frac{\log\beta}{\beta}\right)\right]$$

$$\cancel{(\alpha+\beta)\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\log\alpha\beta}$$
$$= \cancel{8\cos\theta\cdot\frac{\cos 2\theta}{r^2}\log r}$$

$$= \frac{1}{8}\left[\frac{\cos 3\theta + \cos 5\theta}{6r} - \frac{x\log r - y\theta}{r^2} - \frac{2\cos 3\theta}{r}\log r + \frac{2(x\log r + y\theta)}{r^2}\right]$$

$$= \cancel{\frac{1}{48}\left[\frac{\cos 3\theta + \cos 5\theta}{r}\right] - \frac{1}{2}\frac{\cos\theta\cos 2\theta}{r}\log r}$$

$$((v)) = \frac{i}{16}\left[\frac{1}{6}\left(\frac{\alpha}{\beta^2} - \frac{\beta}{\alpha^2}\right) + \frac{1}{6}\left(\frac{\beta^2}{\alpha^3} - \frac{\alpha^2}{\beta^3}\right) - \left(\frac{\log\alpha}{\beta} - \frac{\log\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right)\log\alpha\beta + 3\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)\right]$$
$$= \frac{1}{8}\left[\frac{3\sin\theta}{r} - \frac{\sin 3\theta - \sin 5\theta}{6r} + \frac{\theta x + y\log r}{r^2} - \frac{2\sin 3\theta}{r}\log r\right]$$

$$u = \frac{1}{8r}\left[\frac{\cos 3\theta + \cos 5\theta}{6} + (\cos\theta - 2\cos 3\theta)\log r + 3\theta\sin\theta\right] \qquad + \frac{\cos\theta\log r + \theta\sin\theta}{r}$$

$$v = \frac{1}{8r}\left[\frac{-\sin 3\theta + \sin 5\theta}{6} + 3\sin\theta + (\sin\theta - 2\sin 3\theta)\log r + \theta\cos\theta\right] \quad + \frac{\sin\theta\log r - \theta\cos\theta}{r}$$

Do tego dodać dowolny ruch potencjalny $\quad u = Rf + \cancel{I\varphi}$

$$v = -If + \cancel{R\varphi}$$

$$\frac{\log\alpha}{\alpha} = \frac{\cos\theta\log r + \theta\sin\theta}{r} + i\,\frac{-\sin\theta\log r + \theta\cos\theta}{r}$$

$$u = \frac{1}{8r}\left[\frac{\cos 3\theta + \cos 5\theta}{6} + 2(\cos\theta - \cos 3\theta)\log r + 4\theta\sin\theta\right]$$

$$v = \frac{1}{8r}\left[\frac{-\sin 3\theta + \sin 5\theta}{6} + 3\sin\theta + 2(\sin\theta - \sin 3\theta)\log r\right]$$

Co dalej? ~~Dodając do~~ $\cancel{\begin{cases} u & -\frac{\cos\theta}{3r} \\ v & +\frac{\sin\theta}{3r}\end{cases}}$ Nie można!

starając się by się rozwiązanie $u=v=0$ dla ściany $\theta = \begin{cases}0\\ \pi\end{cases}$ i dla $r=\infty$

Ale czy to jedyne możliwe rozwiązanie?

$\theta = \frac{\pi}{2} \qquad u = \frac{1}{8r}\left[2\pi\right.$

$\qquad\qquad v = \frac{1}{8r}\left[-\frac{1}{3} + 3 + 4\log r\right]$

Więc asymetrja! w u

$$\int (u\cos\theta + v\sin\theta)\, r\,d\theta = \left[1 - (\underbrace{\cos\theta\cos 3\theta + \sin\theta\sin 3\theta}_{})\right]\log r$$

$$1 - \cos 2\theta = 2\sin^2\theta$$

więc całkowita ilość wypływającego byłaby ∞ !? dla lim $r=0$

$\cancel{\int_0^\pi \cos 2\theta\, d\theta}$

$$1 - 2\cos 2\theta = 1 - 2(\cos^2 - \sin^2) = \cancel{\sin^2 - \cos^2 + \ldots}$$

$$= 4\sin^2 - 2$$

$$\int_0^{\frac{\pi}{2}} (2\sin^2 - 1)$$

Uogólnienie

$$u = 2y\, J\left(\frac{2}{\sqrt{z^2-1}}\right) = \frac{\alpha-\beta}{i}\;\frac{\frac{\alpha}{\sqrt{\alpha^2-1}} - \frac{\beta}{\sqrt{\beta^2-1}}}{2i} = \frac{1}{2}(\alpha-\beta)\left(\frac{\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}}\right)$$

$$v = -2x\cdot J\frac{2}{\sqrt{z^2-1}} + 2J\frac{1}{\sqrt{z^2-1}} = -(\alpha+\beta)\frac{\frac{\alpha}{\sqrt{\alpha^2-1}} - \frac{\beta}{\sqrt{\beta^2-1}}}{2i} + \frac{\frac{1}{\sqrt{\alpha^2-1}} - \frac{1}{\sqrt{\beta^2-1}}}{i}$$

$$\frac{\partial u}{\partial x} = \frac{1}{2}(\alpha-\beta)\left[\frac{1}{\sqrt{\beta^2-1}} - \frac{\beta^2}{\sqrt{\beta^2-1}^3} - \frac{1}{\sqrt{\alpha^2-1}} + \frac{\alpha^2}{\sqrt{\alpha^2-1}^3}\right]$$

$$= \frac{\alpha-\beta}{2i}\left(\frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right) + \frac{\sqrt{\beta^2-1}-\sqrt{\alpha^2-1}}{i}$$

$$= \frac{1}{2}(\alpha-\beta)\left[\frac{-1}{\sqrt{\beta^2-1}^3} + \frac{1}{\sqrt{\alpha^2-1}^3}\right]$$

$$\frac{\partial v}{\partial y} = \frac{-1}{2}\left[(\alpha+\beta)\left[\frac{1}{\sqrt{\alpha^2-1}} - \frac{\alpha^2}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}} - \frac{\beta^2}{\sqrt{\beta^2-1}^3}\right] - \left[\frac{\alpha}{\sqrt{\alpha^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}^3}\right]\right.$$

$$= \frac{1}{2}(\alpha+\beta)\left[\frac{1}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}^3}\right] - \left[\frac{\alpha}{\sqrt{\alpha^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}^3}\right]$$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ stimmt

$$\frac{\partial u}{\partial y} = \frac{i}{2}\left[2\left(\frac{\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}}\right) + (\alpha-\beta)\left(\frac{-1}{\sqrt{\alpha^2-1}} + \frac{\alpha^2}{\sqrt{\alpha^2-1}^3} - \frac{1}{\sqrt{\beta^2-1}} + \frac{\beta^2}{\sqrt{\beta^2-1}^3}\right)\right]$$

$$\cancel{= \frac{i}{2}\left[\ldots\right]}$$

$$\frac{\partial v}{\partial x} = \frac{1}{2i}\left[-2\left(\frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right) - (\alpha+\beta)\left(\frac{1}{\sqrt{\alpha^2-1}} - \frac{1}{\sqrt{\beta^2-1}} - \frac{\alpha^2}{\sqrt{\alpha^2-1}^3} + \frac{\beta^2}{\sqrt{\beta^2-1}^3}\right)\right]$$

$$+\frac{1}{i}\left[\frac{-\alpha}{\sqrt{\alpha^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}^3}\right]$$

$$= \frac{1}{2i}\left\{-2\left(\frac{\alpha^3}{\sqrt{\alpha^2-1}^3} - \frac{\beta^3}{\sqrt{\beta^2-1}^3}\right) - (\alpha+\beta)\left(\frac{-1}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}^3}\right)\right\}$$

$$\frac{\partial u}{\partial y} = \frac{i}{2}\left\{2\left(\frac{\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}}\right) + (\alpha-\beta)\left(\frac{1}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}^3}\right)\right\}$$

$u = -\frac{y\xi}{2}$

$v = +\frac{x\xi}{2} + 2\,\mathfrak{I}\,\mathfrak{F}$

$H' - S' = \varphi'(\alpha)$

$\xi = -4\,\mathfrak{I}\varphi'(\alpha)$

$= 2\,\frac{\varphi'(\beta) - \varphi'(\alpha)}{i}$

$\mathfrak{F} = \int (H'' - S'')\,\alpha\, d\alpha$

$= (H' - S')\,\alpha - \int (H' - S')\, d\alpha$

$= \alpha\,(H' - S') - (H - S)$

$\frac{z^2}{\sqrt{z^2-1}} - \sqrt{z^2-1} = \frac{1}{\sqrt{z^2-1}}$

$\mathfrak{I}\,\mathfrak{F} =$

$u = -\frac{\alpha + \beta}{2i}\,\frac{\varphi'(\beta) - \varphi'(\alpha)}{i} = \frac{\alpha - \beta}{2}\left[\varphi'(\beta) - \varphi'(\alpha)\right]$

$v = \frac{\alpha + \beta}{2}\,\frac{\varphi'(\beta) - \varphi'(\alpha)}{i} + \frac{\alpha\,\varphi'(\alpha) - \beta\,\varphi'(\beta)}{i} - \frac{\varphi(\alpha) - \varphi(\beta)}{i}$

$= \frac{\alpha\,\varphi'(\beta) - \beta\,\varphi'(\alpha) + \alpha\,\varphi'(\alpha) - \beta\,\varphi'(\beta)}{2i} - \frac{\varphi(\alpha) - \varphi(\beta)}{i}$

$= \cancel{\alpha\left[\varphi'(\alpha) + \varphi'(\beta)\right] - \beta\,\varphi(\alpha)}$

$= \frac{\alpha - \beta}{2i}\left[\varphi'(\alpha) + \varphi'(\beta)\right] - \frac{\varphi(\alpha) - \varphi(\beta)}{i}$

$u^2 = \frac{(\alpha - \beta)^2}{4}\left[\varphi'(\beta) - \varphi'(\alpha)\right]^2 = \frac{\alpha^2 - 2\alpha\beta + \beta^2}{4}\left[\varphi'(\alpha)^2 - 2\,\varphi'(\alpha)\,\varphi'(\beta) + \varphi'(\beta)^2\right]$

$2u\frac{\partial u}{\partial x} = \cancel{\frac{2\alpha - 2\beta - 2\alpha + 2\beta}{}}\,[\ \cdots\ ] + \frac{(\alpha - \beta)^2}{2}\Big[\varphi'(\alpha)\,\varphi''(\alpha) - \varphi''(\alpha)\,\varphi'(\beta) - \varphi'(\alpha)\,\varphi''(\beta) + \varphi'(\beta)\,\varphi''(\beta)\Big]$

$u\frac{\partial u}{\partial x} = \frac{(\alpha - \beta)^2}{4}\left[\varphi'(\alpha) - \varphi'(\beta)\right]\left[\varphi''(\alpha) - \varphi''(\beta)\right]$

$\alpha^2\ [\alpha\ \ \alpha\ \beta]$

$2\alpha = \frac{\partial\beta}{} - \frac{\partial\alpha}{} + \frac{\partial}{}$

$$\frac{\partial u}{\partial y} = \frac{i}{2}\left[2\left[\varphi'(\beta) - \varphi'(\alpha)\right] + (\alpha-\beta)\left[\varphi''(\alpha) + \varphi''(\beta)\right]\right]$$

$$v\frac{\partial u}{\partial y} = \frac{1}{2}\; i(\alpha-\beta)\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi'(\beta)-\varphi'(\alpha)\right] - \frac{(\alpha-\beta)^2}{4}\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right]$$

$$- \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi'(\beta)-\varphi'(\alpha)\right] + \frac{\alpha-\beta}{2}\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right]$$

$$u\frac{\partial u}{\partial y} = -4\, J\varphi \cdot J\varphi' + 4y\left(J\varphi'\, R\varphi' - J\varphi\, R\varphi''\right) + 4y^2 R\varphi'\, R\varphi''$$

$$= 4\left(-J\varphi + yR\varphi'\right)\left(J\varphi' + yR\varphi''\right)$$

daje mnożniki ar $\frac{4xy^4}{28}$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = +\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi'(\alpha)-\varphi'(\beta)\right] + \frac{\alpha-\beta}{2}\Big\{\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right] -$$

$$-\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi'(\alpha)-\varphi'(\beta)\right]\Big\} + \frac{(\alpha-\beta)^2}{4}\Big\{\left[\varphi'(\alpha)-\varphi'(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right] -$$

$$\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right]\Big\}$$

$$\frac{\partial}{\partial x} \quad = \left[\varphi'(\alpha)-\varphi'(\beta)\right]^2 + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right] + \frac{\alpha-\beta}{2}\Big\{\cancel{\left[\varphi'(\alpha)-\varphi'(\beta)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right]} +$$

$$+\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi'''(\alpha)+\varphi'''(\beta)\right] - \cancel{\left[\varphi''(\alpha)+\varphi''(\beta)\right]\left[\varphi'(\alpha)-\varphi'(\beta)\right]} - \left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]\Big\} +$$

$$+\left(\frac{\alpha-\beta}{2}\right)^2\Big\{\left[\varphi''(\alpha)-\varphi''(\beta)\right]^2 + \left[\varphi'(\alpha)-\varphi'(\beta)\right]\left[\varphi'''(\alpha)-\varphi'''(\beta)\right] - \left[\varphi''(\alpha)+\varphi''(\beta)\right]^2 - \left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi'''(\alpha)+\varphi'''(\beta)\right]\Big\}$$

$$v^2 = -\frac{(\alpha-\beta)^2}{4}\left[\varphi'(\alpha)+\varphi'(\beta)\right]^2 - \left[\varphi(\alpha)-\varphi(\beta)\right]^2 + (\alpha-\beta)\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi(\alpha)-\varphi(\beta)\right]$$

$$\frac{1}{i} v\frac{\partial v}{\partial y} = -\cancel{(\alpha-\beta)\left[\varphi'(\alpha)+\varphi'(\beta)\right]^2} - \frac{(\alpha-\beta)^2}{4}\left[\cancel{2}\varphi'(\alpha)\varphi''(\alpha) + \cancel{2}\varphi''(\alpha)\varphi'(\beta) - \cancel{2}\varphi'(\alpha)\varphi''(\beta)\right.$$

$$\left. - \cancel{2}\varphi'(\beta)\varphi''(\beta)\right] + 2\cancel{\left[\varphi'(\alpha)+\varphi'(\beta)\right]}\left[\varphi(\alpha)-\varphi(\beta)\right] + \frac{\alpha-\beta}{2}\left[\varphi''(\alpha)-\varphi''(\beta)\right]\left[\varphi(\alpha)-\varphi(\beta)\right]$$

$$+\cancel{(\alpha-\beta)\left[\varphi'(\alpha)+\varphi'(\beta)\right]^2} - \cancel{2\varphi(\alpha)\varphi'(\alpha)} + \cancel{2\varphi'(\alpha)\varphi(\beta)} - \cancel{2\varphi(\alpha)\varphi'(\beta)} + \cancel{2\varphi(\beta)\varphi'(\beta)}$$

$$+ \cancel{2\varphi(\alpha)\varphi'(\alpha)} - 2\varphi'(\alpha)\varphi(\beta) + 2\varphi(\alpha)\varphi'(\beta) - \cancel{2\varphi(\beta)\varphi'(\beta)}$$

$$= -\frac{(\alpha-\beta)^2}{4}\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right] + \frac{\alpha-\beta}{2}\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]$$

$$i\, u\frac{\partial v}{\partial x} = \frac{\alpha-\beta}{2}\left[\varphi'(\beta)-\varphi'(\alpha)\right]\left\{\frac{\alpha-\beta}{2}\left[\varphi''(\alpha)+\varphi''(\beta)\right] - \varphi'(\alpha)+\varphi'(\beta)\right\}$$

$$= \frac{(\alpha-\beta)^2}{4}\left[\varphi'(\beta)-\varphi'(\alpha)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right] + \frac{\alpha-\beta}{2}\left[\varphi'(\alpha)-\varphi'(\beta)\right]^2$$

$$i\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right] = \frac{(\alpha-\beta)^2}{4}\left\{\left[\varphi'(\beta)-\varphi'(\alpha)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right] - \left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\beta)-\varphi''(\alpha)\right]\right\}$$

$$+\frac{\alpha-\beta}{2}\left\{\left[\varphi'(\alpha)-\varphi'(\beta)\right]^2 + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]\right\}$$

$$\varphi''(\alpha)\varphi'(\beta) - \varphi''(\beta)\varphi'(\alpha)$$

$$= \frac{(\alpha-\beta)^2}{4}\left\{\cancel{-2\varphi'(\alpha)\varphi''(\alpha) + 2\varphi'(\beta)\varphi''(\beta)}\right\} +$$

$$+\frac{\alpha-\beta}{2}\left\{\left[\varphi'(\alpha)-\varphi'(\beta)\right]^2 - \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]\right\}$$

$\alpha^2 - 2\alpha\beta + \beta^2$

$2\alpha - 2\beta$

$\frac{4y^5}{18}$

$$\frac{\partial}{\partial y}\left[\quad\right] = \left(\cancel{\frac{\alpha-\beta}{2}}\right)\left\{\cancel{-2\varphi'(\alpha)\varphi'(\alpha) + 2\varphi'(\beta)\varphi''(\beta)}\right\} + \left(\frac{\alpha-\beta}{2}\right)^2\left\{-2\varphi'(\alpha)^2 - 2\varphi(\alpha)\varphi''(\alpha)\right.$$

$$\left. -2(\varphi'(\beta))^2 - 2\varphi(\beta)\varphi''(\beta)\right\} + \left[\varphi'(\alpha)-\varphi'(\beta)\right]^2 + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right] +$$

$$(\alpha-\beta)\left\{\cancel{\varphi'(\alpha)\varphi''(\alpha) - \varphi''(\alpha)\varphi'(\beta) + \varphi'(\alpha)\varphi''(\beta) - \varphi'(\beta)\varphi''(\beta)} + \cancel{\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi'(\alpha)-\varphi'(\beta)\right]}\right.$$

$$\left. +\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi'''(\alpha)+\varphi'''(\beta)\right]\right\}$$

$$\cancel{\Delta^2 p = [\varphi(\alpha, \beta + \psi(\beta))]^2}$$

$$\Delta^2 u = \frac{\partial p}{\partial x} + F$$

$$\Delta^2 v = \frac{\partial p}{\partial y} + G$$

$$\Delta^2 p = -\left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y}\right)$$

$$\cancel{4\frac{\partial^2 \varphi}{\partial\alpha\partial\beta} = 4\frac{\partial \varphi}{\partial\alpha} = F + iG}$$

$$4\frac{\partial p}{\partial \alpha} = -\beta\left(\frac{\partial F}{\partial \alpha} + i\frac{\partial G}{\partial \alpha}\right) - F + iG - \frac{\partial}{\partial \alpha}\left[\int F\,d\beta + iG\,d\beta\right]$$

$$4\frac{\partial p}{\partial \beta} = -\alpha\left(\frac{\partial F}{\partial \beta} + i\frac{\partial G}{\partial \beta}\right) - F + iG$$

$$4\frac{\partial^2 p}{\partial\alpha\partial\beta} = -\left(\frac{\partial F}{\partial \alpha} + \frac{\partial F}{\partial \beta} + i\frac{\partial G}{\partial \alpha} - i\frac{\partial G}{\partial \beta}\right)$$

$$16\frac{\partial^2 u}{\partial\alpha\partial\beta} = -\beta\frac{\partial F}{\partial \alpha} - \alpha\frac{\partial F}{\partial \beta} - i\left[\beta\frac{\partial G}{\partial \alpha} - \alpha\frac{\partial G}{\partial \beta}\right] + 2F$$

$$16u = -\frac{\beta^2}{2}F - \frac{\alpha^2}{2}F - i\left[\frac{\beta^2}{2}G - \frac{\alpha^2}{2}G\right] + 2\iint F\,d\alpha\,d\beta$$

$$\cancel{= -\frac{\alpha^2+\beta^2}{2}(F + iG) + 2\iint (F + iG)\,d\alpha\,d\beta + u_0}$$

$$16\frac{\partial^2 v}{\partial\alpha\partial\beta} = i\left[-\beta\frac{\partial F}{\partial \alpha} + \alpha\frac{\partial F}{\partial \beta}\right] - i\left[\beta\frac{\partial G}{\partial \alpha} + \alpha\frac{\partial G}{\partial \beta}\right] + 2G$$

$$16v = \frac{\beta^2}{2}G + \frac{\alpha^2}{2}G + i\left[-\frac{\beta^2}{2}F + \frac{\alpha^2}{2}F\right] + 2\iint G\,d\alpha\,d\beta$$

$$u = \frac{1}{32}\left\{-(\alpha^2+\beta^2)F + i(\alpha^2-\beta^2)G + 4\iint F\,d\alpha\,d\beta\right\} + u_0$$

$$v = \frac{1}{32}\left\{(\alpha^2+\beta^2)G + i(\alpha^2-\beta^2)F + 4\iint G\,d\alpha\,d\beta\right\} + v_0$$

$u = u_{10} + u_{20}$

$v = v_{10} + v_{20}$

$$F = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \alpha}(u+iv) + \frac{\partial u}{\partial \beta}(u-iv)$$

$$G = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial v}{\partial \alpha}(u+iv) + \frac{\partial v}{\partial \beta}(u-iv)$$

$$F + iG = \frac{\partial(u+iv)}{\partial \alpha}(u+iv) + \frac{\partial(u+iv)}{\partial \beta}(u-iv)$$

$$F - iG = \frac{\partial(u-iv)}{\partial \alpha}(u+iv) + \frac{\partial(u-iv)}{\partial \beta}(u-iv)$$

$$\Delta^2\underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}_{\zeta} = \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y}$$

$$\Delta^4\varphi =$$

$$= \frac{\partial}{\partial \alpha}G + \frac{\partial G}{\partial \beta}$$

$$-i\frac{\partial F}{\partial \alpha} + i\frac{\partial F}{\partial \beta}$$

$$\mathfrak{F} = -\frac{(\alpha-\beta)^2}{2}\left[\varphi(\alpha)\varphi''(\beta)+\varphi'(\beta)\varphi'(\alpha)\right] + \frac{\alpha-\beta}{2}\Big\{\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\beta)+\varphi''(\alpha)\right] +$$

$$+\left[\varphi'(\beta)\right]^2-\left[\varphi'(\alpha)\right]^2\Big\} + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi'(\alpha)-\varphi'(\beta)\right]$$

$$i\mathfrak{G} = -\frac{(\alpha-\beta)^2}{2}\left[\varphi'(\alpha)\varphi'(\beta)-\varphi'(\beta)\varphi'(\alpha)\right] + \frac{\alpha-\beta}{2}\Big\{\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi'(\beta)-\varphi'(\alpha)\right] +$$

$$\left[\varphi'(\beta)-\varphi'(\alpha)\right]^2\Big\}$$

$$\cancel{\iint i\mathfrak{G}\,d\alpha\,d\beta = \frac{(\alpha-\beta)^2}{2}\left[\varphi'(\alpha)\varphi(\beta)-\varphi'(\beta)\varphi(\alpha)\right] - (\alpha-\beta)\left[\varphi(\alpha)\Phi(\beta)-\varphi(\beta)\Phi(\alpha)\right] +}$$

$$\cancel{\Phi(\alpha)\Psi(\beta)-\Phi(\beta)\Psi(\alpha)}$$

$$\int\mathfrak{F}\,d\alpha\,d\beta = -\frac{(\alpha-\beta)^2}{2}\left[\varphi(\alpha)\varphi'(\beta)+\varphi'(\alpha)\varphi(\beta)\right] + (\alpha-\beta)\left[\Phi(\alpha)\varphi'(\beta)-\varphi'(\alpha)\Phi(\beta)\right] -$$

$$-\left[\Psi(\alpha)\varphi'(\beta)+\Psi(\beta)\varphi'(\alpha)\right] + \cdots$$

$$\iint\mathfrak{F}\,d\alpha\,d\beta = \iint dr\,d\varphi\begin{vmatrix}\frac{\partial\alpha}{\partial r} & \frac{\partial\alpha}{\partial\varphi}\\ \frac{\partial\beta}{\partial r} & \frac{\partial\beta}{\partial\varphi}\end{vmatrix} = \iint\frac{dr\,d\varphi}{\begin{vmatrix}\frac{\partial r}{\partial\alpha} & \frac{\partial\varphi}{\partial\alpha}\\ \frac{\partial r}{\partial\beta} & \frac{\partial\varphi}{\partial\beta}\end{vmatrix}} = \iint\frac{dr\,d\varphi}{\Big|\quad}$$

$\alpha = re^{i\varphi}$

$\beta = re^{-i\varphi}$

$$\begin{vmatrix}e^{i\varphi} & ire^{i\varphi}\\ e^{-i\varphi} & -ire^{-i\varphi}\end{vmatrix} = -ir-ir \qquad = -2i\iint\mathfrak{F}r\,dr\,d\varphi$$

$$\varphi' = \frac{z}{\sqrt{z^2-1}} \qquad \varphi = \sqrt{z^2-1} \qquad F = \frac{1}{\sqrt{z^2-1}}$$

$$\varphi'' = \frac{1}{\sqrt{z^2-1}} - \frac{z^2}{\sqrt{}^3} = \frac{-1}{\sqrt{z^2-1}^3}$$

$$F = +\frac{(\alpha-\beta)^2}{2}\left[\frac{+\alpha}{\sqrt{\alpha^2-1}\,\sqrt{\beta^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}\,\sqrt{\alpha^2-1}^3}\right] + \frac{\alpha-\beta}{2}\left[\left[\sqrt{\alpha^2-1}+\sqrt{\beta^2-1}\right]\left[\frac{1}{\sqrt{\alpha^2-1}^3}+\frac{1}{\sqrt{\beta^2-1}^3}\right]\right.$$

$$\left. + \frac{\beta^2}{\beta^2-1} - \frac{\alpha^2}{\alpha^2-1}\right] + \left[\sqrt{\alpha^2-1}-\sqrt{\beta^2-1}\right]\left[\frac{\alpha}{\sqrt{\alpha^2-1}} - \frac{\beta}{\sqrt{\beta^2-1}}\right]$$

$$= \frac{(\alpha-\beta)^2}{2}\left[\frac{\alpha}{\sqrt{\alpha^2-1}\,\sqrt{\beta^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}\,\sqrt{\alpha^2-1}^3}\right] + \frac{\alpha-\beta}{2}\left[\frac{-1}{\alpha^2-1} + \frac{\sqrt{\beta^2-1}}{\sqrt{\alpha^2-1}^3} + \frac{1}{\beta^2-1} - \frac{\sqrt{\alpha^2-1}}{\sqrt{\beta^2-1}^3}\right.$$

$$\left. + \frac{\beta^2}{\beta^2-1} - \frac{\alpha^2}{\alpha^2-1}\right] + \alpha + \beta - \beta\frac{\sqrt{\alpha^2-1}}{\sqrt{\beta^2-1}} - \alpha\frac{\sqrt{\beta^2-1}}{\sqrt{\alpha^2-1}}$$

Jednoznaczność rozwiązań ruchu spokojnego stołowego

Rayleigh i Helmholtz: ~~[illegible]~~ w obrębie skończonym

rozwiązanie określone przez prędkości na granicy obrębu

[t. zn. przez $u = f_1(x, y)$

$v = f_2(x, y)$]

W warunkach Poiseuilla etc. wystarczy jedna funkcya p!

Ogólnie np. naczynie z tłokiem

Stosując mój sposób rozwiązania:

i dla tego dwa typy rozwiązań

Dotychczas użyłem znaku X' w sensie funkcyi rzeczywistej; jeśli jednak dopuścimy funkcye złożone z części rzeczywistej i urojonej $Z' = X' + i Y'$

$Z'(\alpha + i\beta) = X'_{\alpha\beta} + i Y'_{\alpha\beta}$

$Z'(\alpha - i\beta) = X'_{\alpha\beta} - i Y'_{\alpha\beta}$

~~$Z'(\alpha) = X'(\alpha) + i Y'(\alpha)$~~

~~$Z'(\beta) = X'(\alpha\beta) - i Y'(\alpha\beta)$~~

to stawiając: $\xi + i p = 8 Z' = 8(X' + i Y')$

otrzymamy: $\xi = 8(R X' - J Y')$

$p = 8(J X' + R Y')$ to znaczy właśnie superpozycyę

rozwiązań I i II, zatem w dalszym ciągu:

$u_1 = -\frac{\xi_1 y}{2} + J \int 2 X'' z\, dz + J f(z)$	$v_1 = \xi_1 \frac{x}{2} - R \int 2 X'' z\, dz + R f(z)$
$u_2 = -\frac{\xi_2 y}{2} + R \int 2 Y'' z\, dz + R f(z)$	$v_2 = \xi_2 \frac{x}{2} + J \int 2 Y'' z\, dz - J f(z)$
$u_1 = -\frac{\xi_1 y}{2} + \int \frac{X'' \alpha\, d\alpha - X'' \beta\, d\beta}{i} + \frac{f(\alpha) - f(\beta)}{2i}$	$v_1 = \frac{\xi_1 x}{2} - \int X'' \alpha\, d\alpha + X'' \beta\, d\beta + \frac{f(\alpha) + f(\beta)}{2}$
$u_2 = -\frac{\xi_2 y}{2} + \int Y'' \alpha\, d\alpha + Y'' \beta\, d\beta + \frac{f\alpha + f\beta}{2}$	$v_2 = \frac{\xi_2 x}{2} + \int \frac{Y'' \alpha\, d\alpha - Y'' \beta\, d\beta}{i} - \frac{f(\alpha) - f(\beta)}{2i}$

$\cancel{u = -\frac{\xi y}{2} + \int \frac{Z''(\alpha)\,\alpha\,d\alpha}{i}}$ $\qquad = R\,Z'(\alpha) + i\,J Z'(\alpha)$

$= A(\alpha) + i\,B(\alpha)$

$Z'(\alpha) = X'(\alpha) + i\,Y'(\alpha) = M'(\alpha) + i\,N'(\alpha) + i\,\mathcal{M}(\alpha) - i \dots$

$Z'(\beta) = X'(\beta) + i\,Y'(\beta) = M'(\alpha_1) - i\,N'(\alpha_1) + i\,(\mathcal{M}'(\alpha_1) - i \dots)$

$$u = -\frac{\xi y}{2} + \int \underbrace{\left(\frac{X''_\alpha}{i} + Y''(\alpha)\right)}_{\frac{1}{i}\left[iB + A\right]} \alpha\,d\alpha + \frac{1}{i}\int \underbrace{\left(-X'' + iY''\right)}_{\frac{1}{i}\left[-M + iN + i\mathcal{M}' + \mathcal{N}'\right]} \beta\,d\beta$$

$$\frac{1}{i}\left[iB - A\right]$$

$$= -\frac{\xi y}{2} + \int (B + iA)\,\alpha\,d\alpha + (B - iA)\,\beta\,d\beta$$

$$= -\frac{\xi y}{2} + \frac{1}{i}\int \underbrace{(A + iB)}_{f(x+iy)}\,\alpha\,d\alpha - \underbrace{(A - iB)}_{f(x-iy)}\,\beta\,d\beta$$

$\cancel{\frac{1}{i}\left[f_1(\alpha) + i f_2(\alpha) - f_1(\beta) + i f_2(\beta)\right]}$

$= \frac{1}{i}\left[\underbrace{(f_1 + i f_2)(\alpha)}_{f(x+iy)} - \underbrace{(f_1 - i f_2)(\beta)}_{f(x-iy)}\right]$

$= \frac{1}{i}$

$$= -\frac{\xi y}{2} + 2\,J\int Z'(\alpha)\,\alpha\,d\alpha + J f_1(\alpha) + R f_2(\alpha)$$

Czy to ——— może być zawsze wyrażone jako $R\,Z(\alpha)$?

$R f_2(\alpha) = J[i f_2(\alpha)]$ zatem $J f_1(\alpha) + R f_2(\alpha) = J\left[(f_1 + i f_2)\alpha\right]$

$-X''_\alpha + i Y''(\alpha)\ \alpha\,d\alpha \qquad -X''(\beta) + i Y''(\beta)$

$-Z'_{(+i)}(\alpha)\,\alpha\,d\alpha \qquad -Z'_{(-i)}(\beta)\,\beta\,d\beta = -2\,R\,Z'(\alpha)\,\alpha\,d\alpha$

$R f_1(\alpha) + J f_2(\alpha) = R\left[(f_1 + i f_2)\alpha\right]$

Zatem rozwiązanie: $\quad \xi = 4\left[Z'(\alpha) + Z'(\beta)\right]$

$$\xi + i p = 8 Z'(x+iy)$$

$$u = -\frac{\xi y}{2} + J\int 2 Z'' z\, dz + \overbrace{J f(z)}^{u_0}$$

$$v = +\frac{\xi x}{2} - R\int 2 Z'' z\, dz + \overbrace{R f(z)}^{v_0}$$

obejmuje przypadek I i II jeśli się Z' dopuści jako funkcyę złożoną z części rzeczywistej i urojonej

funkcya p będzie jednoznacznie określona jeśli wartości jej na obrzeżu będą dane zatem wystarczy to do określenia Z'

Wogóle zatem zadanie jednoznacznie określone jeśli p i prędkości u albo v dane na obrzeżu. A jeśli u_0, v_0 (pochodzące od potęg z) nieskończenie może dla ∞ urosnąć jednak dane w kształcie warunków powierzchniowych, to są one znowu określone tylko dla Z' rzeczyw. funkcyi!

Do obliczenia ψ z danych Z', f

$$u = -i\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right) \qquad v + iu = 2\frac{\partial\psi}{\partial\alpha} = \frac{\xi\beta}{2} - 2\int Z''_\beta \beta\, d\beta + f(\alpha)$$

$$v = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta} \qquad v - iu = 2\frac{\partial\psi}{\partial\beta} = \frac{\xi\alpha}{2} - 2\int Z''_\alpha \alpha\, d\alpha + f(\beta)$$

$$\frac{\partial\psi}{\partial\alpha} = \beta\left[Z'(\alpha) + Z'(\beta)\right] - \int Z''(\beta)\,\beta\, d\beta + \tfrac{1}{2}f(\alpha)$$

$$\psi = \beta Z(\alpha) + \alpha\beta Z'(\beta) - \alpha\underbrace{\int Z''(\beta)\,\beta\, d\beta}_{=\beta Z'(\beta) - \int Z'(\beta)} + \tfrac{1}{2}\int f(\alpha)\, d\alpha$$

$$\psi = \beta Z(\alpha) + \alpha Z(\beta) + \tfrac{1}{2}\int f(\alpha)\, d\alpha + \tfrac{1}{2}\int f(\beta)\, d\beta$$

Wystarczające warunki: 1) Jeśli p dany w nieskończoności

2) jeśli $-\frac{\xi y}{2} + J\int\ldots$, $\frac{\xi x}{2} - R\int\ldots$ są skończone w nieskończ.

3) Jeśli $\lim u, v = 0$ w nieskończ.

wtedy w każdym razie zadanie jednoznacznie określone

$\Delta^2\Delta^2\psi = 0$

czyli ξ ma być niezupełne

$$\Delta^2\psi = f_1(\alpha) + f_2(\beta) = 4\frac{\partial^2\psi}{\partial\alpha\,\partial\beta}$$

$$4\psi = \beta\underbrace{\int f_1(\alpha)\,d\alpha}_{F_1(\alpha)} + \alpha\underbrace{\int f_2(\beta)\,d\beta}_{F_2(\beta)} + g_1(\alpha) + g_2(\beta)$$

$$u = -\frac{\partial\psi}{\partial y} = i\left[F_1(\alpha) - F_2(\beta) - \beta F_1'(\alpha) + \alpha F_2'(\beta) + g_1'(\alpha) - g_2'(\beta)\right]$$

$$v = \frac{\partial\psi}{\partial x} = F_1(\alpha) + F_2(\beta) + \beta F_1'(\alpha) + \alpha F_2'(\beta) + g_1'(\alpha) + g_2'(\beta)$$

$$F'(\alpha) = M + iN \quad \Big|\ +x-iy \qquad = 2i(yM - xN) = -2i\,\mathfrak{I}\left[\beta F'(\alpha)\right]$$

$$F'(\beta) = M - iN \quad +x+iy \qquad 2(xM + yN) = 2R\left[\beta F'(\alpha)\right]$$

$$u = \mathfrak{I}\left[F_1(\alpha) - \beta F'(\alpha)\right]$$

$$(M+iN)(x-iy) - (M-iN)(x+iy) =$$
$$= 2i(Nx - My)$$
$$(M+iN)(x-iy) + (M-iN)(x+iy) =$$
$$= Mx + Ny$$

$$u = \mathfrak{I}\left[-G(\alpha) + \beta F(\alpha)\right]$$

$$v = R\left[G(\alpha) + \beta F(\alpha)\right]$$

$$\psi = \beta F(\alpha) + \alpha F(\beta) + g(\alpha) + g(\beta)$$

$$u = -\frac{\partial\psi}{\partial y} = -i\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right) = \frac{1}{i}\left[\beta F'(\alpha) + F(\beta) - F(\alpha) - \alpha F'(\beta) + g'(\alpha) - g'(\beta)\right]$$

$$v = \frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta} = \left[\beta F'(\alpha) + F(\beta) + F(\alpha) + \alpha F'(\beta) + g'(\alpha) + g'(\beta)\right]$$

$$u = 2\,\mathfrak{I}\left[\beta F'(\alpha) - F(\alpha) + g'(\alpha)\right]$$

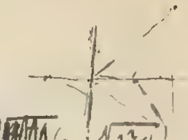
$$v = 2R\left[\beta F'(\alpha) + F(\alpha) + g'(\alpha)\right]$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 4\frac{\partial^2\psi}{\partial\alpha\,\partial\beta} = 4\left[F'(\alpha) + F'(\beta)\right] = 8R\,F'(\alpha) = \zeta$$

wychodzi na to samo co moją metodą.

W naszym przykładzie mamy : 5 -

$$Z' = \frac{i\,z}{2\sqrt{z^2-1}} \quad \Big\| \quad f = -\frac{i}{\sqrt{z^2-1}} \qquad Z = \frac{i}{2}\sqrt{z^2-1}$$



$$\int \frac{dz}{\sqrt{z^2-1}} = \lg\left(z+\sqrt{z^2-1}\right).$$

$$\psi = \frac{i}{2}\left[\beta\sqrt{\alpha^2-1} + \alpha\sqrt{\beta^2-1} - \sqrt{\alpha^2-1} + \sqrt{\beta^2-1}\right]$$

$$= \frac{i}{2}\Big[\underbrace{r e^{-i\theta}\sqrt{r_1 r_2}\, e^{\frac{i(\theta_1+\theta_2)}{2}} + r e^{i\theta}\sqrt{r_1 r_2}\, e^{-i\frac{\theta_1+\theta_2}{2}}}_{r\sqrt{r_1 r_2}\left[e^{-i\left(\theta - \frac{\theta_1+\theta_2}{2}\right)} + e^{i\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}\right]} - \sqrt{r_1 r_2}$$

$$r\sqrt{r_1 r_2}\left[e^{-i\left(\theta - \frac{\theta_1+\theta_2}{2}\right)} + e^{i\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}\right] - \sqrt{r_1 r_2}\ e$$

Próba $\displaystyle \frac{\partial\psi}{\partial x} = \frac{i}{2}\left[\frac{\beta\alpha}{\sqrt{\alpha^2-1}} + \sqrt{\alpha^2-1} + \sqrt{\beta^2-1} + \frac{\alpha\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right]$

$$= \frac{i}{2}\left[\cancel{\frac{(\beta-1)\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta(\alpha-1)}{\sqrt{\beta^2-1}} + \sqrt{\alpha^2-1} + \sqrt{\beta^2-1}}\right]$$

$$= \frac{i}{2}\left[\cancel{r^2\left(\frac{1}{\sqrt{\alpha^2-1}} + \frac{1}{\sqrt{\beta^2-1}}\right)} - 2\frac{z}{\sqrt{z^2-1}} + 2\sqrt{z^2-1}\right]$$

$$= \frac{i}{2}\Big[$$

$$-\frac{z-1}{z+1} \quad \overline{(z-1)}^2$$

$$\psi = \frac{1}{2}\left[\beta\sqrt{\alpha^2-1} + \alpha\sqrt{\beta^2-1} + \frac{1}{2}\left(\lg\frac{(\alpha+1)}{(\alpha-1)} - \lg\frac{\beta+1}{\beta-1}\right)\right]$$

$$\frac{\partial\psi}{\partial x} = \frac{1}{2}\left[\frac{\beta\alpha}{\sqrt{\alpha^2-1}} + \frac{\alpha\beta}{\sqrt{\beta^2-1}} + \sqrt{\alpha^2-1} + \sqrt{\beta^2-1} + \frac{1}{\sqrt{\alpha^2-1}} + \frac{1}{\sqrt{\beta^2-1}}\right]$$

$$= \frac{1}{2}\left[\frac{\alpha(\alpha+\beta)}{\sqrt{\alpha^2-1}} + \frac{\beta(\alpha+\beta)}{\sqrt{\beta^2-1}} - \frac{2}{\sqrt{\alpha^2-1}} + \frac{2}{\sqrt{\beta^2-1}}\right]$$

$$= \frac{1}{2}\left[2x - 2\beta - \frac{2\,2}{\sqrt{r_1 r_2}} \cos\frac{\theta_1+\theta_2}{2}\right.$$

$p - i\zeta = 8Z'$ $\qquad \cancel{p} + \zeta = 4\frac{(Z'\alpha + Z'\beta)}{\cancel{p}}\, i$

$u = -\frac{\zeta y}{2} + R\int 2Z''z\,dz + Rf(z) \quad = -i\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right) \quad i$

$v = \frac{\zeta x}{2} + I\int \qquad - If(z) \quad = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta}$

$2\frac{\partial\psi}{\partial\alpha} = \zeta\frac{(x - iy)}{2} + (I + iR)\int 2Z''z\,dz \; + i(R + iI)f$

$= \zeta\frac{\beta}{2} + 2\int Z''\alpha\, d\alpha + i\, f(\alpha)$

$\frac{\partial\psi}{\partial\alpha} = i\left[Z'(\alpha) - Z'(\beta)\right]3 + \alpha Z'(\alpha) - Z + \frac{i}{2} f(\alpha)$

$\psi = i\left[\beta Z(\alpha) - \alpha\, 3\, Z'(\beta)\right] + \alpha Z(\alpha) - 2\int Z(\alpha)\,d\alpha + \frac{i}{2}\int f(\alpha)\,d\alpha$

$\frac{\partial u}{\partial x} + \cancel{\frac{\partial\zeta}{\partial y}} = 0$ $\qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$

$u = \zeta\frac{y^2}{2}\ldots$ $\qquad = a$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right) = 0$ $\qquad \frac{\partial^2 u}{\partial x^2} = -\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right) = -\frac{\partial\zeta}{\partial y} = \frac{\partial p}{\partial x}$

$\cancel{\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial\zeta}{\partial x} = 0}, \quad \frac{\partial\zeta}{\partial y}$

$$\frac{\partial p}{\partial x} = \Delta^2 u$$
$$\frac{\partial p}{\partial y} = \Delta^2 v \qquad \Delta^2 p = 0$$
$$\frac{\partial p}{\partial z} = \Delta^2 w$$

zatem rozkład p jest jednoznacznie określony w ~~przestrz~~ obszarze jednowartym przez podanie wartości p na ~~[illegible]~~ powierzchni obszaru [także jeśli obszar ∞] (Kirchhoff p. 189 Lamb p. 47)

to nie jest wystarczające! Implicite założenie że przegrody nie sięgają do nieskończoności! zatem tylko przestrzeń ∞ w śród której wydzielone ciało jednowarte o ściankach stałych.

Otóż jednak interesuje nas także inny przypadek: kanał nieskończony t. j. przestrzeń jednowarta, sięgająca do nieskończ. i opodal aż do nieskończ. ~~przeprowadzący~~ ściany. Wtedy p samo przez się równanie $\Delta p = 0$ i warunki graniczne w nieskończ. nie jest dostatecznie określone. Mimo to tylko jedno rozwiązanie możliwe, ~~[illegible]~~

Jeśli bowiem dwa, to różnica $p_1 - p_2 = P$ będzie nieskończenie mała w ∞

$$\frac{\partial p_1}{\partial x} = \Delta^2 u_1$$
$$\frac{\partial p_2}{\partial x} = \Delta^2 u_2$$
$$\frac{\partial (p_1 - p_2)}{\partial x} = \Delta^2 (u_1 - u_2)$$

$$\xi_1 - \xi_2 = \frac{\partial (v_1 - v_2)}{\partial x} - \frac{\partial (u_1 - u_2)}{\partial y}$$

$p_1 - p_2$ } nieskończ. małe w odległości ∞

$u_1 - u_2$, $v_1 - v_2$, $w_1 - w_2$, $\xi_1 - \xi_2$ } ~~nie~~ muszą być nieskończenie małe w ∞?

Więc chodzi o to: czy pod warunkiem nieskończ. małej wielkości może powstać w przypadku naszym ruch skończony w skończonej przestrzeni?

Funkcya dysypacyjna Φ

$$\text{Praca } W = \iint (p_{xn} u + p_{yn} v + p_{zn} w)\, dS = \iiint_{-\infty}^{\infty} \Phi \, dx\, dy\, dz$$

$$p_{xn} = p_{xx} \cos nx + p_{xy} \cos ny + p_{xz} \cos nz$$
$$= \left(p - 2\mu \frac{\partial u}{\partial x}\right) \cos nx - \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \cos ny - \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \cos nz$$

Czy p, ξ przez warunki 0 w nieskończ. oznaczone?

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$$

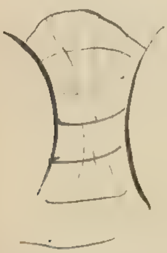
$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = 0$$

$$\frac{\partial p}{\partial x} = -\frac{\partial \xi}{\partial y}$$

Założenie: przestrzeń sięgająca do nieskończoności, ograniczona (częściowo) ścianami stałemi

p i ξ znikające w nieskończ. odległości

- prąd cieczy odpowiedni $\varphi + i\psi$ określony temi warunkami jednoznacznie, $=0$ tutaj jednak różnica to na ściankach nie ma warunku analogicznego do $\frac{\partial \varphi}{\partial n} = 0$



W każdym razie: $p + i\xi = f(z) = 0$ dla $z=\infty$

więc jeżeli nie ma być wszędzie $=0$ to musi mieć punkty nieciągłości w skończoności (ale wewnątrz poza ścianami!)

N. p. $f(z) = \frac{1}{z_1} - \frac{1}{z_2}$ $\quad p = \frac{\cos\theta_1}{r_1} - \frac{\cos\theta_2}{r_2}$

$$\xi = \frac{\sin\theta_2}{r_2} - \frac{\sin\theta_1}{r_1}$$

$$= W = \iint p\,(u\cos nx + v\cos ny + w\cos nz)\,dS +$$

$$\cancel{\mu \iint \left(u\frac{\partial u}{\partial x}\,dy\,dz + v\frac{\partial v}{\partial x}\,dx\,dz + w\frac{\partial w}{\partial z}\,dx\,dy\right) dS}$$

$$\underbrace{-\mu\iint\left[u\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + 2v\frac{\partial v}{\partial y} + w\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right] dS.\cos ny}\; + \dots$$

$$= \mu\iiint \frac{\partial u}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + 2\frac{\partial v}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \;\Big|\; + u\left(\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 u}{\partial y^2}\right) + 2v\frac{\partial^2 v}{\partial y^2} + w\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y\partial z}\right)$$

$$\frac{\partial v}{\partial z}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) + 2\frac{\partial w}{\partial z}\frac{\partial w}{\partial z} + \frac{\partial u}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \;\Big|\; + v\left(\frac{\partial^2 w}{\partial y\partial z} + \frac{\partial^2 v}{\partial z^2}\right) + 2w\frac{\partial^2 w}{\partial z^2} + u\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x\partial z}\right)$$

$$\frac{\partial w}{\partial x}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \;\Big|\; + w\left(\frac{\partial^2 u}{\partial z\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + 2u\frac{\partial^2 u}{\partial x^2} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y\partial x}\right)$$

$$W = \iint p(ul+vm+wn)\,dS - \mu\iiint\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}+\ldots\right)^2\right\}$$

$$\left.\begin{aligned}-\mu\iiint u\left(\cancel{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}}\right) &+ u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)\\ +v&\\ +w&\end{aligned}\right\} = -\mu\iiint (u\Delta^2 u + v\Delta^2 v + w\Delta^2 w)\,dx\,dy\,dz$$

$$= \iint p(ul+vm+wn)\,dS - \mu\iiint \Phi\,dx\,dy\,dz - \iiint\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right)dx\,dy\,dz$$

$$= \cancel{\iiint p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)dx\,dy\,dz} - \mu\iiint \Phi\,dx\,dy\,dz$$

Z drugiej strony możemy rozdzielić całą przestrzeń w rurki prądu (wtórne: poprzeczny przekrój stały) zatem ilość $Vq = \text{const}$ (w skończoności)

zatem $(ul+vm+wn)\,dS = V_0\,ds$

Przeprowadzając zatem przekrój w skończoności i rozdzielając go zaś na takie rurki prądu aby $V_0\,ds$ było dla nich jednakowe otrzymamy

$\sum V_0\,ds$ = skończone jeśli przekrój skończony w skończoności i zgodnie.

$$W = \sum P.V_0\,ds - \mu\underbrace{\sum\left(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)ul\,dS}+\ldots$$

$$< \sum\left(2\frac{\partial u}{\partial x}+\ldots\right)V_0\,ds+\ldots$$

lepiej obierając końcowe elementa normalnie do prądu!

oznaczając największe wartości przez P_M i DV_M

$$\left.\begin{aligned}\sum\ldots &< P_M\sum V_0\,ds\\ \sum\ldots &< DV_M\sum V_0\,ds\end{aligned}\right\}$$ każda z nich nieskończenie mała

zatem $\iiint \Phi\,dx\,dy\,dz$ znikające zatem $u=v=w=0$

Do tego dowodu wymagam : $\lim_{\infty} P = 0$

$$\lim_{\infty} \left(2\frac{\partial V}{\partial n} + \cdots\right) = 0$$

zatem v albo ~~punktowy końcowy w odległości tak że~~ $\lim_{\infty} V = 0$ co wymaga ∞ wielki końcowy punktowy

albo $\lim_{\infty} V^2 = \text{const}$ co wymaga skończonego punktowego ale ścianą [illegible] niezależnego

w razie zachowania wpływu bezwładności

$$W = -n \iiint \Phi \, dx\,dy\,dz - \rho \iiint \Big[\underbrace{u\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + v\left(u\frac{\partial v}{\partial x} + \cdots\right) + w(\cdots)}_{\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\frac{u^2+v^2+w^2}{2}\right)}\Big] dx\,dy\,dz$$

$$= \iint \frac{V^2}{2}\underbrace{(ul + vm + wn)}_{V_n} dS - \cancel{\iiint \frac{V^2}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dx\,dy\,dz}$$

obliczając końcowe elementy $\perp$ prędkości $= \iint \frac{V^3}{2} dS$ co znika oczywiście pod owemi założeniami

a zatem pozostaje owe rozwiązanie także ważnem ~~dla~~ przy uwzględnieniu bezwładności.

W tym razie nie można jednak wnioskować z tego o jednoznaczności ogólnego rozwiązania ponieważ superpozycja nie możliwa!

To samo pozostaje ważnem w razie kanałów połączonych (wielokrotnie wartości zbiegające do ∞)

Ale ~~pytanie czy~~ ogólne zadanie ruchu nie jest ~~było~~ określone dostatecznie przez warunek $\lim_{\infty} \frac{v}{u} = 0$ $\lim_{\infty} P = f$ dane ~~tylko prędkość~~ (bo punktowy nieskończony jako zadanie!)

N. p. $p = -4 \frac{\sin n\theta}{r^n}$ $\quad u = -2\frac{y \cos n\theta}{r^n} - \frac{n}{n-1}\frac{\sin(n-1)\theta}{r^{n-1}} + \mathcal{I}$; $\quad$ u zupełnie dowolne!

$v = 2\frac{x\cos n\theta}{r^n} - \frac{n}{n-1}\frac{\cos(n-1)\theta}{r^{n-1}} + \theta$; $\quad f$ dowolne!

Tak samo w ogólnym wypadku podanie wartości u, v, w ~~…~~ na powierzchni obrotu ~~…~~ co wielkości
nie jest wystarczające do określenia ruchu

Dowód Rayleigha niestosowny

Czy to się zgadza z ową ogólną prawdą? Może

dając $\theta = 0$, $= \frac{\pi}{2}$, $= \pi$ $u = v = \infty$ $\lim_{\infty} p = 0!$ O punkt osobliwy gdzie p skacze.

zatem tam trzeba uważać i w rachunkach ... co

Drugie $\frac{p - i\xi}{4} = \frac{1}{z^2} = \frac{\cos 2\theta}{r^2} - i\frac{\sin 2\theta}{r^2}$

$\int -\frac{2}{z^2}\,dz = \frac{2}{z} = \frac{2\cos\theta}{r} - i\,\frac{2\sin\theta}{r}$

$u = -2\frac{\sin 2\theta \sin\theta}{r} + \frac{2\cos\theta}{r}$

$v = 2\frac{\sin 2\theta\cos\theta}{r} - 2\frac{\sin\theta}{r}$

θ

$\frac{\xi + i p}{4} = \frac{\cos 2\theta}{r^2} - i\frac{\sin 2\theta}{r^2}$

$u = -2\frac{\cos 2\theta\sin\theta}{r} - 2\frac{\sin\theta}{r}$

$v = \frac{2\cos 2\theta\cos\theta}{r} - \frac{2\cos\theta}{r}$

$\xi = \frac{\lg r\cos 2\theta + \theta\sin 2\theta}{r^2}$

$u = 2\frac{\lg r\cos 2\theta + \theta\sin 2\theta}{r}\sin\theta + \ldots$

$v = 2\frac{\lg r\cos 2\theta + \theta\sin 2\theta}{r}\cos\theta - \frac{2(\cos\theta\lg r + \theta\sin\theta) + \cos\theta}{r}$

th maximum co do względnej?

$$p = -\frac{1}{4r^2}\left[3 - \frac{\cos 4\theta}{3} - 4\theta \sin 2\theta + 2\pi \sin 2\theta\right]$$

$\theta = \frac{\pi}{2};\ p = -\frac{1}{4r^2}\left[3 - \frac{1}{3}\right]$

$\theta = \frac{3\pi}{2}$

$\theta = \frac{\pi}{4};\ p = -\frac{1}{4r^2}\left[3 + \frac{1}{3} - \pi + 2\pi\right]$

$= -\frac{1}{4r^2}\left[3 + \frac{1}{3} + \pi\right]$

$$2\,\frac{\cos 2\theta \log r + \theta \sin 2\theta}{r^2} = \frac{\log\alpha}{\alpha^2} + \frac{\log\beta}{\beta^2}$$

$$2\,\frac{\theta \sin 2\theta}{r^2} = \frac{\log\alpha}{\alpha^2} + \frac{\log\beta}{\beta^2} - \frac{1}{2}\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\log\alpha\beta$$

$$4\theta\,\frac{\sin 2\theta}{r^2} = \left[\log\alpha - \log\beta\right]\left[\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right]$$

$$2\,\frac{\cos 4\theta}{r^2} = \frac{\alpha}{\beta^3} + \frac{\beta}{\alpha^3}$$

$$\frac{1}{r^2} = \frac{1}{\alpha\beta}$$

$$2\,\frac{\sin 2\theta}{r^2} = \frac{1}{i}\left[\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right]$$

Biorąc $\frac{p - iq}{4} = \frac{\log z}{z^2} = \frac{\log r \cos 2\theta + \theta \sin 2\theta}{r^2} + i\,\frac{\theta \cos 2\theta - \log r \sin 2\theta}{r^2}$

$$\int\left(\frac{1}{z^2} - \frac{2\log z}{z^2}\right)dz = -\frac{1}{z} + \frac{2}{z}(\log z + 1) = \frac{2\log z}{z} + \frac{1}{z}$$

$$= \frac{2\,(\log r \cos\theta + \theta \sin\theta) + \cos\theta}{r} + i\,\frac{2(-\sin\theta \log r + \theta\cos\theta) - \sin\theta}{r}$$

$$u = 2\,\frac{\theta \cos 2\theta - \log r \sin 2\theta}{r^2}\sin\theta + \frac{2\,(\log r \cos\theta + \theta \sin\theta) + \cos\theta}{r}$$

$$v = -2\,\frac{\left[\theta\cos 2\theta - \log r \sin 2\theta\right]\cos\theta}{r} + \frac{2(-\log r \sin\theta + \theta\cos\theta) - \sin\theta}{r}$$

$$-\cancel{2\sin 2\theta \cos\theta}\cos\theta + 2\cos^2\theta + \cancel{2\sin 2\theta}\cos\theta\sin\theta - 2\sin^2\theta = 2\int_0^\pi \cos 2\theta\, d\theta = \int_0^{2\pi}\cos\varphi\, d\varphi = 0$$

$$\int_0^\pi \theta \sin 2\theta\, d\theta = -\frac{\cos 2\theta \cdot \theta}{2}\Big| + \frac{1}{2}\int_0^\pi \cos 2\theta\, d\theta = 0$$

$-\cancel{\Delta^2 p} = t$

$$4\frac{\partial^2 t}{\partial\alpha\,\partial\beta} = -\left(\frac{\partial \mathfrak{F}}{\partial\alpha} + \frac{\partial \mathfrak{F}}{\partial\beta} + i\frac{\partial \mathfrak{G}}{\partial\alpha} - i\frac{\partial \mathfrak{G}}{\partial\beta}\right)$$

$$4\frac{\partial t}{\partial\alpha} = -\mathfrak{F} + i\mathfrak{G} - \frac{\partial}{\partial\alpha}\int(\mathfrak{F}+i\mathfrak{G})\,d\beta$$

$$4\frac{\partial t}{\partial\beta} = -\mathfrak{F} - i\mathfrak{G} - \frac{\partial}{\partial\beta}\int(\mathfrak{F}-i\mathfrak{G})\,d\alpha$$

$$16\frac{\partial^2 u}{\partial\alpha\,\partial\beta} = 2\mathfrak{F} - \frac{\partial}{\partial\alpha}\int(\mathfrak{F}+i\mathfrak{G})\,d\beta - \frac{\partial}{\partial\beta}\int(\mathfrak{F}-i\mathfrak{G})\,d\alpha$$

$$16u = 2\iint \mathfrak{F}\,d\alpha\,d\beta - \iint(\mathfrak{F}+i\mathfrak{G})\,d\beta^2 - \iint(\mathfrak{F}-i\mathfrak{G})\,d\alpha^2$$

$$16\frac{\partial^2 v}{\partial\alpha\,\partial\beta} = 2\mathfrak{G} - i\frac{\partial}{\partial\alpha}\int(\mathfrak{F}+i\mathfrak{G})\,d\beta + i\frac{\partial}{\partial\beta}\int(\mathfrak{F}-i\mathfrak{G})\,d\alpha$$

$$16v = 2\iint \mathfrak{G}\,d\alpha\,d\beta - i\iint(\mathfrak{F}+i\mathfrak{G})\,d\beta^2 + i\iint(\mathfrak{F}-i\mathfrak{G})\,d\alpha^2$$

$$16u = -\iint \mathfrak{F}\left[d\alpha^2 - 2\,d\alpha\,d\beta + d\beta^2\right] + i\iint \mathfrak{G}\,(d\alpha^2 - d\beta^2)$$

$$16v = \iint \mathfrak{G}\left[d\alpha^2 - 2\,d\alpha\,d\beta + d\beta^2\right] + i\iint \mathfrak{F}(d\alpha^2 - d\beta^2)$$

$$\Delta^2 u = \frac{\partial p}{\partial x} + F \qquad u = -\frac{\partial \psi}{\partial y}$$

$$\Delta^2 v = \frac{\partial p}{\partial x} + G \qquad v = \frac{\partial \psi}{\partial x}$$

$$\Delta^2\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\partial G}{\partial y} - \frac{\partial F}{\partial x}$$

$$\Delta^2 \Delta^2 \psi = \qquad = 16\,\frac{\partial^4 \psi}{\partial \alpha^2 \partial \beta^2} = \frac{\partial}{\partial \alpha}(iG - F) + \frac{\partial}{\partial \beta}(-iG - F)$$

$$F = -\frac{4y^4 x}{r^8} \qquad F - iG = +\frac{4y^4}{r^8}(x - iy) = -\frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 \beta$$

$$G = -\frac{4y^5}{r^8} \qquad F + iG = -\frac{4y^4}{r^8}(x + iy) = \frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 \alpha$$

$$2\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^3\left(-\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) = 2\left(\frac{\beta - \alpha}{\alpha\beta}\right)^3 \frac{\alpha^3 - \beta^3}{\alpha^2\beta^2}$$

$$= \frac{2}{(\alpha\beta)^5}\begin{array}{l} \beta^3\alpha^3 - 3\beta^2\alpha^4 + 3\beta\alpha^5 - \alpha^6 \\ -\beta^6 + 3\beta^5\alpha - 3\beta^4\alpha^2 + \beta^3\alpha^3 \end{array}$$

$$16\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} = 2\left[\frac{2}{(\alpha\beta)^2} - 3\left(\frac{1}{\alpha\beta^3} + \frac{1}{\alpha^3\beta}\right) + 3\left(\frac{1}{\alpha^4} + \frac{1}{\beta^4}\right) - \left(\frac{\alpha}{\beta^5} + \frac{\beta}{\alpha^5}\right)\right]$$

$$\psi = \frac{1}{8}\left[2\log\alpha \cdot \log\beta - \frac{3}{2}\left[\frac{\alpha}{\beta}(\log\alpha - 1) + \frac{\beta}{\alpha}(\log\beta - 1)\right] + \frac{1}{4}\left(\frac{\beta^2}{\alpha^2} + \frac{\alpha^2}{\beta^2}\right) - \frac{1}{72}\left(\frac{\alpha^3}{\beta^3} + \frac{\beta^3}{\alpha^3}\right)\right]$$

$$= \frac{1}{8}\left[2\left[(\log r)^2 + \theta^2\right] - \frac{3}{\text{[illegible]}}\left[\log r \cos 2\theta - \theta \sin 2\theta\right] - 3\cos 2\theta + \frac{1}{4}\cos 4\theta - \frac{1}{72}\cos 6\theta\right]$$

$$v = \frac{1}{8}\Big[2\left(\frac{\log\beta}{\alpha} + \frac{\log\alpha}{\beta}\right) - \frac{3}{2}\left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right) - \cancel{\frac{\alpha}{\beta}\frac{1}{\alpha}} + \frac{3}{2}\left(\frac{\alpha\log\alpha}{\beta^2} + \frac{\beta\log\beta}{\alpha^2}\right) - \cancel{\frac{3}{2}\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right)}$$

$$\frac{3}{2}\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) - \frac{1}{2}\left(\frac{\beta^2}{\alpha^3} + \frac{\alpha^2}{\beta^3}\right) - \frac{1}{24}\left(\frac{\alpha^2}{\beta^3} + \frac{\beta^2}{\alpha^3}\right) + \frac{1}{24}\left(\frac{\alpha^3}{\beta^4} + \frac{\beta^3}{\alpha^4}\right)\Big]$$

$$= \frac{1}{8}\Big[\frac{1}{2}\left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right) + \frac{3}{2}\left(\frac{\alpha\log\alpha}{\beta^2} + \frac{\beta\log\beta}{\alpha^2}\right) - \left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) - \frac{13}{24}\left(\frac{\alpha^2}{\beta^3} + \frac{\beta^2}{\alpha^3}\right)$$

$$+ \frac{1}{24}\left(\frac{\alpha^3}{\beta^4} + \frac{\beta^3}{\alpha^4}\right)\Big]$$

$$\beta F(\alpha) + \alpha F(\beta) \quad + F(\alpha) + F(\beta) \quad + g(\alpha) + g(\beta)$$

$$-\frac{3}{2}\left(\beta\frac{\log\alpha}{\alpha^2} + \alpha\frac{\log\beta}{\beta^2}\right) + \frac{3}{2}\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) + \frac{3}{2}\left(\frac{\log\alpha}{\alpha} + \frac{\log\beta}{\beta}\right) - 2\left(\frac{\log\alpha}{\alpha} + \frac{\log\beta}{\beta}\right)$$

$$u = -\frac{i}{8}\Big[2\left(\frac{\log\beta}{\alpha} - \frac{\log\alpha}{\beta}\right) - \frac{3}{2}\left(\frac{\log\alpha}{\beta} - \frac{\log\beta}{\alpha}\right) + \frac{3}{2}\left(\frac{\beta\log\beta}{\alpha^2} - \frac{\alpha\log\alpha}{\beta^2}\right) - 2\left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right)$$

$$+ \cancel{\frac{1}{2}\left(\frac{\alpha}{\beta^2} - \frac{\beta}{\alpha^2}\right)} - \cancel{\frac{1}{2}\left(\frac{\beta^2}{\alpha^3} - \frac{\alpha^2}{\beta^3}\right)} + \frac{11}{24}\left(\frac{\alpha^2}{\beta^3} - \frac{\beta^2}{\alpha^3}\right) + \frac{1}{24}\left(\frac{\beta^3}{\alpha^4} - \frac{\alpha^3}{\beta^4}\right)\Big]$$

$$+\frac{1}{i}\Big[\left(\beta\frac{\log\alpha}{\alpha^2} - \alpha\frac{\log\beta}{\beta^2}\right)\frac{3}{2} + \frac{3}{2}\left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right) + \frac{3}{2}\left(\frac{\log\beta}{\beta} - \frac{\log\alpha}{\alpha}\right) - 2\left(\frac{\log\alpha}{\alpha} - \frac{\log\beta}{\beta}\right)\Big]$$

$$v = \frac{1}{8r}\Big[\frac{1}{2}\left(\log r.\cos\theta - \theta\sin\theta\right) + \frac{3}{2}\left(\log r\cos 3\theta - \theta\sin 3\theta\right) - \cos 3\theta - \frac{13}{24}\cos 5\theta$$

$$+ \frac{1}{24}\cos 7\theta$$

$$-\frac{3}{2}\left(\log r.\cos 3\theta + \theta\sin 3\theta\right) + \frac{3}{2}\cos 3\theta + \frac{1}{2}\left(\log r.\cos\theta + \theta\sin\theta\right)$$

$$u = \frac{1}{8r}\Big[-\frac{7}{2}\left(\theta\cos\theta + \sin\theta\log r\right) - \frac{3}{2}\left(\theta\cos 3\theta + \sin 3\theta\log r\right) + 2\sin 3\theta + \frac{11}{24}\sin 5\theta$$

$$+ \frac{1}{24}\sin 7\theta$$

$$+ \frac{3}{2}\left(\sin 3\theta\log r - \theta\cos 3\theta\right) - \frac{3}{2}\sin 3\theta - \frac{7}{2}\left(\theta\cos\theta - \sin\theta\log r\right)\Big]$$

$\theta = 0 \quad u = v = 0$

$\theta = \frac{\pi}{2} \quad u = 0$

$\theta = \alpha \quad u = 10\pi \;, v = 0$

$$\Delta^2\Delta^2\psi = \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \quad = \frac{\partial}{\partial\alpha}(G - iF) + \frac{\partial}{\partial\beta}(G + iF)$$

$$G - iF = -\frac{4y^4}{r^8}(y - ix) \quad = -\frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 \frac{\alpha}{i}$$

$$G + iF = -\frac{4y^4}{r^8}(y + ix) \quad = +\frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 \frac{\beta}{i}$$

$$i\Delta^4\psi = -\frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 + \frac{1}{2}\left(\quad\right)^4 + 2\frac{\alpha}{\alpha^2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^3 + \frac{2\beta}{\beta^2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^3$$

$$= 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^3$$

$$= 2\left(\frac{1}{\alpha^4} - \frac{3}{\alpha^3\beta} + \frac{3}{\alpha^2\beta^2} - \frac{1}{\alpha\beta^3}\right.$$

$$\left.\frac{1}{\alpha^3\beta} - \frac{3}{\alpha^2\beta^2} + \frac{3}{\alpha\beta^3} - \frac{1}{\beta^4}\right)$$

$$16\frac{d^4\psi}{\partial\alpha^2\partial\beta^2}i = 2\left[\left(\frac{1}{\alpha^4} - \frac{1}{\beta^4}\right) - 2\left(\frac{1}{\alpha^3\beta} - \frac{1}{\alpha\beta^3}\right)\right]$$

$$\psi = \frac{1}{8i}\left[\frac{1}{4}\left(\frac{\beta^2}{\alpha^2} - \frac{\alpha^2}{\beta^2}\right) - \left(\frac{\beta\log\beta}{\alpha} - \frac{\alpha\log\alpha}{\beta}\right) + \left(\frac{\beta}{\alpha} - \frac{\alpha}{\beta}\right)\right]$$

$$= -\frac{1}{8}\left[\frac{1}{2}\sin 4\theta + 2\sin 2\theta - 2(\theta\cos 2\theta + \log r\sin 2\theta)\right]$$

$$2(-\theta\cos 2\theta + \log r\sin 2\theta) = \left[\frac{\beta i\log\alpha}{\alpha} - \frac{\alpha i\log\beta}{\beta}\right]$$

$$+4\theta \qquad\qquad = 4i\left[\log\alpha - \log\beta\right]$$

$$(\psi) = -\frac{1}{8}\left[\frac{1}{2}\sin 4\theta + 2\sin 2\theta + 4\theta(1 - \cos 2\theta)\right]$$

$$v = \frac{1}{8i}\Big[-\frac{1}{2}\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big) - \frac{1}{2}\Big(\frac{\beta^2}{\alpha^3}-\frac{\alpha^2}{\beta^3}\Big) - \Big(\frac{\log\beta}{\alpha}-\frac{\log\alpha}{\beta}\Big) - \cancel{\Big(\frac{1}{\alpha}-\frac{1}{\beta}\Big)} + \Big(\frac{\beta\log\beta}{\alpha^2}-\frac{\alpha\log\alpha}{\beta^2}\Big)$$

$$+ \cancel{\Big(\frac{1}{\alpha}-\frac{1}{\beta}\Big)} - \cancel{\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)}\Big]$$

$$= \frac{1}{8r}\Big[\sin 3\theta + \sin 5\theta + 2\big[\theta\cos\theta + \log r\sin\theta\big] - 2\big[\theta\cos 3\theta + \log r\sin 3\theta\big]\Big]$$

$$u = -\frac{1}{8}\Big[-\frac{3}{2}\Big(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\Big) - \frac{1}{2}\Big(\frac{\beta^2}{\alpha^3}+\frac{\alpha^2}{\beta^3}\Big) + \Big(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}\Big) + \cancel{\Big(\frac{1}{\alpha}+\frac{1}{\beta}\Big)} + \Big(\frac{\beta\log\beta}{\alpha^2}+\frac{\alpha\log\alpha}{\beta^2}\Big)$$

$$- \cancel{\Big(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\Big)} - \cancel{\Big(\frac{1}{\alpha}+\frac{1}{\beta}\Big)}\Big]$$

$$= \frac{1}{8r}\Big[3\cos 3\theta + \cos 5\theta + 2(\log r\cos\theta - \theta\sin\theta) - 2(\log r\cos 3\theta - \theta\sin 3\theta)\Big]$$

$$\cancel{-4\cos 2\theta}$$

$$u = \frac{\partial\psi}{\partial y} = \frac{1}{8}\Big[2\cos 4\theta + \cancel{4\cos 2\theta} + 4 + 8\theta\sin 2\theta\Big]\frac{\cos\theta}{r}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{8}\Big[\qquad\Big]\frac{\cos\theta}{r}$$

$\theta = 0$	$\theta = \frac{\pi}{2}$	$\theta = \pi$
$u = 4(1-\log r)$	$u = 0$	$u = -4(1-\log r)$
$v = 0$	$v =$	$v = 0$

$$+ 2(2) - 6(5)$$

$F = -\frac{4x^5}{r^8}$ $\qquad G - iF = -\frac{4x^4}{r^8}(y - ix) = -\frac{1}{2i}\frac{(\alpha+\beta)^4}{(\alpha\beta)^4}\alpha = -\frac{\alpha}{2i}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^4$

$G = -\frac{4x^4 y}{r^8}$ $\qquad G + iF = -\frac{4x^4}{r^8}(y + ix) = \frac{1}{2i}\frac{(\alpha+\beta)^4}{(\alpha\beta)^4}\beta = \frac{\beta}{2i}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^4$

$$\frac{2}{i}\left[+\frac{\alpha}{\alpha^2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^3 - \frac{\beta}{\beta^2}\left(\quad\right)\right] = \frac{2}{i}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^3 = 16\,\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}$$

$$\frac{1}{\alpha^4}+\frac{3}{\alpha^3\beta}+\cancel{\frac{3}{\alpha^2\beta^2}}+\frac{1}{\alpha\beta^3}-\frac{1}{\alpha^3\beta}-\cancel{\frac{3}{\alpha^2\beta^2}}-\frac{3}{\alpha\beta^3}-\frac{1}{\beta^4}$$

$$\frac{1}{\alpha^4}-\frac{1}{\beta^4}+2\left(\frac{1}{\alpha^3\beta}-\frac{1}{\alpha\beta^3}\right) = 8i\,\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}$$

$$\psi = \frac{1}{8i}\left[\frac{1}{2}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right)+\left(\frac{\beta\log\beta}{\alpha}-\frac{\alpha\log\alpha}{\beta}\right)-\left(\frac{\beta}{\alpha}-\frac{\alpha}{\beta}\right)\right]$$

$+\frac{1}{3}\sin2\theta$

$$= -\frac{1}{8}\left[+\frac{1}{6}\sin4\theta - 2\sin2\theta + 2\left(\theta\cos2\theta+\log r\sin2\theta\right)\right] + \quad]$$

$(\theta\cos2\theta - \log r\sin2\theta)$

$-2\sin3\theta + \cancel{2\sin\theta} - \frac{8}{3}\sin\theta$

$$v = \frac{1}{8r}\left[\frac{\sin3\theta+\sin5\theta}{3} - 2\left[\theta\cos\theta+\log r\sin\theta\right] + 2\left[\theta\cos3\theta+\log r\sin3\theta\right]\right]$$

$\cancel{2\cos\theta} - 2\cos3\theta + \frac{8}{3}\cos\theta$

$$u = \frac{1}{8r}\left[\frac{\cos3\theta+\cos5\theta}{3} + 2\left[\log r\cos\theta-\theta\sin\theta\right] + 2\left[\log r\cos3\theta-\theta\sin3\theta\right]\right]$$

$\theta = 0$:	$v = 0$
$\theta = \frac{\pi}{2}$:	$v = -4\log r$, $u = 0$
$\theta = -\frac{\pi}{2}$:	$v = 4\log r$, $u = 0$

$$\frac{\xi + i\eta}{4} = \frac{\log z}{z^2} = \frac{\log r \cos 2\theta + \theta \sin 2\theta}{r^2} + i\,\frac{\theta \cos 2\theta - \log r \sin 2\theta}{r^2}$$

$$u = -\frac{2\left[\log r \cos 2\theta + \theta \sin 2\theta\right]\sin\theta}{r} + \frac{2(-\sin\theta \log r + \theta\cos\theta) - 2\sin\theta}{r}$$

$$v = \frac{2\left[\log r \cos 2\theta + \theta \sin 2\theta\right]\cos\theta}{r} - \frac{2(\cos\theta \log r + \theta\sin\theta) \cancel{+\cos\theta}}{r}$$

$\theta = 0:\quad u = 0,\quad v = 0$	$\theta = \frac{\pi}{2}:\quad u = \frac{2\log r - 2}{r},\quad v = -\frac{\pi}{r}$	$\theta = -\frac{\pi}{2}:\quad u = -\frac{2\log r + 2}{r},\quad v = -\frac{\pi}{r}$

$$u = -\frac{2\sin\theta\left[(\frac{\pi}{2}-\theta)\cos 2\theta - \log r \sin 2\theta\right]}{r} + \frac{2\left(-\log r \cos\theta + (\frac{\pi}{2}-\theta)\sin\theta\right) - 2\cos\theta}{r}$$

$$v = \frac{2\cos\theta\left[-(\frac{\pi}{2}-\theta)\cos 2\theta - \log r \sin 2\theta\right]}{r} + \frac{2\left(\log r \sin\theta + (\frac{\pi}{2}-\theta)\cos\theta\right) + \cancel{\sin\theta}}{r}$$

$$u = -\frac{2\sin\theta\left[\theta\cos 2\theta - \log r \sin 2\theta\right]}{r} - \frac{2(\log r \cos\theta + \theta\sin\theta) + 2\cos\theta}{r} \quad \cancel{+2\pi \sin\theta\cos^2\theta}$$

$$v = 2\cos\theta\left[\theta\cos 2\theta - \log r \sin 2\theta\right] + 2(\log r \sin\theta - \theta\cos\theta) + \cancel{\sin\theta} \quad \cancel{+2\pi\cos\theta\sin^2\theta}$$

$\theta = 0:\quad u = -2\log r + 2,\quad v = 0$	$\theta = \frac{\pi}{2}:\quad u = 0,\quad v = 2\log r$	$\theta = -\frac{\pi}{2}:\quad u = 0,\quad v = -2\log r$

$$v = \frac{1}{8r}\left[\frac{\overset{-2\sin 3\theta - \frac{8}{3}\sin\theta}{\sin 3\theta} + \sin 5\theta}{3} - 16\,\theta\,\sin^2\theta\cos\theta\right]$$

$$u = \frac{1}{8r}\left[\frac{\overset{-2\cos 3\theta + \frac{8}{3}\cos\theta}{\cos 3\theta} + \cos 5\theta}{3} - 16\,\theta\,\sin\theta\cos^2\theta + 4\cos\theta\right]$$

$\theta = 0, \pm\frac{\pi}{2}:$

$u = v = 0$

$$\psi = \int (v\cos\varphi - u\sin\varphi)\, r\, d\varphi =$$

$$-\frac{1}{8}\int\Big[\underbrace{(\sin 3\varphi\cos\varphi - \cos 3\varphi\sin\varphi)}_{\frac{\sin 2\varphi}{3}} - \frac{2\cos 3\varphi\sin\varphi}{3} + \underbrace{(\sin 5\varphi\cos\varphi - \cos 5\varphi\sin\varphi)}_{\frac{\sin 4\varphi}{3}} - 4\cos\varphi\sin\varphi\Big]\,d\varphi$$

$$= 2\sin 3\varphi\cos\varphi \qquad = 2\sin 2\varphi \cdot 0 = \int_0^\varphi \ldots = \frac{\sin^2\varphi}{2} \qquad \frac{10}{3}\,\frac{\sin^2\varphi}{2}$$

$$v = \frac{1}{16i}\left[\frac{1}{3}\left(\frac{\alpha}{\beta^2} - \frac{\beta}{\alpha^2}\right) + \frac{1}{3}\left(\frac{\alpha^2}{\beta^3} - \frac{\beta^2}{\alpha^3}\right) - \frac{4}{\pi}(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right] - 2\left(\frac{\alpha}{\beta^2} - \frac{\beta}{\alpha^2}\right) - \frac{8}{3}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$

$$u = \frac{1}{16}\left[\frac{1}{3}\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + \frac{1}{3}\left(\frac{\alpha^2}{\beta^3} + \frac{\beta^2}{\alpha^3}\right) + \frac{2}{\pi}(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 4\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right]$$

$$-2\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + \frac{8}{3}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ stimmt!

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial\alpha}(v - iu) + \frac{\partial}{\partial\beta}(v + iu)$$

$$-2\frac{\alpha}{\beta^2} + \frac{8}{3}\frac{1}{\alpha}$$

$$v - iu = \frac{1}{8i}\left[\frac{\frac{\beta}{\alpha^2} + \frac{\alpha^2}{\beta^3}}{3} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\frac{1}{\beta}\right]$$

$$v + iu = \frac{1}{8i}\left[\frac{-\frac{\alpha}{\beta^2} - \frac{\beta^2}{\alpha^3}}{3} - 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) - 2(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\frac{1}{\alpha}\right]$$

$$+2\frac{\beta}{\alpha^2} - \frac{8}{3}\frac{1}{\beta}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{1}{8i}\left[\frac{-2\left(\frac{\beta}{\alpha^3} - \frac{\alpha}{\beta^3}\right) + 2\left(\frac{\alpha}{\beta^3} - \frac{\beta}{\alpha^3}\right)}{3} - 2\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right) - 2\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)\right. + \frac{8}{3}\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)$$

$$\left. + \underbrace{2\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\left(\frac{1}{\alpha\beta} + \frac{1}{\beta\alpha}\right)}_{4\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)} + 2(\lg\alpha - \lg\beta)\left[\frac{1}{\beta^2} + \frac{1}{\alpha^2} + \frac{1}{\alpha^2} + \frac{1}{\beta^2}\right]\right]$$

$$\zeta = \frac{1}{8i}\left[\frac{10}{3}\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right) + \frac{4}{3}\left(\frac{\alpha}{\beta^3} - \frac{\beta}{\alpha^3}\right) + 4(\lg\alpha - \lg\beta)\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\right] = \Delta^2\varphi$$

$$= 4\frac{\partial^2\varphi}{\partial\alpha\partial\beta}$$

$$\frac{\partial^2\zeta}{\partial\alpha\partial\beta} = \frac{1}{8i}\left[4\left(\frac{1}{\alpha^4} - \frac{1}{\beta^4}\right) + 8\left(\frac{1}{\beta\alpha^3} - \frac{1}{\alpha\beta^3}\right)\right]$$

$$\psi = \frac{1}{8i}\left[\cancel{\frac{1}{3}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right) + \frac{1}{12}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right) + (\lg\beta - \lg\alpha)\left(\frac{\beta}{\alpha}+\frac{\alpha}{\beta}\right)}\right]$$

$$= \frac{1}{4}\left[\cancel{\frac{1}{3}\sin 2\theta - \frac{1}{12}\sin 4\theta - 2\theta\cos 2\theta}\right]$$

$$\psi = -\frac{1}{8}\left[\frac{1}{6}\sin 4\theta - \frac{5}{3}\sin 2\theta + 4\,\theta\cos 2\theta\right]$$

$$= -\frac{1}{8i}\left[\frac{1}{12}\left(\frac{\alpha^2}{\beta^2}-\frac{\beta^2}{\alpha^2}\right) - \frac{5}{6}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right) + (\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)\right]$$

$$\psi = \frac{1}{8i}\left[\frac{1}{12}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right) + \frac{5}{6}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right) + \left(\frac{\alpha\lg\alpha}{\beta} + \beta\frac{\lg\beta}{\alpha} - \beta\frac{\lg\alpha}{\alpha} + \frac{\alpha\lg\beta}{\beta}\right)\right]$$

$$\psi_0 = \frac{1}{8i}\left[\frac{1}{12}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right) + \left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right) + \left(-\frac{\alpha\lg\alpha}{\beta} + \beta\frac{\lg\beta}{\alpha}\right)\right]$$

$$\psi - \psi_0 = \frac{1}{8i}\left[-\frac{1}{6}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right) + \left(\alpha\frac{\lg\beta}{\beta} - \beta\frac{\lg\alpha}{\alpha}\right)\right]$$

$$= \frac{1}{8i}\left[\alpha\left(\frac{\lg\beta}{\beta} - \frac{1}{6\beta}\right) - \beta\left(\frac{\lg\alpha}{\alpha} - \frac{1}{6\alpha}\right)\right]$$

$$v = +\frac{1}{8}\left[\frac{2}{3}(\cos 4\theta + \cos 2\theta) - 8\theta\sin 2\theta\right]\frac{\sin\theta}{r}$$

$$u = +\frac{1}{8}\left[\frac{2}{3}(\cos 4\theta + \cos 2\theta) - 8\theta\sin 2\theta\right]\frac{\cos\theta}{r}$$

$$= \frac{1}{6r}(\cos 3\theta - 12\theta\sin\theta)\cos\theta$$

$$V_r = v\sin\theta + u\cos\theta \quad = +\frac{1}{8r}\left[\frac{2}{3}(\cos 4\theta + \cos 2\theta) - 8\theta\sin 2\theta\right]$$

$$V_{\perp r} = \cancel{v\cos\theta - u\sin\theta = \frac{1}{8}\left[\frac{2}{3}(\cos 4\theta + \cos 2\theta) - 8\theta\sin 2\theta\right]\frac{\sin 2\theta}{r}} = 0$$

$$V_r = 0: \qquad \frac{2}{3}\,(\underbrace{\cos 4\theta + \cos 2\theta}) = 8\theta\sin 2\theta$$

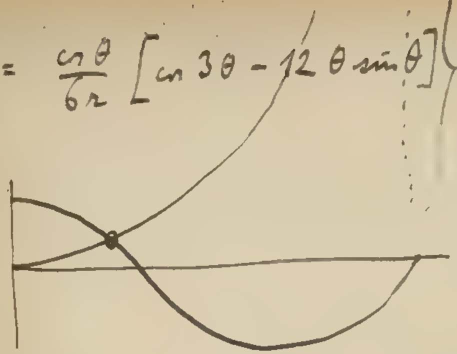
$$\cos 3\theta\cos\theta = 6\theta\sin 2\theta = 12\,\theta\sin\theta\cos\theta$$

$$\cos 3\theta = 12\theta\sin\theta$$

$$V_r = \frac{\cos\theta}{6r}\left[\cos 3\theta - 12\,\theta\sin\theta\right] \Big\} 6r$$

$y = \theta\sin\theta$

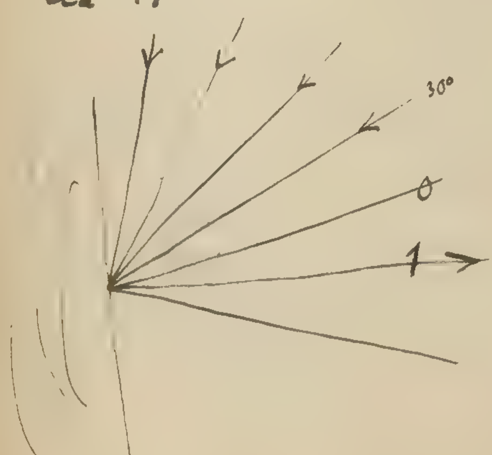
$\frac{\partial y}{\partial \theta} = \sin\theta + \theta\cos\theta$



Jeden pierwiastek w pierwszym kwadrancie $< \frac{\pi}{6} = 30^\circ$

	$\cos 3\theta$	$12\theta\sin\theta$
20°	0·5	1·2
15°	0·707	0·811
14°	0·743	0·703

cca 14°20'



12. 0·342 . 0·349

12. 0·259 . 0·262

przybliżenie při: $\cos 3\theta = 12\sin^2\theta$

12. 0·0676
135
0·811

12. 0·242² = 586 . 12
117
703

12. ½ 0·524
3·144 . 0·866
2515
88
19
2·822

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2-\beta x^4}\,dx = \int_{-\infty}^{+\infty} e^{-\beta\left(x^2+\frac{\alpha}{2\beta}\right)^2+\frac{\alpha^2}{4\beta}}\,dx$$

$$= e^{\frac{\alpha^2}{4\beta}}$$

$$e^{-a(x^2+b)^2}\,dx$$

$$x^2+b=\sqrt{y}$$

$$2x\,dx=\frac{1}{2}\frac{dy}{\sqrt{y}}$$

$$dx=\frac{1}{4}\frac{dy}{\sqrt{y}\sqrt{\sqrt{y}-b}}$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2-\beta x^4}\,dx = F(\alpha,\beta) = F(\alpha,0)+\beta\left(\frac{\partial F}{\partial\beta}\right)_0+\cdots$$

$$= \int_{-\infty}^{+\infty} e^{-\alpha x^2}\,dx - \beta\int x^4 e^{-\alpha x^2}(-\beta x^4)\,dx + \frac{\beta^2}{1.2}\int x^8 e^{-\alpha x^2}\,dx \ldots$$

$$\frac{\beta}{\alpha^2} \ll 1$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$$

$$= \sqrt{\frac{\pi}{\alpha}}\left[1-\frac{3\beta}{4\alpha^2}+\frac{\beta^2}{1.2}\cdot\frac{3.5.7}{2^4\alpha^4}-\frac{\beta^3}{3!}\frac{1.3.5.7.9.11}{2^6.\alpha^6}\ldots\right]$$

$$\int x^2 e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\neq \sqrt{\frac{\pi}{\alpha}}\left[1-\frac{3\beta}{4\alpha^2}\right]$$

$$\int x^4 \ldots = \frac{3}{4}\sqrt{\frac{\pi}{\alpha^5}}$$

$$-\int x^2 e^{-\alpha x^2-\beta x^4}\,dx = -\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}\left[1-\frac{3\beta}{4\alpha^2}\right]+\sqrt{\frac{\pi}{\alpha}}\frac{3\beta}{2\alpha^3}$$

$$\int x^6 = \frac{3.5}{8}\sqrt{\frac{\pi}{\alpha^7}}$$

$$= \sqrt{\frac{\pi}{\alpha}}\left[\frac{3\beta}{2\alpha^3}+\frac{3\beta}{8\alpha^3}-\frac{1}{2\alpha}\right] = \sqrt{\frac{\pi}{\alpha}}\Big[15$$

$$\int x^8 = \frac{3.5.7}{16}\sqrt{\frac{\pi}{\alpha^9}}$$

$$\int x^2 e^{-\alpha x^2-\beta x^4}\,dx = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}\left[1-\frac{15\beta}{8\alpha^2}\right]$$

$$\int x^4 e^{-\alpha x^2-\beta x^4} = \sqrt{\frac{\pi}{\alpha}}\left[\frac{3}{4\alpha^2}-\beta\frac{3.5.7}{16\alpha^4}\right] = \frac{3}{4\alpha^2}\sqrt{\frac{\pi}{\alpha}}\left[1-\frac{35\beta}{4\alpha^2}\right]$$

$$\int e^{-\alpha x^2 - \beta x^4} dx = \underset{\int e^{-\beta x^4} dx}{F(\alpha\beta)} + \alpha \left(\frac{\partial F}{\partial \alpha}\right)_0 + \dots$$

$$= \frac{1}{4\sqrt[4]{\beta}} \Gamma\left(\frac{1}{4}\right) + \alpha \int x^2 e^{-\beta x^4} dx + \frac{\alpha^2}{2} \int x^4 e^{-\beta x^4} dx - \dots$$

$$\frac{\alpha}{\sqrt{\beta}} \leqq 1 \qquad = \quad \frac{\alpha^2}{\beta} \ll 1 \qquad\qquad + \frac{(-1)^n}{n!} \alpha^n \int x^{2n} e^{-\beta x^4} dx$$

$$\int x^{2n} e^{-\beta x^4} dx = \qquad \cancel{x\sqrt[4]{\beta} = y} \qquad \beta x^4 = z \qquad x = \left(\frac{z}{\beta}\right)^{\frac{1}{4}}$$

$$\cancel{dx = \frac{dy}{\sqrt{\beta}}} \qquad 4\beta x^3 dx = dz$$

$$\cancel{= \int y^{2n} e^{-y}} \qquad dx = \frac{dz\, \beta^{3/4}}{4\beta\, z^{3/4}} = \frac{dz}{4 z^{3/4} \beta^{1/4}}$$

$$= \int \frac{z^{\frac{n}{2}}}{\beta^{\frac{n}{2}}} e^{-z} \frac{dz}{4\beta^{1/4} z^{3/4}} = \frac{1}{4\beta^{\frac{2n+1}{4}}} \int e^{-z} z^{\frac{2n-3}{4}} dz$$

$$= \frac{1}{4\beta^{\frac{2n+1}{4}}} \Gamma\left(\frac{2n+1}{4}\right)$$

$$\int e^{-\alpha x^2 - \beta x^4} dx =$$

$$\int V_2 \, r\,d\varphi = \frac{1}{8}\int_0^{\frac{\pi}{2}} \frac{2}{3}(\cos 4\theta + \cos 2\theta)\,d\theta - \int_0^{\frac{\pi}{2}} \theta \cos 2\theta \, d\theta =$$

$$= \frac{1}{12}\Big[\frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2} + \theta\frac{\cos 2\theta}{2} - \underbrace{\int \frac{\cos 2\theta}{2}\,d\theta}_{-\frac{\sin 2\theta}{4}}\Big]$$

$$= \frac{1}{12}\Big[\frac{\sin 4\theta + \sin 2\theta}{4} + \frac{\theta \cos 2\theta}{2}\Big]_0^{\frac{\pi}{2}} = -\frac{1}{24}\,\frac{\pi}{2} = -\frac{\pi}{48}$$

ujemny wypływ, zatem całkowita przy danym wpadku warstwa istniejąca ilość wypływu zostaje zmniejszona wskutek bieżności!

$$\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = \frac{\pi}{4}$$

$$(V_r^r) = 2\frac{\cos^2\theta}{r} + \frac{c}{\mu}\frac{\cos\theta}{6r}(\cos 3\theta - 12\theta\sin\theta)$$

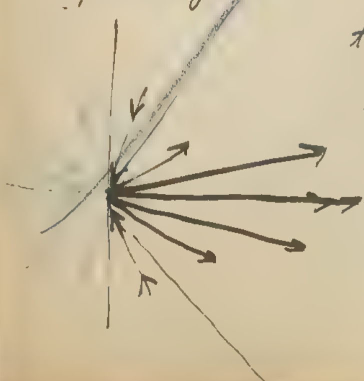
$$= \frac{\cos\theta}{r}\Big[2\cos\theta + \frac{c}{6\mu}\underbrace{(\cos 3\theta - 12\theta\sin\theta)}_{\text{dla } \frac{\pi}{2} > \theta > 15^\circ \text{ ujemne skończone}}\Big]$$

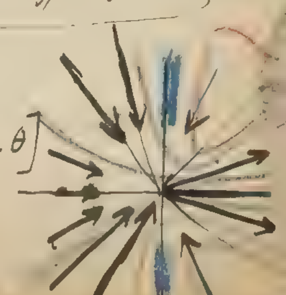
zatem zawsze istnieje (dla każdego $\frac{c}{\mu}$) taki obszar, gdzie prąd przeciwny!

Można otrzymać rozkład prądu bez przyjęcia że taka ilość $-\frac{\pi}{48}$ znika w źródle 0, superponując unoszenie (czyli $\Delta\psi = \frac{1}{6}[\log x - \log_1] + (\frac{x}{3} - \frac{2}{x})$)

$$V_2 = +\frac{\cos^2\theta}{12 r}$$ zatem:

$$V_r^r = \frac{\cos\theta}{6r}\Big[\frac{\cos\theta}{2} + \cos 3\theta - 12\theta\sin\theta\Big]$$





Arrige $\psi = \frac{1}{2i}\left[\alpha \frac{\lg \beta}{\beta} - \beta \frac{\lg \alpha}{\alpha}\right] + \cancel{\tfrac{1}{2}\left[\lg \alpha + \lg \beta\right]}$

$$= (\lg r - i\theta)(\cos 2\theta + i \sin 2\theta) - \dots$$

$$\psi = -\theta \cos 2\theta + \sin 2\theta \lg r$$

$$-u = \frac{\partial \psi}{\partial y} = \left[-\cos 2\theta + 2\theta \sin 2\theta + 2 \cos 2\theta \lg r\right] \frac{\cos\theta}{r} + \frac{\sin 2\theta}{r} \sin\theta$$

$$v = \frac{\partial \psi}{\partial x} = -\left[-\cos 2\theta + 2\theta \sin 2\theta + 2 \cos 2\theta \lg r\right] \frac{\sin\theta}{r} + \frac{\sin 2\theta}{r} \cos\theta$$

$\theta = 0$	$u =$ $v = 0$
$\theta = \pm\frac{\pi}{2}$	$u = 0$ $v = \pm\left(\frac{-1 + 2 \lg r}{r}\right)$

$$-\sqrt{z^2+1} + \frac{z^2}{\sqrt{z^2+1}} = \frac{-1}{\sqrt{z^2+1}}$$

---

x

$$F' = -\frac{x}{\sqrt{z^2+1}} \qquad F = -\sqrt{z^2+1} \qquad g' = \frac{1}{\sqrt{z^2+1}}$$

$$u = \frac{4 r^2}{\sqrt{r_1 r_2}} \sin\theta \sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right) - \frac{4}{\sqrt{r_1 r_2}} \cos\frac{\theta_1+\theta_2}{2}$$

$$v = -\frac{4 r^2}{\sqrt{r_1 r_2}} \cos\theta \sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right)$$

$$F'' = -\frac{1}{\sqrt{z^2+1}} + \frac{z^2}{\sqrt{z^2+1}^3} = \frac{-1}{\sqrt{z^2+1}^3}$$

$$F = \frac{1}{z} \quad F' = -\frac{1}{z^2} \quad F'' = \frac{2}{z^3}$$

$$g' = -\frac{2}{z}$$

$$u = \frac{2\cos^3\theta}{r}$$

$$v = \frac{2 \sin\theta \cos^2\theta}{r}$$

(2 sources)

$$F . F'' = \frac{1}{\alpha^2+1} \qquad \psi = i\left[\frac{\beta}{\alpha} - \frac{\alpha}{\beta} - 2 \lg \frac{\alpha}{\beta}\right]$$

$$\alpha F(\beta) F''(\alpha) = \frac{\alpha \beta}{\sqrt{\beta^2+1}\sqrt{\alpha^2+1}^3} = -\sin 2\theta - 2\theta$$

$$g(\beta) F''(\alpha) =$$

Z

$$\Delta^2\Delta^2\psi = \frac{\partial}{\partial\alpha}(\zeta - i\mathfrak{F}) + \frac{\partial}{\partial\beta}(\zeta + i\mathfrak{F}) = \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)$$

$$\mathfrak{F} + i\zeta = (u+iv)\frac{\partial}{\partial\alpha}(u+iv) + (u-iv)\frac{\partial(u+iv)}{\partial\beta}$$

$$\mathfrak{F} - i\zeta = (u+iv)\frac{\partial}{\partial\alpha}(u-iv) + (u-iv)\frac{\partial(u-iv)}{\partial\beta}$$

$$= \cancel{\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}}$$

$$\cancel{-\frac{\partial u}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\frac{\partial u}{\partial y}}$$

$$+ u\frac{\partial^2 v}{\partial x^2} + v\frac{\partial^2 v}{\partial x\partial y} - u\frac{\partial^2 u}{\partial x\partial y} - v\frac{\partial^2 u}{\partial y^2}$$

$$= u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\zeta$$

$$= \left(u\frac{\partial}{\partial\alpha} + u\frac{\partial}{\partial\beta} + iv\frac{\partial}{\partial\alpha} - iv\frac{\partial}{\partial\beta}\right)\zeta$$

$$= (u+iv)\frac{\partial\zeta}{\partial\alpha} + (u-iv)\frac{\partial\zeta}{\partial\beta}$$

$$\psi_0 = \frac{1}{i}\left[\beta\,\mathfrak{F}(\alpha) - \alpha\,\mathfrak{F}(\beta) + g(\alpha) - g(\beta)\right]$$

$$u = -\frac{\partial\psi}{\partial y} = -i\left(\frac{\partial\psi_0}{\partial\alpha} - \frac{\partial\psi_0}{\partial\beta}\right) = -\left[\beta\,\mathfrak{F}'(\alpha) - \mathfrak{F}(\beta) + g'(\alpha) - \mathfrak{F}(\alpha) + \alpha\,\mathfrak{F}'(\beta) + g'(\beta)\right]$$

$$v = \frac{\partial\psi}{\partial x} = \frac{\partial\psi_0}{\partial\alpha} + \frac{\partial\psi_0}{\partial\beta} = \frac{1}{i}\left[\beta\,\mathfrak{F}'(\alpha) - \mathfrak{F}(\beta) + g'(\alpha) + \mathfrak{F}(\alpha) - \alpha\,\mathfrak{F}'(\beta) - g'(\beta)\right]$$

$$u+iv = 2\left[\mathfrak{F}(\alpha) - \alpha\,\mathfrak{F}'(\beta) - g'(\beta)\right]$$

$$u-iv = +2\left[\mathfrak{F}(\beta) - \beta\,\mathfrak{F}'(\alpha) - g'(\alpha)\right]$$

$$\zeta = \Delta^2\psi = \frac{4}{i}\left[\mathfrak{F}'(\alpha) - \mathfrak{F}'(\beta)\right]$$

$$\frac{\partial\zeta}{\partial\alpha} = \frac{4}{i}\,\mathfrak{F}''(\alpha)$$

$$\frac{\partial\zeta}{\partial\beta} = -\frac{4}{i}\,\mathfrak{F}''(\beta)$$

$$Z = \frac{8}{i}\left\{\left[\mathfrak{F}(\alpha)\,\mathfrak{F}''(\alpha) - \mathfrak{F}(\beta)\,\mathfrak{F}''(\beta)\right] - \left[\alpha\,\mathfrak{F}'(\beta)\,\mathfrak{F}''(\alpha) - \beta\,\mathfrak{F}'(\alpha)\,\mathfrak{F}''(\beta)\right] - \left[g'(\beta)\,\mathfrak{F}''(\alpha) - g'(\alpha)\,\mathfrak{F}''(\beta)\right]\right\}$$

$$\mathfrak{F}(\alpha) - \alpha\,\mathfrak{F}'(\beta) - g'(\beta) = -\sqrt{\alpha^2+1} + \frac{\alpha\beta}{\sqrt{\beta^2+1}} - \frac{1}{\sqrt{\beta^2+1}} = \frac{r^2 - r_1 r_2 - 1}{\sqrt{r_1 r_2}}\overbrace{\left(\cos\frac{\theta_1+\theta_2}{2} + i\sin\frac{\theta_1+\theta_2}{2}\right)}^{e^{i\frac{\theta_1+\theta_2}{2}}}$$

$$\frac{\partial\zeta}{\partial\alpha} = -\frac{4}{i\sqrt{\alpha^2+1}^3} = -\frac{4}{i\sqrt{r_1 r_2}^3}\,e^{-\frac{3}{2}(\theta_1+\theta_2)i}$$

$$\boxed{Z = 8\,\frac{r^2 - r_1 r_2 - 1}{(r_1 r_2)^2}\sin(\theta_1+\theta_2)} = \frac{\partial\xi}{\partial\eta} - \frac{\partial\eta}{\partial x}$$

$$\frac{1}{8}\int Z\,d\alpha\,d\beta = \left[\beta\int F(\alpha)\,F''(\alpha)\,d\alpha - \right]$$

$$-\left[F(\beta)\int \alpha F''(\alpha)\,d\alpha - \right]$$

$$-\left[g(\beta)\,F(\alpha) - \right]$$

$$Z = 16\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} = \frac{8}{i}\left[\,\right.$$

$$\psi = \frac{1}{2i}\iint d\alpha^2\,d\beta^2 \dots$$

$$\int F F''\,dz = \int\frac{dz}{z^2+1} = \operatorname{arctg} z$$

$$\iint^{FF''} dz^2 = \int \operatorname{arctg} z\,dz = z\operatorname{arctg} z - \int\frac{z\,dz}{1+z^2} = z\operatorname{arctg} z - \frac{1}{2}\lg(1+z^2)$$

$$\int F\,dz = -\int\sqrt{z^2+1}\,dz = -z\sqrt{z^2+1} + \int\frac{z^2}{\sqrt{z^2+1}}\,dz$$

$$= -\lg(z+\sqrt{1+z^2}) + \int\frac{z^2}{\sqrt{z^2+1}}$$

$$\int\frac{z^2}{\sqrt{\ }} = \frac{1}{2}z\sqrt{z^2+1} - \frac{1}{2}\lg(z+\sqrt{1+z^2})$$

$$\cancel{\ } = -\frac{1}{2}\sqrt{1+z^2} + \frac{1}{2}\frac{z^2}{\sqrt{1+z^2}} - \frac{1}{2}\frac{1}{\sqrt{1+z^2}}$$

$$\cancel{\frac{1}{2}\left[-\sqrt{1+z^2} - \frac{z^2}{\sqrt{1+z^2}} - \frac{1}{\sqrt{1+z^2}}\right]} = \frac{-1-z^2-z^2-1}{2\sqrt{1+z^2}}$$

$$= -\sqrt{1+z^2}$$

$$\int F\,dz = -\frac{z\sqrt{z^2+1}}{2} - \frac{1}{2}\lg(z+\sqrt{1+z^2})$$

$$\int \alpha F''\,d\alpha = -\int\frac{z\,dz}{\sqrt{1+z^2}^{\,3}} = \frac{1}{\sqrt{1+z^2}} \qquad \iint \alpha F'' = \lg(z+\sqrt{1+z^2})$$

$$g = \int g'\,dz = \int\frac{dz}{\sqrt{1+z^2}} = \lg(z+\sqrt{1+z^2}) \qquad \int g\,dz = z\lg(z+\sqrt{1+z^2}) - \int\frac{z\,dz}{\sqrt{1+z^2}}$$

$$= z\lg(z+\sqrt{1+z^2}) - \sqrt{1+z^2}$$

$$\psi = \frac{1}{2i}\left\{\frac{\beta\alpha}{2}\left[\beta\operatorname{arctg}\alpha - \alpha\operatorname{arctg}\beta\right] - \frac{1}{4}\left[\beta^2\lg(1+\alpha^2) - \alpha^2\lg(1+\beta^2)\right]\right.$$

$$+ \lg(\alpha+\sqrt{1+\alpha^2})\left[\frac{\beta\sqrt{\beta^2+1}}{2} + \cancel{\frac{1}{2}\lg(\beta+\sqrt{1+\beta^2})}\right] - \lg(\beta+\sqrt{1+\beta^2})\left[\frac{\alpha\sqrt{\alpha^2+1}}{2} + \cancel{\frac{1}{2}\lg(\alpha+\sqrt{1+\alpha^2})}\right]$$

$$+ \cancel{\lg(\beta+\sqrt{1+\beta^2})\cdot\sqrt{1+\alpha^2} - \sqrt{1+\beta^2}\lg(\alpha+\sqrt{1+\alpha^2})}\Big\}$$

$$+ \sqrt{1+\alpha^2}\,\beta\lg(\beta+\sqrt{1+\beta^2}) - \sqrt{1+\beta^2}\,\alpha\lg(\alpha+\sqrt{1+\alpha^2})$$

$$\cancel{\pm\sqrt{1+\alpha^2}\sqrt{1+\beta^2}} \quad + \cancel{\sqrt{1+\alpha^2}\sqrt{1+\beta^2}}$$

$$\psi = \frac{1}{2i}\left\{ \frac{\beta\alpha}{2}\left[\beta \operatorname{arctg}\alpha - \alpha \operatorname{arctg}\beta\right] - \frac{1}{2}\left[\beta^2 \lg\sqrt{1+\alpha^2} - \alpha^2 \lg\sqrt{1+\beta^2}\right] + \lg(\alpha+\sqrt{1+\alpha^2})\left[\frac{\beta\sqrt{\beta^2+1}}{2} - \alpha\sqrt{1+\beta^2}\right] - \lg(\beta+\sqrt{1+\beta^2})\left[\frac{\alpha\sqrt{1+\alpha^2}}{2} - \beta\sqrt{1+\alpha^2}\right]\right\}$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left\{ \frac{\beta^2}{2}\operatorname{arctg}\alpha + \frac{\alpha^2}{2}\operatorname{arctg}\beta + \frac{\beta^2\alpha}{2(1+\alpha^2)} + \frac{\alpha^2\beta}{2(1+\beta^2)} - \alpha\beta(\operatorname{arctg}\alpha + \operatorname{arctg}\beta) \right.$$

$$- \left[\frac{\beta^2\alpha}{2(1+\alpha^2)} + \frac{\alpha^2\beta}{2(1+\beta^2)} - \beta\alpha\lg\sqrt{1+\beta^2} - \alpha\beta\lg\sqrt{1+\alpha^2}\right] +$$

$$+ \frac{\beta\sqrt{\beta^2+1}}{2\sqrt{1+\alpha^2}} + \frac{\alpha\sqrt{1+\alpha^2}}{2\sqrt{1+\beta^2}} - \frac{\alpha\sqrt{1+\beta^2}}{\sqrt{1+\alpha^2}} - \beta\frac{\sqrt{1+\alpha^2}}{\sqrt{1+\beta^2}} -$$

$$- \sqrt{1+\beta^2}\lg(\alpha+\sqrt{1+\alpha^2}) - \sqrt{1+\alpha^2}\lg(\beta+\sqrt{1+\beta^2}) +$$

$$+ \frac{\sqrt{1+\beta^2}}{2}\lg(\alpha+\sqrt{1+\alpha^2}) + \frac{\sqrt{1+\alpha^2}}{2}\lg(\beta+\sqrt{1+\beta^2})$$

$$+ \frac{\beta^2}{2\sqrt{1+\beta^2}}\lg(\alpha+\sqrt{1+\alpha^2}) + \frac{\alpha^2}{2\sqrt{1+\alpha^2}}\lg(\beta+\sqrt{1+\beta^2})$$

$$\left. + \frac{\alpha\beta}{\sqrt{1+\beta^2}}\lg\alpha+\sqrt{\ } + \frac{\alpha\beta}{\sqrt{1+\alpha^2}}\lg\ldots \right\}.$$

$$\frac{-3(1+\beta^2) - \beta^2 + 2\alpha\beta}{2\sqrt{1+\beta^2}} = \cancel{\frac{-3}{2\sqrt{1+\beta^2}} - \frac{4\beta^2}{}}$$

$$= \frac{1}{2}\left\{ \frac{r^2}{2}\left[a\cos 2\theta + b\sin 2\theta - 2a\right] + \frac{r^3}{r_1 r_2}\left[\cos 2\theta \cos(\theta - (\theta_1+\theta_2)) + \sin 2\theta \sin(\theta - (\theta_1+\theta_2))\right] \right.$$

$$+ \frac{r}{2}\left[\cos\theta \cdot \lg r_1 r_2 + \sin\theta \cdot (\theta_1+\theta_2)\right] +$$

$$+ \frac{r}{2}\cos(\theta+\theta_1+\theta_2) - r\cos[\theta - (\theta_1+\theta_2)] -$$

$$-\left\{\frac{3}{2}\sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{2\sqrt{r_1 r_2}}\left[\cos\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) - 2\cos\frac{\theta_1+\theta_2}{2}\right]\right\} A -$$

$$\left. -\left\{\frac{3}{2}\sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{2\sqrt{r_1 r_2}}\left[\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) + 2\sin\frac{\theta_1+\theta_2}{2}\right]\right\} B \right\}$$

$\theta = \pm\frac{\pi}{2}$: $\cancel{-\frac{3\pi y^2}{4}}$

$$-u = \frac{\pi y}{2} - \frac{3}{2}\sqrt{y^2-1}\cdot\frac{\pi}{2} - \frac{3y^2}{2\sqrt{y^2-1}}\cdot\frac{\pi}{2}$$

$$= \frac{\pi}{2}\left[y - \frac{3(2y^2-1)}{2\sqrt{y^2-1}}\right]$$

$a = \Re \operatorname{arctg} \alpha = \frac{1}{2} \operatorname{arctg} \frac{2x}{1-r^2} \qquad b = \Im \operatorname{arctg} \alpha = \frac{1}{2} \operatorname{arctgh} \frac{2y}{1+r^2}$

$A = \lg \sqrt{r^2 + r_1 r_2 + 2r\sqrt{r_1 r_2} \cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}$

$B = \arcsin \frac{r \sin\theta + \sqrt{r_1 r_2} \sin\frac{\theta_1+\theta_2}{2}}{\sqrt{r^2 + r_1 r_2 + 2r\sqrt{r_1 r_2}\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}}$

$y = 0 \;;\; \theta = 0 \;;\; \theta_2 = -\theta_1 \;;$

$a = \frac{1}{2} \operatorname{arctg} \frac{2x}{1-x^2} \quad ; \quad b = 0 \,;$

$A = \lg \sqrt{x^2 + x^2 + 1 + 2x\sqrt{1+x^2}} = \lg\left(x + \sqrt{1+x^2}\right)$

$B = \arcsin 0 = 0$

$x = \pm 0 \,; \quad \theta = \theta_1 = \theta_2 = \pm \frac{\pi}{2}$

$a = \cancel{0} \;;\quad b = \pm\frac{1}{2} \operatorname{arctgh} \frac{2y}{1+y^2} \;;$

$A = \lg \sqrt{y^2 + y^2 - 1 + 2y\sqrt{y^2-1}} = \pm\lg\left(y + \sqrt{y^2-1}\right)$

$B = \pm \arcsin \frac{y + \sqrt{y^2-1}}{y + \sqrt{y^2-1}} = \pm\frac{\pi}{2} \qquad = \operatorname{arctg} \frac{y+\sqrt{y^2-1}}{0} = \infty$

$\theta = 0 \,; \quad v = \frac{\pi}{2} \quad \frac{2y}{\sqrt{y^2-1}}$

$v = \frac{r^2}{2}\left[b \cos 2\theta - a \sin 2\theta + 2b\right] - r\left[\frac{\theta_1+\theta_2}{2}\cos\theta - \frac{1}{2}\lg r_1 r_2 \cdot \sin\theta\right]$

$- \frac{r}{2} \sin(\theta + \theta_1 + \theta_2) - \frac{r}{2} \sin(\theta - \theta_1 - \theta_2) - \frac{1}{2\sqrt{r_1 r_2}}\left[A \sin\frac{\theta_1+\theta_2}{2} + B \cos\frac{\theta_1+\theta_2}{2}\right]$

$\cancel{- \frac{\sqrt{r_1 r_2}}{2}\left[B \cos\frac{\theta_1+\theta_2}{2} - A \sin\frac{\theta_1+\theta_2}{2}\right]}$

$- \frac{r^2}{\sqrt{r_1 r_2}}\left[A \sin\frac{\theta_1+\theta_2}{2} + B \cos\frac{\theta_1+\theta_2}{2}\right]$

$\theta = \pm\frac{\pi}{2} :$

$v = \pm\frac{r^2}{2}\,\frac{1}{2}\operatorname{arctgh}\frac{2y}{1+y^2} \pm \frac{y}{2}\lg(y^2-1) \pm \frac{y}{2} \pm y \mp \frac{\left(\frac{1}{2} + r^2\right)}{\sqrt{r_1 r_2}} \lg\left(y + \sqrt{y^2-1}\right) \cancel{\pm \frac{\sqrt{y^2-1}}{2} \lg\left(y+\sqrt{y^2-1}\right)}$

$$\psi = \frac{1}{2i}\left[\beta\,\alpha^2 \operatorname{arctg}\alpha - \alpha\,\beta^2 \operatorname{arctg}\beta\right]$$

$$\begin{matrix}-u\\ v\end{matrix} = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[2\alpha\beta(\operatorname{arctg}\alpha \pm \operatorname{arctg}\beta) + \frac{\beta\alpha^2}{1+\alpha^2} \pm \frac{\alpha\beta^2}{1+\beta^2} \mp (\alpha^2 \operatorname{arctg}\alpha \pm \beta^2 \operatorname{arctg}\beta)\right]$$

$$-u = \left[2r^2 a - r^2\cos 2\theta . a + r^2 \sin 2\theta . b + \frac{r^3}{r_1 r_2}\cos\left(\theta - (\theta_1+\theta_2)\right)\right]$$

$$v = \left[2r^2 b + r^2(a\sin 2\theta + b\cos 2\theta) + \frac{r^3}{r_1 r_2}\sin(\theta - (\theta_1+\theta_2))\right]$$

1).

$\theta = \pm\frac{\pi}{2}$: $-u = \frac{3\pi y^2}{2}$		$\theta = 0$:
~~$-u = \mp\theta^2$~~	$v = \pm\frac{\pi^2}{2}\operatorname{arctgh}\frac{2y}{1+y^2} \pm \frac{y^3}{y^2-1}$	$v = 0$

$$\psi = \frac{1}{2i}\left(\alpha^3 \operatorname{arctg}\alpha - \beta^3 \operatorname{arctg}\beta\right)$$

$$\begin{matrix}-u\\ v\end{matrix} = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[3(\alpha^2 \operatorname{arctg}\alpha \pm \beta^2 \operatorname{arctg}\beta) + \frac{\alpha^3}{1+\alpha^2} \pm \frac{\beta^3}{1+\beta^2}\right]$$

2).

$$= \left[3r^2(a\cos 2\theta - b\sin 2\theta) + \frac{r^3}{r_1 r_2}\cos\left(3\theta - (\theta_1+\theta_2)\right)\right].$$

$$v = 3r^2(a\sin 2\theta + b\cos 2\theta) + \frac{r^3}{r_1 r_2}\sin(3\theta - (\theta_1+\theta_2))$$

$\theta = \pm\frac{\pi}{2}$:		$\theta = 0$:
$-u = -3y^2\frac{\pi}{2}$	$v = \mp\frac{3}{2}\pi^2 \operatorname{arctgh}\frac{2y}{1+y^2} \pm \frac{y^3}{y^2-1}$	$v = 0$

$$\psi = \frac{1}{2i}\left[\beta\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) - \alpha\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{1+\beta^2})\right]$$

3).

$$\begin{matrix}-u\\ v\end{matrix} = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\frac{\alpha\beta}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{1+\alpha^2}) \pm \frac{\alpha\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{1+\beta^2}) \pm \alpha+\beta \mp \sqrt{1+\alpha^2}\,\lg(\alpha+\ldots) - \sqrt{\ldots}\right.$$

$$-u = \frac{r^2}{\sqrt{r_1 r_2}}\left(A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_1+\theta_2}{2}\right) + r\cos\theta - \sqrt{r_1 r_2}\left[A\cos\frac{\theta_1+\theta_2}{2} - B\sin\frac{\theta_1+\theta_2}{2}\right]$$

$$v = \frac{r^2}{\sqrt{r_1 r_2}}\left(B\cos\frac{\theta_1+\theta_2}{2} - A\sin\frac{\theta_1+\theta_2}{2}\right) - r\sin\theta + \sqrt{r_1 r_2}\left[A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right]$$

$\theta = \pm\frac{\pi}{2}$:	~~$\theta = 0$: $v = \frac{y^2}{\sqrt{y^2-1}}\frac{\pi}{2} + \sqrt{y^2-1}\frac{\pi}{2}$~~ $-u = \theta$, $v = \mp\frac{\pi}{2}\left(\frac{y^2}{\sqrt{1-y^2}} + \sqrt{1-y^2}\right) = \frac{\pi}{2}\frac{1}{\sqrt{1-y^2}} - y$
$-u = \frac{y^2}{\sqrt{y^2-1}}\cdot\frac{\pi}{2} + \sqrt{y^2-1}\,\frac{\pi}{2}$; $= \frac{\pi}{2}\,\frac{2y^2-1}{\sqrt{y^2-1}}$	$v = -\frac{y^2}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1}) - y + \sqrt{y^2-1}\,\lg(y+\sqrt{y^2-1}) = -y - \frac{1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$

$$\psi = \frac{1}{2i}\left[\alpha\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) - \beta\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{1+\beta^2})\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\Big[\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) + \frac{\alpha^2}{\sqrt{1+\alpha^2}}\lg \alpha+\sqrt{1+\alpha^2} + \alpha$$
$$+\sqrt{1+\beta^2}\,\lg\ldots \quad + \ldots \quad + \beta\Big]$$

$$-u = \sqrt{r_1 r_2}\left[A\cos\frac{\theta_1+\theta_2}{2} - B\sin\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[A\cos\left(2\theta-\frac{\theta_1+\theta_2}{2}\right) - B\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right)\right]$$
$$+ r\cos\theta$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\Big[\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{\;})$$
$$-\sqrt{1+\beta^2}\,\lg\ldots \qquad (4)$$

$$v = \sqrt{r_1 r_2}\left[A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[A\sin\left(2\theta-\frac{\theta_1+\theta_2}{2}\right) + B\cos\left(2\theta-\frac{\theta_1+\theta_2}{2}\right)\right]$$
$$+ r\sin\theta$$

$$\boxed{\theta = \pm\frac{\pi}{2}}$$

$$\cancel{\theta = 0: \quad v = \frac{\pi}{2}\left(\sqrt{y^2-1} + \frac{y^2}{\sqrt{y^2-1}}\right) = \frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}}$$

$$-u = -\frac{\pi}{2}\sqrt{r_1 r_2} - \frac{r^2}{\sqrt{r_1 r_2}}\frac{\pi}{2} = -\frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}$$

$\theta_1 = \frac{\pi}{2},\ \theta_2 = -\frac{\pi}{2}$

$u = 0$

$v = \frac{\pi}{2}\left(\sqrt{1-y^2} - \frac{y^2}{\sqrt{1-y^2}}\right) = -\frac{\pi}{2}\frac{1-2y^2}{\sqrt{y^2-1}}$

$$v = \sqrt{r_1 r_2}\,\lg(y+\sqrt{y^2-1}) + \frac{r^2}{\sqrt{r_1 r_2}}\lg(y+\sqrt{y^2-1}) + r = \frac{2y^2-1}{\sqrt{y^2-1}}\lg(y+\sqrt{\;}) + y$$

(4a)	(4b)	(3)
$-u = -\frac{\pi}{2}\sqrt{y^2-1}$	$-u = -\frac{y^2}{\sqrt{y^2-1}}\frac{\pi}{2}$	$-u = \frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}$
$v = \sqrt{y^2-1}\,(\lg y+\sqrt{y^2-1})$	$v = \frac{y^2}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$	$v = -y - \frac{2y^2-1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$

$(4a-4b)$: $-u = +\frac{\pi}{2}\frac{1}{\sqrt{y^2-1}}$	$(3-4a+4b)$: $-u = 2\sqrt{y^2-1}$	$(3+4b+4a)$: $-u = 0$
$v = \frac{-1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$	$v = -y$	$v = 2\sqrt{y^2-1}\,\lg(y+\sqrt{\;})$

$$\psi = \frac{1}{2i}\left[\beta\alpha \lg\sqrt{1+\alpha^2} - \alpha\beta \lg\sqrt{1+\beta^3}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\beta \lg\sqrt{1+\alpha^2} + \alpha \lg\sqrt{1+\beta^2} - \alpha \lg\sqrt{1+\alpha^2} - \beta \lg\sqrt{1+\beta^2} + \frac{\beta\alpha^2}{1+\alpha^2} + \frac{\alpha\beta^2}{1+\beta^2}\right]$$

$$= (\theta_1+\theta_2)\, r \sin\theta + \frac{r^3}{r_1 r_2}\left[\underbrace{\cos\theta \cos\frac{\theta_1+\theta_2}{2} + \sin\theta \sin\frac{\theta_1+\theta_2}{2}}_{\cos\left(\theta - \frac{(\theta_1+\theta_2)}{2}\right)}\right]$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\beta \lg\sqrt{1+\alpha^2} - \alpha \lg\sqrt{1+\beta^2} + \frac{\beta\alpha^2}{1+\alpha^2} - \frac{\alpha\beta^2}{1+\beta^2} + \alpha \lg\sqrt{1+\alpha^2} - \beta \lg\sqrt{1+\beta^2}\right]$$

$$= (\theta_1+\theta_2)\, r \cos\theta + \frac{r^3}{r_1 r_2} \sin\left(\theta - (\theta_1+\theta_2)\right) \qquad (5)$$

$$\boxed{\theta = \pm\frac{\pi}{2}}$$

$-u = y\pi + \cancel{\quad}$	$v = \pm\dfrac{y^3}{y^2-1}$

$$\psi = \frac{1}{2i}\left[\alpha^2 \lg\sqrt{1+\alpha^2} - \beta^2 \lg\sqrt{1+\beta^2}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[2\alpha \lg\sqrt{1+\alpha^2} + 2\beta \lg\sqrt{1+\beta^2} + \frac{\alpha^3}{1+\alpha^2} + \frac{\beta^3}{1+\beta^2}\right]$$

$$= r\left[\cos\theta \lg r_1 r_2 - \sin\theta\cdot(\theta_1+\theta_2)\right] + \frac{r^3}{r_1 r_2}\cos\left(3\theta - (\theta_1+\theta_2)\right) \qquad (6)$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[2\alpha \lg\sqrt{1+\alpha^2} - 2\beta \lg\sqrt{--} + \frac{\alpha^3}{1+\alpha^2} - \frac{\beta^3}{1+\beta^2}\right]$$

$$= r\left[\sin\theta \lg r_1 r_2 + \cos\theta\cdot(\theta_1+\theta_2)\right] + \frac{r^3}{r_1 r_2}\sin\left(3\theta - (\theta_1+\theta_2)\right)$$

$$\boxed{\theta = \pm\frac{\pi}{2}}$$

$-u = -\pi y$	$v = \pm\left(y \lg(y^2-1) + \dfrac{y^3}{y^2-1}\right)$

$$\psi = \frac{1}{2i}\left[-\beta\sqrt{\alpha^2+1} + \alpha\sqrt{\beta^2+1}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2i}\left[\sqrt{\beta^2+1}+\sqrt{\alpha^2+1} - \frac{\alpha\beta}{\sqrt{\alpha^2+1}} - \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] =$$

$$= \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} - \frac{r^2}{\sqrt{r_1 r_2}}\cos\frac{\theta_1+\theta_2}{2}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\sqrt{\beta^2+1}-\sqrt{\alpha^2+1} - \frac{\alpha\beta}{\sqrt{\alpha^2+1}} + \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] =$$

$$= -\sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\sin\frac{\theta_1+\theta_2}{2}$$

$\theta = \frac{\pi}{2}$:

$$u = 0$$

$$v = -\sqrt{y^2-1} + \frac{y^2}{\sqrt{y^2-1}} = \frac{1}{\sqrt{y^2-1}}$$

$\theta_1 = \frac{\pi}{2},\ \theta_2 = -\frac{\pi}{2}$:

$$-u = \sqrt{1-y^2} - \frac{y^2}{\sqrt{1-y^2}} + \frac{1-2y^2}{\sqrt{1-y^2}}$$

$$v = 0$$

$$\psi = \frac{1}{2}\left[\beta\sqrt{\alpha^2+1} + \alpha\sqrt{\beta^2+1}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{i}{2}\left[\sqrt{\beta^2+1}-\sqrt{\alpha^2+1} + \frac{\alpha\beta}{\sqrt{\alpha^2+1}} - \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] = \frac{1}{2i}\left[\sqrt{\alpha^2+1}-\sqrt{\beta^2+1} + \frac{\alpha\beta}{\sqrt{\beta^2+1}} - \frac{\alpha\beta}{\sqrt{\alpha^2+1}}\right]$$

$$= \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\sin\frac{\theta_1+\theta_2}{2}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2}\left[\sqrt{\alpha^2+1}+\sqrt{\beta^2+1} + \frac{\alpha\beta}{\sqrt{\alpha^2+1}} + \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] =$$

$$= \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\cos\frac{\theta_1+\theta_2}{2}$$

$\theta = \pm\frac{\pi}{2}$:

$$u = \sqrt{y^2-1} + \frac{y^2}{\sqrt{y^2-1}} = \pm\frac{2y^2-1}{\sqrt{y^2-1}}$$

$$v = 0$$

$$\cancel{\psi = \frac{1}{2i}\left[-\beta\sqrt{\alpha^2+1} + \alpha\sqrt{\beta^2+1} + 2\lg(\alpha+\sqrt{1+\alpha^2}) - 2\lg(\beta+\sqrt{1+\beta^2})\right]}$$

$$-u = \frac{1}{2}\sqrt{\alpha^2+1} + \sqrt{\beta^2}$$

$$\psi = \frac{1}{2i}\left[\lg(\alpha + \sqrt{1+\alpha^2}) - \lg(\beta + \sqrt{1+\beta^2})\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\frac{1}{\sqrt{1+\alpha^2}} + \frac{1}{\sqrt{1+\beta^2}}\right] = \frac{1}{2\sqrt{r_1 r_2}} \cos\frac{\theta_1+\theta_2}{2}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\qquad - \qquad\right] = \frac{1}{2\sqrt{r_1 r_2}} \sin\frac{\theta_1+\theta_2}{2}$$

$\theta = \frac{\pi}{2}$	$\theta_1 = \frac{\pi}{2}$ | $\theta_2 = -\frac{\pi}{2}$
$u = 0$	$-u = \frac{1}{2\sqrt{1^2-y^2}}$
$v = \frac{1}{2\sqrt{y^2-1}}$	$v = 0$

$$\psi = \frac{1}{2i}\left[\alpha\sqrt{\alpha^2+1} - \beta\sqrt{\beta^2+1}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\sqrt{\alpha^2+1} + \sqrt{\beta^2+1} + \frac{\alpha^2}{\sqrt{\alpha^2+1}} + \frac{\beta^2}{\sqrt{\beta^2+1}}\right]$$

$$= \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\cos\left(2\theta - \frac{\theta_1+\theta_2}{2}\right)$$

$$v = \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right)$$

$\theta = \frac{\pi}{2}$, $\theta_1 = \frac{\pi}{2}$, $\theta_2 = -\frac{\pi}{2}$ (a)	(b)
$-u = \sqrt{1^2 - y^2}$	$\frac{-y^2}{\sqrt{1-y^2}}$
$v = 0$	$v = 0$

$$\psi = \frac{1}{2i}\left[\left[\lg(\alpha + \sqrt{\alpha^2+1})\right]^2 - \left[\lg(\beta + \sqrt{1+\beta^2})\right]^2\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\frac{1}{\sqrt{1+\alpha^2}}\lg(\alpha + \sqrt{\cdots}) + \frac{1}{\sqrt{1+\beta^2}}\lg \cdots\right] = \frac{1}{\sqrt{r_1 r_2}}\left[A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_1+\theta_2}{2}\right]$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\qquad - \qquad\right] = \frac{1}{\sqrt{r_1 r_2}}\left[-A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right]$$

$\boxed{\theta = \frac{\pi}{2}:}$ $\quad -u = \frac{\pi}{2}\frac{1}{\sqrt{y^2-1}}$

$$= (4a) - (4b)$$

$$v = \frac{-1}{\sqrt{y^2-1}}\lg(y + \sqrt{y^2-1})$$

$\theta_1 = \frac{\pi}{2}$, $\theta_2 = -\frac{\pi}{2}$, $\theta = \frac{\pi}{2}$
$-u = 0$
$v = \frac{\pi}{2}\frac{1}{\sqrt{1^2-y^2}}$

$$\psi = \frac{1}{2i}\left[\sqrt{1+\alpha^2}\,\lg\alpha \quad -\sqrt{\cdot}\,\lg\beta\right]$$

$$-u = \frac{1}{2}\left[\sqrt{1+\alpha^2} + \cdots \quad + \sqrt{1+\alpha^2}\,\lg\alpha + \frac{\alpha}{\sqrt{1+\alpha^2}}\lg\alpha\right.$$

$$-u = \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \sqrt{r_1 r_2}\left[\lg r\cdot\cos\frac{\theta_1+\theta_2}{2} - \theta\sin\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[\lg r\cdot\cos\left(2\theta-\frac{\theta_1+\theta_2}{2}\right) - \theta\sin(\quad)\right]$$

$\theta_1=\theta_2=\theta=\frac{\pi}{2}$: $\quad -u = -\frac{\pi}{2}\sqrt{y^2-1} - \frac{\pi}{2}\frac{y^2}{\sqrt{y^2-1}}$

$$v = \frac{1}{2i}\left[\sqrt{1+\alpha^2}\cdot \;\; \cdots \quad\right]$$

$$= \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \sqrt{r_1 r_2}\left[\lg r\sin\frac{\theta_1+\theta_2}{2} + \theta\cos\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[\lg r\sin(2\theta-\theta_1) + \theta\cos(\cdots)\right]$$

$$v = \sqrt{y^2-1} + \sqrt{y^2-1}\,\lg r + \frac{y^2}{\sqrt{y^2-1}}\lg r$$

$$\psi = \frac{1}{2i}\left[\beta\sqrt{1+\alpha^2}\,\lg\alpha - \alpha\sqrt{1+\beta^2}\,\lg\beta\right]$$

$$-u = \frac{1}{2}\left[\frac{\beta\sqrt{1+\alpha^2}}{\alpha} + \frac{\beta\alpha}{\sqrt{1+\alpha^2}}\lg\alpha - \sqrt{1+\alpha^2}\,\lg\alpha\right.$$

$$= \sqrt{r_1 r_2}\cos\left(2\theta-\frac{\theta_1+\theta_2}{2}\right) + \frac{r^2}{\sqrt{r_1 r_2}}\left[\lg r\cdot\cos\frac{\theta_1+\theta_2}{2} + \theta\cdot\sin\frac{\theta_1+\theta_2}{2}\right] - \sqrt{r_1 r_2}\left[\lg r\cos\frac{\theta_1+\theta_2}{2} - \theta\sin\frac{\theta_1+\theta_2}{2}\right]$$

$\theta=\frac{\pi}{2}$: $\quad -u = \frac{y^2}{\sqrt{y^2-1}}\frac{\pi}{2} + \sqrt{y^2-1}\,\frac{\pi}{2}$

$$v = \frac{1}{2i}\left[\beta\frac{\sqrt{1+\alpha^2}}{\alpha} - \cdots \quad + \sqrt{1+\alpha^2}\,\lg\alpha - \cdots \quad + \frac{\beta\alpha\lg\alpha}{\sqrt{1+\alpha^2}} - \cdots\right]$$

$$= -\sqrt{r_1 r_2}\sin\left(2\theta-\frac{\theta_1+\theta_2}{2}\right) + \frac{r^2}{\sqrt{r_1 r_2}}\left[-\lg r\sin\frac{\theta_1+\theta_2}{2} + \theta\cos\frac{\theta_1+\theta_2}{2}\right] + \sqrt{r_1 r_2}\left[\lg r\cdot\sin\frac{\theta_1+\theta_2}{2} + \theta\cos\frac{\theta_1+\theta_2}{2}\right]$$

$\theta=\frac{\pi}{2}$: $\quad v = -\sqrt{y^2-1} + \frac{y^2}{\sqrt{y^2-1}}\lg r + \sqrt{y^2-1}\,\lg r$

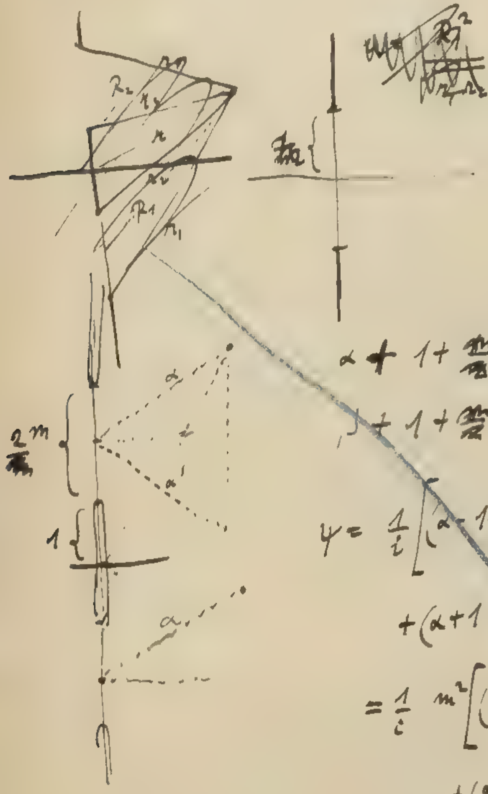
$$\psi = \frac{1}{2i}\left[(\alpha+\beta)\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) - (\alpha+\beta)\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{1+\beta^2})\right]$$

$\theta = \pm\frac{\pi}{2}$: $\quad u = v = 0$

$$\Delta^2\psi = \frac{1}{2i}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{1+\alpha^2}) + \cancel{1} - \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{1+\beta^2}) - \cancel{1}\right] = \zeta$$

$$p = \frac{1}{2}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) + \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ })\right] \qquad p = \infty \text{ dla } r = \infty$$

$$= \frac{r}{\sqrt{r_1 r_2}}\left[A\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right) - B\sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right)\right]$$



$$\psi = \frac{1}{i}\left[\alpha\sqrt{\beta^2+m^2} - \beta\sqrt{\alpha^2+m^2} + \lg(\alpha+\sqrt{\alpha^2+m^2}) - \lg\ldots\right]$$

$\alpha + 1 + m = \alpha'$ $\qquad \alpha - 1 - m = \alpha''$

$\beta + 1 + m = \beta'$

$$\psi = \frac{1}{i}\left[(\alpha-1-m)\sqrt{(\beta-1-m)^2+m^2} - (\beta-1-m)\sqrt{(\alpha-1-m)^2+m^2} + \lg\ldots\right.$$

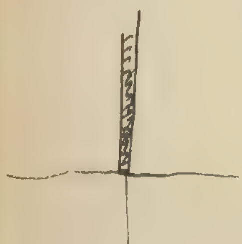
$$\left. + (\alpha+1+m)\sqrt{(\beta+1+m)^2+m^2} - (\beta+1+m)\sqrt{\ldots} \quad + \ldots\right]$$

$$= \frac{1}{i}\, m^2\left[\left(\frac{\alpha-1}{m}-1\right)\sqrt{\left[\frac{\beta-1}{m}-1\right]^2+1} - \left(\frac{\beta-1}{m}-1\right)\sqrt{\ } + \lg\left[\frac{\alpha-1}{m}-1+\sqrt{\left(\frac{\alpha-1}{m}-1\right)^2}\ldots\right.\right.$$

$$+ \left(\frac{\alpha+1}{m}+1\right)\sqrt{\left[\frac{\beta+1}{m}+1\right]^2+1} - \ldots$$

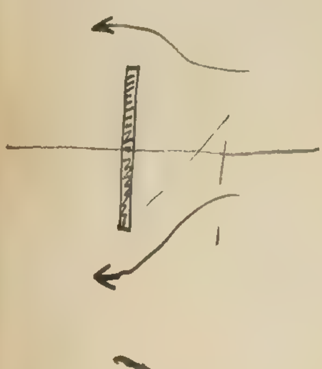
$$v = \sqrt{r} \cos\theta \sin\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$$

$$u = -\sqrt{r}\left[\sin\theta + 1\right] \sin\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$$



$$v = \sqrt{r} \cos\theta \sin\left(\frac{\theta}{2} - \frac{\pi}{4}\right)$$

$$u = -\sqrt{r}\left[\sin\theta - 1\right] \sin\left(\frac{\theta}{2} - \frac{\pi}{4}\right)$$



$$v = \sqrt{r_1} \cos\theta_1 \sin\left(\frac{\theta_1}{2} - \frac{\pi}{4}\right) + \sqrt{r_2} \cos\theta_2 \sin\left(\frac{\theta_2}{2} + \frac{\pi}{4}\right)$$

$$= r\cos\theta \left[\frac{1}{\sqrt{r_1}} \sin\left(\frac{\theta_1}{2} - \frac{\pi}{4}\right) + \frac{1}{\sqrt{r_2}} \sin\left(\frac{\theta_2}{2} + \frac{\pi}{4}\right)\right]$$

$$u = \sqrt{r_1}\left(1 - \sin\theta_1\right) \sin\left(\frac{\theta_1}{2} - \frac{\pi}{4}\right) - \sqrt{r_2}\left(1 + \sin\theta_2\right) \sin\left(\frac{\theta_2}{2} + \frac{\pi}{4}\right)$$

$$= \frac{\left(r_1 - 1 - y\right)}{\sqrt{r_1}} \sin\left(\frac{\theta_1}{2} - \frac{\pi}{4}\right) - \frac{\left(r_2 + y - 1\right)}{\sqrt{r_2}} \sin\left(\frac{\theta_2}{2} \cdots\right)$$

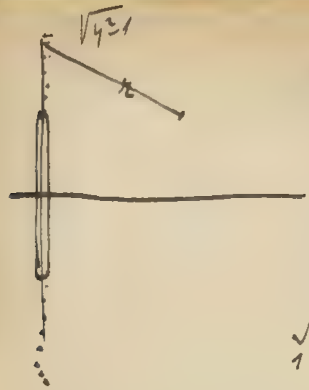
$\theta = 0$: $v = 0$

$$u = 2\sqrt{r}\;\left(1 + \sin\theta_2\right) \sin\left(\frac{\theta_2}{2} + \frac{\pi}{4}\right)$$

$\theta = \theta_1 = \theta_2 = \frac{\pi}{2}$: $v = 0$

$$u = -2\sqrt{y - 1}$$

$$p = \frac{2}{\sqrt{r_1}} \cos\left(\frac{\theta_1}{2} - \frac{\pi}{4}\right) + \frac{2}{\sqrt{r_2}} \cos\left(\frac{\theta_2}{2} + \frac{\pi}{4}\right)$$



$$u = \frac{\cos^3\theta}{r}$$

$$v = \frac{\sin\theta\cos^2\theta}{r}$$

$$\int_1^Y \sqrt{y^2-1}\,\frac{\cos^3\theta}{r}\,dy = \int_1^Y \frac{\sqrt{\eta^2-1}\cdot x^3}{\left[x^2+(y-\eta)^2\right]^2}\,d\eta$$

$$+ \int_1^Y \frac{\sqrt{\eta^2-1}}{\left[x^2+(y+\eta)^2\right]^2}\,d\eta$$

$$\xi = \Delta^2\psi = 4\,\frac{\partial\psi}{\partial\alpha\,\partial\beta} = \frac{2}{i}\left\{ \cdots\cdots \right\} \qquad \beta = \cdots$$

$$\frac{(2)-(1)}{8} = \qquad v = -\frac{x^2}{4}\,\operatorname{arctg}\frac{2y}{1+y^2} \qquad u = 0$$

$$\frac{5-6}{2} = \qquad v = -\tfrac{1}{2}\lg(y^2-1) \qquad -u = \pi y$$

$$\psi = \frac{1}{2i}\left\{\left[\left(\frac{\beta^2\alpha}{2}+\frac{\alpha^3}{8}-\frac{\beta\alpha^2}{8}\right)\operatorname{arctg}\alpha - \cdots\cdots\right] - \frac{1}{2}\left[\left(\beta^2+2\alpha\beta-\frac{\alpha^2}{2}\right)\lg\sqrt{1+\alpha^2} - \cdots\right]\right.$$

stimmt

$$\xi = \frac{2}{i}\left\{\underbrace{\left[\beta\operatorname{arctg}\alpha - \alpha\operatorname{arctg}\beta\right]} + \left[\frac{\sqrt{1+\beta^2}}{\sqrt{1+\alpha^2}} - \frac{\sqrt{1+\alpha^2}}{\sqrt{1+\beta^2}}\right] + \left[\frac{\alpha}{\sqrt{\alpha^2+1}}\lg\left(\beta+\sqrt{1+\beta^2}\right) - \frac{\beta}{\sqrt{\beta^2+1}}\lg(\alpha+\sqrt{\ })\right]\right.$$

$$4\left\{r\left(\beta\cos\theta - \alpha\sin\theta\right) - \sin(\theta_1+\theta_2) + \frac{r^4}{\sqrt{r_1 r_2}}\left[A\sin\left(\theta-\frac{\theta_1+\theta_2}{2}\right) + B\cos\left(\theta-\frac{\theta_1+\theta_2}{2}\right)\right]\right\}$$

$$\lim_{\infty} = \infty!$$

$$\xi_{(3)} = \frac{2}{i}\left\{\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ \cdots}) - \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ \cdots})\right\}$$

$$= 4\left\{\frac{r}{\sqrt{r_1 r_2}}\left[A\sin\left(\theta-\frac{\theta_1+\theta_2}{2}\right) + B\cos\left(\theta-\frac{\theta_1+\theta_2}{2}\right)\right]\right\}$$ inne ξ takiego rodzaju nie istnieją?

zatem w każdym razie musi być (3) odjęte, aby ∞ zniknęło

$$\psi = \frac{A}{2i}\left[\frac{\beta}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) - \frac{\alpha}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ })\right]$$

(7)

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\frac{-\alpha\beta}{\sqrt{1+\alpha^2}^3}\lg(\alpha+\sqrt{\ }) + \frac{\beta}{1+\alpha^2} - \frac{1}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) \ldots\right.$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\qquad + \right.$$

$$-u = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[A\cos 3\frac{\theta_1+\theta_2}{2} + B\sin 3\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\cos(\theta - \theta_1+\theta_2) - \frac{1}{\sqrt{r_1 r_2}}\left[A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_1+\theta_2}{2}\right]$$

$$v = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[-A\sin 3\frac{\theta_1+\theta_2}{2} + B\cos 3\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\sin(\theta+\theta_1+\theta_2) + \frac{1}{\sqrt{r_1 r_2}}\left[-A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right]$$

$\theta = \frac{\pi}{2}$:

$$-u = \left(\frac{y^2}{\sqrt{y^2-1}^3} - \frac{1}{\sqrt{y^2-1}}\right)\frac{\pi}{2} = \frac{y^2-y^2+1}{\sqrt{\ }^3} = \frac{1}{\sqrt{y^2-1}^3}\frac{\pi}{2}$$

$$v = -\frac{y^2}{\sqrt{y^2-1}^3}\lg(y+\sqrt{\ }) + \frac{y}{y^2-1} - \frac{1}{\sqrt{y^2-1}}\lg(y+\sqrt{\ }) = \frac{y}{y^2-1} - \frac{2y^2-1}{\sqrt{y^2-1}^3}\lg(y+\sqrt{y^2-1})$$

$$\psi = \frac{1}{2i}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) - \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ })\right]$$

(8)

$$-u = \frac{1}{2}\left[\frac{-\alpha^2}{\sqrt{1+\alpha^2}^3}\lg(\alpha+\sqrt{\ }) + \frac{1}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) + \frac{\alpha}{1+\alpha^2}\right.$$

$$v = \frac{1}{2i}\left[\right.$$

$$-u = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[A\cos\left(2\theta - 3\frac{\theta_1+\theta_2}{2}\right) - B\sin\left(2\theta - 3\frac{\theta_1+\theta_2}{2}\right)\right] + \frac{1}{\sqrt{r_1 r_2}}\left[A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\cos(\theta-\theta_1-\theta_2)$$

$\pi - 3\frac{\pi}{2}$

$\pi -$

$$v = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[A\sin\left(2\theta - 3\frac{\theta_1+\theta_2}{2}\right) + B\cos(\qquad)\right] + \frac{1}{\sqrt{r_1 r_2}}\left[-A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\sin(\theta-\theta_1-\theta_2)$$

$$-u = -\frac{y^2}{\sqrt{y^2-1}^3}\frac{\pi}{2} + \frac{1}{\sqrt{y^2-1}}\frac{\pi}{2} = \frac{-y^2+y^2-1}{\sqrt{\ }^3} = \frac{-1}{\sqrt{y^2-1}^3}\frac{\pi}{2}$$

$$v = +\frac{y^2}{\sqrt{y^2-1}^3}\lg(y+\sqrt{\ }) - \frac{1}{\sqrt{y^2-1}}\lg(y+\sqrt{\ }) - \frac{y}{y^2-1} = -\frac{y}{y^2-1} + \frac{1}{\sqrt{y^2-1}^3}\lg(y+\sqrt{y^2-1})$$

$(7+8)$: $u = 0$

$$v = \frac{-2}{\sqrt{y^2-1}} \log(y + \sqrt{y^2-1})$$

$$+ \frac{1}{2}\frac{2y^2+1}{\sqrt{y^2-1}} = \sqrt{y^2-1}\,a + \frac{y^2}{\sqrt{y^2-1}}\,b - \frac{c}{\sqrt{y^2-1}} - \frac{2d}{\sqrt{y^2-1}}$$

$$\frac{3}{2}\left[\frac{2y^2-1}{\sqrt{y^2-1}}\right] = -\sqrt{y^2-1}\,a - \frac{y^2}{\sqrt{y^2-1}}\,b + \frac{2y^2-1}{\sqrt{y^2-1}}\,c$$

$$2\frac{2y^2-1}{\sqrt{y^2-1}} = \frac{2y^2-2}{\sqrt{y^2-1}}\,c - \frac{2d}{\sqrt{y^2-1}} \qquad c = 2 \qquad d = -1$$

$$\zeta_7 = \Delta(\psi_7) = \frac{2}{2}\left\{\frac{1}{1+\alpha^2} + \frac{\alpha}{\sqrt{1+\alpha^2}^3}\log(\alpha+\sqrt{\ }) \ \ldots\ldots\right.$$

$$\left.= 2\left\{\frac{1}{r_1 r_2}\sin(\theta_1+\theta_2) - \underbrace{\frac{r}{\sqrt{r_1 r_2}^3}\left[A \sin\left[\theta - \tfrac{3}{2}(\theta_1+\theta_2)\right]\right.}_{\text{rozważ } \lim_{x=\infty} \frac{\log x}{x} = 0} \left.+ B\cos\left[\theta - \tfrac{3}{2}(\theta_1+\theta_2)\right]\right]\right\}\right\}$$

~~N ...~~ rozważ ζ_3: $\lim_{r=\infty} \log r \underbrace{\sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}_{= \frac{\sin 2\theta}{r^2}} = 0$

Cytanie tylko, czy ρ nie stanie się ∞!!

Inaczej dadzą się spełnić warunki wszystkie za pomocą $(7+8)$ 1, 2, 3, 5, 6

Inny sposób: $\bar{X} = \dfrac{\displaystyle\iint_{-\infty}^{\infty}\int_0^{\infty} -\frac{\partial U}{\partial x}\, e^{-hU}\, dx\; dy\, dz}{\displaystyle\iint_{-\infty}^{\infty} dy\, dz \int_0^{\infty} e^{-hU}\, dx}$

To nie naprężenie, tylko

$$\frac{\partial U}{\partial x} = 2\varphi x\left[4 - 3\delta\right] + 2\varphi' x\left[2 + 3\delta\right] + 2\varphi'' x\,\delta$$

$$= 2x\left\{\varphi(4-3\delta) + \varphi'(2+3\delta) + \varphi''\delta\right\}$$

= siła wypadkowa (tak dalece wytężenia [illegible] sprężystości)

$$U = \alpha + \gamma(x^2+y^2+z^2)$$

$$\frac{\int_0^{\infty} x\, e^{-h(\alpha+\gamma x^2)}\, dx}{\int_0^{\infty} e^{-h(\alpha+\gamma x^2)}\, dx} = \frac{\frac{1}{2\gamma h}\, e^{-\gamma x^2}\Big|_{\infty}^{0}}{\frac{1}{2}\sqrt{\frac{\pi}{h\gamma}}}$$

$$= \frac{1}{\sqrt{\gamma h \pi}}$$

= przeciętne jednostronne wychylenie

W stanie zerowym $(x = y = z = 0)$

$$X = \frac{4\,(1+\delta)}{\sqrt{2}\sqrt{1+\delta}} \cdot \varphi(l\sqrt{1+\delta}) = \frac{4}{\sqrt{2}}\left(1+\frac{\delta}{2}\right)\left[\varphi + l\frac{\delta}{2}\varphi'\right] = \frac{4}{\sqrt{2}}\varphi\left(1+\frac{\delta}{2}\right) + \frac{2}{\sqrt{2}}\varphi' l\,\delta$$

$$Y = \frac{2}{\sqrt{2}\sqrt{1+\delta}}\varphi(l\sqrt{1+\delta}) + \frac{2}{\sqrt{2}}\varphi = \frac{2}{\sqrt{2}}\left[\left(1-\frac{\delta}{2}\right)\left[\varphi + \frac{l\delta}{2}\varphi'\right] + \varphi\right] = \frac{2}{\sqrt{2}}\varphi\left(2-\frac{\delta}{2}\right) + \frac{1}{\sqrt{2}}\varphi' l\,\delta$$

$$r_1 = \sqrt{\left(\underset{+x_1}{\frac{1+\delta}{\sqrt2}}-x\right)^2+\left(\underset{+y_1}{-\frac{1}{\sqrt2}}-y\right)^2+\underset{(z_1-z)^2}{z^2}} = \sqrt{(1+\delta)-(1+\delta)x\sqrt2+y\sqrt2+x^2y^2z^2}$$

$$r_{10} = \sqrt{\left(-\frac{1+\delta}{\sqrt2}-x\right)^2+\cdots} = \sqrt{1+\delta+(1+\delta)x\sqrt2+y\sqrt2+x^2y^2z^2}$$

$$r_2 = \sqrt{\left(\frac{1+\delta}{\sqrt2}-x\right)^2+\left(\frac{1}{\sqrt2}-y\right)^2+z^2} = \sqrt{(1+\delta)-(1+\delta)x\sqrt2-y\sqrt2+x^2y^2z^2}$$

$$r_9 = \sqrt{\cdots} \qquad \sqrt{(1+\delta)+(1+\delta)x\sqrt2-y\sqrt2+x^2y^2z^2}$$

$$r_3 = \sqrt{\left(\frac{1+\delta}{\sqrt2}-x\right)^2+y^2+\left(-\frac{1}{\sqrt2}-z\right)^2} = \sqrt{(1+\delta)-(1+\delta)x\sqrt2+z\sqrt2+x^2y^2z^2}$$

$$r_4 = \sqrt{\left(\frac{1+\delta}{\sqrt2}-x\right)^2+y^2+\left(\frac{1}{\sqrt2}-z\right)^2} = \sqrt{(1+\delta)-(1+\delta)x\sqrt2-z\sqrt2+\cdots}$$

---

r_1	$=\sqrt{(1+\delta)}$	$-(1+\delta)x\sqrt2$	$+y\sqrt2$		$+x^2y^2z^2$
2		$-$	$-$		"
3		$-$		$+z\sqrt2$	"
4		$-$		$-$	"

---

r_1	$=\sqrt{(1+\delta)}$	$-(1+\delta)x\sqrt2$	$+y\sqrt2$		$+\cdots$
10	$(1+\delta)$	$+$	$+$		$+\cdots$
5	1		$+$	$-z\sqrt2$	$+x^2y^2z^2$
7	1		$+$	$+$	$+\cdots$

$$\cancel{4}-2\rho x\sqrt2-\cancel{2y\sqrt2}+\cancel{2(x^2y^2z^2)} \qquad -1+\frac{\rho}{2}$$

$$-4\Big[1-\frac{x\rho}{2\sqrt2}-\cancel{\frac{y}{\sqrt2}}+\cancel{\frac{x^2\cdots}{2}}-\frac{x^2}{4}-\frac{z^2}{4}+\frac{xy\rho}{4}-\frac{x(\cdots)\rho}{4\sqrt2}+\frac{y(x\rho\cdots)}{2\sqrt2}-\frac{x^3\rho}{8\sqrt2}$$

$$-\frac{y^3}{4\sqrt2}-\frac{3x^2y}{4\sqrt2}+\frac{9xy^2\rho}{8\sqrt2}\Big\}$$

$$+\frac{x^2y\rho}{\sqrt2} \pm \frac{3}{2}\frac{xy^2\rho}{\sqrt2}$$

$$\frac{(r_1-l)^3}{2}+(r_{10}-l)^3 = \frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right) + \frac{3}{4}x^2\delta + \frac{3}{4}y^2\delta + \frac{3x^2y}{2\sqrt2}\left(1+\frac{\delta}{2}\right) - \frac{3x^2y\delta}{2\sqrt2}$$

$$+\frac{3y(x^2+y^2+z^2)}{2\sqrt2}\delta - \frac{3x^2y}{4\sqrt2}\delta - \frac{3y^3}{4\sqrt2}\delta$$

$$=2\left[\frac{3x^2\delta}{4} + \frac{3y^2\delta}{4} + \frac{3x^2y}{2\sqrt2} + \frac{y^3}{2\sqrt2} + \frac{3y^3\delta}{4\sqrt2} + \frac{3yz^2\delta}{2\sqrt2}\right] - \frac{3y^3\delta}{4\sqrt2}$$

$$(r_{1}-l)^3 + \dots = 2\left[\frac{3z^2\delta}{4} + \frac{3y^2\delta}{4} + \frac{3x^2y}{2\sqrt2} + \frac{y^3}{2\sqrt2} + \frac{3y^3\delta}{4\sqrt2} + \frac{3yz^2\delta}{2\sqrt2}\right] = 2\left(\frac{3z^2y}{2\sqrt2}+\frac{y^3}{2\sqrt2}\right)$$

$$\frac{\partial}{\partial y} = 2\left\{\frac{3y\delta}{2} + \frac{3(x^2+2y^2+z^2)}{2\sqrt2} + \frac{3(\cancel{x^2}+3y^2+z^2)\delta}{2\sqrt2}\right\}$$

$$-\bar{Y} = 2\varphi\left\{\sqrt2 - \frac{\delta}{2\sqrt2} + y + \frac{y\delta}{4} - \frac{x^2}{4\sqrt2} - \frac{3y^2}{2\sqrt2} - \frac{z^2}{4\sqrt2} + \frac{3x^2\delta}{8\sqrt2} + \frac{3y^2\delta}{8\sqrt2} + \frac{3z^2\delta}{4\sqrt2}\right\}$$

$$+\varphi'\left\{\frac{\delta}{\sqrt2} + 2y - \frac{y\delta}{2} + \frac{x^2}{2\sqrt2} + \frac{3y^2}{\sqrt2} + \frac{z^2}{2\sqrt2} - \frac{3(x^2+y^2+2z^2)\delta}{4\sqrt2}\right\}$$

$$+\varphi''\left\{\frac{y\delta}{2} + \frac{x^2+2y^2+z^2}{2\sqrt2} + \frac{(\dots+z^2)\delta}{2\sqrt2}\right\}$$ stimmt

$$-\bar{Y} = 4\varphi\left(\frac{1}{\sqrt2} - \frac{\delta}{4\sqrt2}\right) + \varphi'\frac{\delta l}{\sqrt2} +$$

$$+\iiint x^2\left[\varphi\left(-\frac{1}{2\sqrt2}+\frac{3\delta}{4\sqrt2}\right) + \varphi'\left(-\frac{1}{\sqrt2}-\frac{3\delta}{4\sqrt2}\right) + \varphi''\left(\frac{1}{2\sqrt2}+\frac{\delta}{2\sqrt2}\right)\right]$$

$$+y^2\left[\varphi\left(-\frac{3}{2\sqrt2}+\frac{3\delta}{4\sqrt2}\right) + \varphi'\left(\frac{3}{\sqrt2}-\frac{3\delta}{4\sqrt2}\right) + \varphi''\left(\frac{1}{\sqrt2}+\frac{3\delta}{2\sqrt2}\right)\right]$$

$$+z^2\left[\varphi\left(-\frac{1}{2\sqrt2}+\frac{3\delta}{2\sqrt2}\right) + \varphi'\left(\frac{1}{\sqrt2}-\frac{3\delta}{2\sqrt2}\right) + \varphi''\left(\frac{1}{2\sqrt2}+\frac{\delta}{2\sqrt2}\right)\right]$$

$$\frac{9}{2} - 6 = -\frac{3}{2}$$

$$-\frac{7}{2}+1 = -\frac{5}{2}$$

$$2\left\{-\frac{\beta}{\sqrt2} + x - \frac{y\beta}{2} + \frac{9x^2\beta}{4\sqrt2} + \frac{xy}{\sqrt2} - y^2\frac{7\beta}{2\sqrt2} + \frac{z^2\beta}{2\sqrt2}\right.$$

$$\left. - \frac{3x^2\beta}{\sqrt2} - \frac{2xy}{\sqrt2} + \frac{\beta y^2}{2\sqrt2} + \frac{\beta z^2}{2\sqrt2}\right\}$$

W celu otrzymania V:

$$\left.\begin{matrix} r_1 \\ r_2 \end{matrix}\right\} 1+\delta-(1+\delta)x\sqrt2 \pm y\sqrt2 + x^2+y^2+z^2 \qquad \left.\begin{matrix} r_1 \\ r_{10} \end{matrix}\right\} 1+\delta-(1+\delta)x\sqrt2 \pm y\sqrt2 + x^2+y^2+z^2$$

$$\left.\begin{matrix} r_3 \\ r_4 \end{matrix}\right\} 1+\delta-(1+\delta)x\sqrt2 \mp z\sqrt2 + x^2+y^2+z^2 \qquad \left.\begin{matrix} r_5 \\ r_7 \end{matrix}\right\} 1+y\sqrt2 \mp z\sqrt2 + (x^2+y^2+z^2)$$

$$+\left.\begin{matrix} r_1-l \\ r_{10}-l \end{matrix}\right\} = \frac{\delta}{2}+\frac{y}{\sqrt2}\left(1-\frac{\delta}{2}\right)+\frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right)-\frac{x^2}{4}\left(1+\frac{\delta}{2}\right)-\frac{y^2}{4}\left(1-\frac{3\delta}{2}\right)-\frac{y(x^2+y^2+z^2)}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)$$

$$+\frac{y^3}{4\sqrt2}\left(1-\frac{5\delta}{2}\right)+\frac{3x^2y}{4\sqrt2}\left(1-\frac{\delta}{2}\right)$$

$$+\left.\begin{matrix} r_5 \\ r_7 \end{matrix}-1\right\} = \frac{y}{\sqrt2}+\frac{x^2+y^2+z^2}{2}-\frac{z^2}{4}-\frac{y^2}{4}-\frac{y(x^2+y^2+z^2)}{2\sqrt2}$$

$$+\frac{y^3}{4\sqrt2}+\frac{3z^2y}{4\sqrt2}$$

$$\begin{matrix} r_1+r_{10}+r_5+r_7 \\ -l \end{matrix} = 2\Big\{\frac{\delta}{2}+\frac{y}{\sqrt2}\left(2-\frac{\delta}{2}\right)+\frac{x^2+y^2+z^2}{2}\left(2-\frac{\delta}{2}\right)-\frac{x^2}{4}\left(1+\frac{\delta}{2}\right)-\frac{y^2}{4}\left(2-\frac{3\delta}{2}\right)-\frac{z^2}{4}$$

$$-\frac{y(x^2+y^2+z^2)}{2\sqrt2}\left(2-\frac{3\delta}{2}\right)+\frac{y^3}{4\sqrt2}\left(2-\frac{5\delta}{2}\right)+\frac{3x^2y}{4\sqrt2}\left(1-\frac{\delta}{2}\right)+\frac{3z^2y}{4\sqrt2}\Big\}$$

$$\frac{\partial}{\partial y}(\quad) = 2\Big\{\frac{1}{\sqrt2}\left(2-\frac{\delta}{2}\right)+y\left(2-\frac{\delta}{2}\right)-\frac{y}{2}\left(2-\frac{3\delta}{2}\right)-\frac{x^2+3y^2+z^2}{2\sqrt2}\left(2-\frac{3\delta}{2}\right)$$

$$+\frac{3y^2}{4\sqrt2}\left(2-\frac{5\delta}{2}\right)+\frac{3x^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right)+\frac{3z^2}{4\sqrt2}\Big\}$$

$$=2\Big\{\sqrt2-\frac{\delta}{2\sqrt2}+y+\frac{y\delta}{4}-\frac{x^2}{4\sqrt2}-\frac{3y^2}{2\sqrt2}-\frac{z^2}{4\sqrt2}+\frac{3x^2\delta}{8\sqrt2}+\frac{3y^2\delta}{8\sqrt2}+\frac{3z^2\delta}{4\sqrt2}\Big\}$$

$$(r_1-l)^2+(r_{10}-l)^2 = 2\Big\{\frac{x^2}{2}(1+\delta)+\frac{y^2}{2}(1-\delta)+\frac{y\delta}{\sqrt2}+\frac{x^2+y^2+z^2}{2}\delta-\frac{x^2}{4}\delta-\frac{y^2\delta}{4}-\frac{y(x^2+y^2+z^2)}{2\sqrt2}\delta+\frac{y^3\delta}{4\sqrt2}+$$

$$+\frac{3x^2y\delta}{4\sqrt2}-\frac{3x^2y}{2\sqrt2}+\frac{y(x^2+y^2+z^2)}{\sqrt2}(1-\delta)-\frac{y^3}{2\sqrt2}(1-2\delta)\Big\}$$

$$\cdots\cdots = 2\Big\{\frac{z^2}{2}+\frac{y^2}{2}-\frac{3z^2y}{\sqrt2}+\frac{y(x^2+y^2+z^2)}{\sqrt2}-\frac{y^3}{2\sqrt2}\Big\}$$

$$\frac{\partial}{\partial y} = 2\left[\frac{\delta}{\sqrt2}+2y-\frac{y\delta}{2}+\frac{x^2}{\sqrt2}+\frac{3y^2}{\sqrt2}+\frac{z^2}{\sqrt2}-\frac{3(x^2+y^2+2z^2)\delta}{4\sqrt2}\right]$$

Ściśle uwzględnić wpływ temperatury : $l = l_0[1+\alpha\, dT]$

$$\varphi = \varphi_0 + l_0 \alpha\, dT\, \varphi'$$
$$= \varphi_0 + \varphi' l \alpha t$$

$$X = \frac{4\varphi_0}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + 2\frac{\varphi_0' l \delta}{\sqrt{2}} - \frac{m^2 R^2 T_0^{\,2}}{4\sqrt{2}}(A_0 + B_0\delta) +$$
$$+ \left[\frac{4\varphi' l}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{2\varphi'' l^2 \delta}{\sqrt{2}} - \frac{m^2 R^2 T_0^2}{4\sqrt{2}}(A_0' + B_0'\delta) l\right]\alpha t$$
$$-\frac{2m^2 R^2 t}{4\sqrt{2}}(A_0 + B_0\delta)$$

$$\overbrace{\frac{4\varphi_0}{\sqrt{2}} - \frac{m^2 R^2 T_0^2 A_0}{4\sqrt{2}}}^{X_{00}} + \left\{\left[\frac{4\varphi' l}{\sqrt{2}} - \frac{m^2 R^2 T_0^2 A_0' l}{4\sqrt{2}}\right]\alpha - \frac{2m^2 R^2 A_0}{4\sqrt{2}}\right\} t =$$
$$= X_{00}(1 - 6\alpha t)$$

$$\left[\frac{4\varphi'}{\sqrt{2}} - \frac{m^2 R^2 T_0^2 A_0'}{4\sqrt{2}}\right]\alpha l - \frac{m^2 R^2 A_0}{2\sqrt{2}} = -6\left(\frac{4\varphi_0}{\sqrt{2}} - \frac{m^2 R^2 T_0^2 A_0}{4\sqrt{2}}\right)\alpha$$

$$\left[4\frac{(\varphi' l + 6\varphi_0)}{\sqrt{2}} - \frac{m^2 R^2 T_0^2 (A_0' l + 6A_0)}{4\sqrt{2}}\right]\alpha = \frac{m^2 R^2 A_0}{2\sqrt{2}}$$

$$\alpha = \frac{\dfrac{m^2 R^2 A_0}{2\sqrt{2}}}{\dfrac{4(6\varphi_0 + \varphi' l)}{\sqrt{2}} - \dfrac{m^2 R^2 T_0^2 (6A_0 + A_0' l)}{4\sqrt{2}}}$$

$$2\left\{\frac{\delta}{\sqrt{2}} + 2y - \cancel{y\delta} - \frac{y\delta}{2} - \frac{x^2+3y^2+z^2}{2\sqrt{2}}\delta + \frac{3y^2\delta}{4\sqrt{2}} + \frac{3x^2\delta}{4\sqrt{2}} - \frac{3x^2}{\sqrt{2}} + \frac{x^2+3y^2+z^2}{\sqrt{2}}(1-\delta) - \frac{3y^2}{2\sqrt{2}}(1-2\delta)\right.$$
$$\left. - \frac{3z^2}{2\sqrt{2}} + \frac{x^2+3y^2+z^2}{\sqrt{2}} - \frac{3y^2}{2\sqrt{2}}\right\}$$

$-\frac{1}{2} + \frac{3}{4} - 1$ $\quad -\frac{1}{2} - 1$ $\quad -3+1+1$

$-\frac{3}{2} + \frac{3}{4} - 3 + 3$

$1 - \frac{3}{2} + 1$

$$= -\frac{1}{4\sqrt{2}\,k^2}\left\{\frac{2(\varphi-\varphi' l)-\varphi'' l^2}{[2\varphi+\varphi' l]^2} + \delta\,\frac{2\varphi^2+23\varphi'^2 l^2+3\varphi''^2 l^4-35\varphi\varphi' l-27\varphi\varphi'' l^2+8\varphi'\varphi'' l^3}{4[2\varphi+\varphi' l]^3}\right\}$$

$$k = \frac{3N}{2(E-U_0)} = \frac{1}{mRT}$$

$A<0$ (inaczej term. Druck współczynnik nie byłby dodatni!)

$$\bar{X} = \frac{4\varphi}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{2\varphi' l\,\delta}{\sqrt{2}} - \overbrace{\frac{m^2R^2T^2}{4\sqrt{2}}\left\{A+B\delta\right\}}^{\text{To stanowi różnicę względem dawniejszego przypadku}}$$

$$\bar{X}_0 = a\rho_0^2$$

$$\bar{X} + \underset{E\delta}{\underset{\parallel}{\bar{\Xi}}} = a\rho_0^2(1-\delta)^2 = \bar{X}_0(1-2\delta)$$

$$\bar{\Xi} = \frac{E}{1+\mu}\left[\frac{\partial u}{\partial x} + \frac{\mu}{1-2\mu}\nabla\right]$$

przy $\frac{\partial u}{\partial x} = \nabla = \delta$

$$\Xi = \frac{E}{1+\mu}\,\frac{1-\mu}{1-2\mu}\,\delta$$

$$= E\delta$$

$$E = \frac{\bar{X}_0(1-2\delta)-\bar{X}}{\delta} = -\left(2\bar{X}_0 + \frac{\bar{X}-\bar{X}_0}{\delta}\right)$$

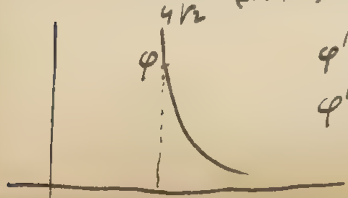
$$= -\left[\frac{8}{\sqrt{2}}\varphi - \frac{m^2R^2T^2}{2\sqrt{2}}A + \frac{2\varphi}{\sqrt{2}} + \frac{2\varphi' l}{\sqrt{2}} - \frac{m^2R^2T^2}{4\sqrt{2}}B\right]$$

$$E = -\left[\frac{10}{\sqrt{2}}\varphi + \frac{2\varphi' l}{\sqrt{2}} - \frac{m^2R^2T^2}{4\sqrt{2}}\left[2A+B\right]\right]$$

← przy tem jednak należy podstawić każdorazowo $\varphi(l)$ dla odpowiedniej temp. T

$$\frac{\partial E}{\partial T} = -\left[\frac{10}{\sqrt{2}}\varphi' + \frac{2\varphi'' l}{\sqrt{2}}\right]\alpha + \frac{m^2R^2T}{2\sqrt{2}}\left[2A+B\right]$$

$$-\frac{m^2R^2T^2}{4\sqrt{2}}(2A'+B')$$



$\varphi' < 0$

$\varphi'' > 0$

$$\frac{\partial}{\partial x}\left(\frac{u}{v^2}\right) = \frac{v\frac{\partial u}{\partial x} - 2u\frac{\partial v}{\partial x}}{v^3}$$

$$-\frac{1}{\sqrt{2}\,l^2}\left\{\frac{3\varphi\left(1-\frac{3\delta}{2}\right) - 3\varphi' l\left(1-\frac{3\delta}{2}\right) - \varphi'' l^2(1+3\delta)}{2\frac{h}{l}\left[\varphi(4-3\delta) + \varphi' l(2+3\delta) + \varphi'' l^2\delta\right]} + \right.$$

$$\left. + \frac{\frac{\varphi}{2}\left(1+\frac{5\delta}{2}\right) - \frac{\varphi' l}{2}\left(1+\frac{5\delta}{2}\right) - \frac{\varphi'' l^2}{2}}{2\frac{h}{l}\left[\varphi\left(4-\frac{\delta}{2}\right) + \varphi' l\left(2+\frac{\delta}{2}\right) + \varphi'' l^2\frac{\delta}{2}\right]}\right\} =$$

$$= -\frac{1}{2\sqrt{2}\,h^2}\left\{\frac{3(\varphi-\varphi' l) - \varphi'' l^2 + \varphi - \varphi' l - \varphi'' l^2}{[4\varphi + 2\varphi' l]^2} + \ldots\right.$$

$$\delta\,\frac{[4\varphi + 2\varphi' l]\left[-\frac{9\varphi}{2} + 9\frac{\varphi' l}{2} - 3\varphi'' l^2 + \frac{5\varphi}{2} - \frac{5\varphi' l}{2}\right] - 2\left[3(\varphi - \varphi' l) - \varphi'' l^2\right][}{[4\varphi + 2\varphi' l]^3}$$

$$\left.\frac{\left[-3\varphi + 3\varphi' l + \varphi'' l^2\right] - 2\left[\varphi - \varphi' l - \varphi'' l^2\right]\left[-\varphi + \varphi'\frac{l}{2} + \varphi''\frac{l^2}{2}\right]}{[4\varphi + 2\varphi' l]^3}\right\}$$

$$= -\frac{1}{2\sqrt{2}\,h^2}\left\{\frac{4(\varphi - \varphi' l) - 2\varphi'' l^2}{[4\varphi + 2\varphi' l]^2} + \delta\,\frac{\cancel{[4\varphi + 2\varphi' l]}\left[-2\varphi + \cancel{2\varphi' l - 3\varphi'' l^2}\right] -}{}\right.$$

$$-2\left[-10\varphi^2 - \frac{19}{2}\varphi'^2 - \frac{3}{2}\varphi''^2 + \frac{39}{2}\varphi\varphi' + \frac{15}{2}\varphi\varphi'' - \cancel{7\varphi'\varphi''}\right]$$

$$\bar{X} = \iiint \frac{X\, e^{-hU}\, dx\, dy\, dz}{\iiint e^{-hU}\, dx\, dy\, dz}$$

$$X = A + B(x,y,z) + C(x^2, y^2, z^2)$$

$$U = \alpha + \cancel{\beta(x,y,z)} + \gamma(x^2, y^2, z^2)$$

$$\bar{X} = A + \frac{\iiint \left[B(x,y,z) + C(x^2y^2z^2)\right] e^{-h[A + \cancel{\beta} + \gamma(x^2y^2z^2)]}\, dx\, dy\, dz}{\iiint e^{-h[A + \cancel{\beta} + \gamma(x^2 \ldots)]}\, dx\, dy\, dz}$$

$$= A + \iiint \frac{C(x^2y^2z^2)\, e^{-h\gamma}}{\iiint e^{-h\gamma}}$$

$$= 4\varphi\left\{\frac{1}{\sqrt{2}} + \frac{\delta}{2\sqrt{2}}\right\} + \frac{2\varphi' l \delta}{\sqrt{2}} \quad -$$

$$-\frac{\iiint\left\{\frac{x^2}{\sqrt{2}\, l^2}\left[3\varphi\left(1 - \frac{3\delta}{2}\right) + 3\varphi' l\left(-1 + \frac{3\delta}{2}\right) + \varphi'' l^2 (1 + 3\delta)\right] + \frac{y^2+z^2}{\sqrt{2}\, l^2}\left[\frac{\varphi}{2}\left(1 + \frac{5\delta}{2}\right) - \frac{\varphi' l}{2}\left(1 + \frac{5\delta}{2}\right) - \frac{\varphi'' l^2}{2}\right]\right\} e^{-h\ldots}}{\int e^{-\frac{h}{l^2}\left[x^2 (\varphi[4 - 3\delta] + \varphi' l\,[2 + 3\delta] + \varphi'' l^2 \delta\right)} dx \cdot \iint e^{-\frac{h}{l^2}(y^2+z^2)\left(\varphi\left[4 - \frac{\delta}{2}\right] + \varphi' l \left[2 + \frac{\delta}{2}\right] + \varphi'' l^2 \frac{\delta}{2}\right)]}\, dy\, dz}$$

$$+X = 4\varphi\left\{-\frac{1}{\sqrt{2}} - \frac{\delta}{2\sqrt{2}} + \frac{x}{2} - \frac{3x\delta}{4} + \frac{3x^2}{4\sqrt{2}} + \frac{y^2+z^2}{8\sqrt{2}} - \frac{9\delta x^2}{8\sqrt{2}} + \frac{5(y^2+z^2)\delta}{16\sqrt{2}}\right\}$$

$$+2\varphi' l\left\{-\frac{\delta}{\sqrt{2}} + x + \frac{3x\delta}{2} - \frac{3x^2}{2\sqrt{2}} - \frac{y^2+z^2}{4\sqrt{2}} + \frac{9x^2\delta}{4\sqrt{2}} - 5\frac{(y^2+z^2)\delta}{8\sqrt{2}}\right\}$$

$$+\frac{2\varphi'' l^2}{\cancel{8}}\left\{ \frac{x\delta}{2} - \frac{x^2}{2\sqrt{2}} - \frac{(y^2+z^2)}{4\sqrt{2}} - 3\frac{x^2\delta}{2\sqrt{2}}\right\}$$

$\cancel{2\varphi\sqrt{2} + 2\varphi + \varphi' l}$

$$\sum_1^{12}(r-l) = 4\delta + 4(x^2+y^2+z^2) - \frac{3x^2\delta}{} - \frac{(y^2+z^2)}{2}\delta$$

$$\sum_1^{12}(r-l)^3 = \quad 6x^2\delta + 3(y^2+z^2)\delta$$

$$U = 12\Phi(l) + \varphi l\left[4\delta + 4\frac{(x^2+y^2+z^2)}{l^2} - 3x^2\delta - \frac{y^2+z^2}{2}\delta\right]$$

$$+\varphi' l\left[\quad 2\frac{(x^2+y^2+z^2)}{l^2} + 3x^2\delta + \frac{y^2+z^2}{2}\delta\right]$$

$$+\varphi'' l^3\left[\qquad x^2\delta + \frac{y^2+z^2}{2}\delta\right]$$

φ derivative dla

$$\varphi l\left[4\delta + \left(4-\frac{\delta}{2}\right)(y^2+z^2) - \frac{5x^2\delta}{2}\right]$$

$$\left(2+\frac{\delta}{2}\right)(y^2+z^2) + \frac{5x^2\delta}{2}\Big]$$

$$U_0 = 12\Phi(l) + 4\varphi\frac{r^2}{l} + 2\frac{\varphi' r^2}{l}$$

$$-\frac{\partial U}{\partial r} = -4(2\varphi+\varphi')\frac{r}{l}$$

$$(2\varphi+\varphi') \gg 0$$

$$-2 + 6\left(1+\frac{\delta}{3}\right) + x^2\left(2-\frac{3\delta}{2}\right) + (y^2+z^2)\left(2-\frac{\delta}{4}\right)$$

$$-6\left[1+\frac{2\delta}{3} + x^2 \qquad + y^2+z^2\right]$$

$$+2\left[1+\delta\right] + 3(x^2+y^2+z^2)\left(1+\frac{\delta}{3}\right) + x^2\left(1+\frac{3\delta}{2}\right) + (y^2+z^2)\left(1-\frac{\delta}{4}\right)$$

$$\cancel{=2\delta-4\delta+2\delta} \quad x^2\left(2-\frac{3\delta}{2} - \cancel{6} + \cancel{6} + 2\delta + 2 + 3\delta\right)$$

$$(y^2+z^2)\left(2-\frac{\delta}{4} + 1 - \frac{\delta}{4} - 6\right)$$

$$x^2\left(4+\frac{7\delta}{2}\right)$$

$$(r_1-l)^3 = -\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right)+\frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right) + 3\left[\frac{x^2(\ldots)}{2}\frac{\delta}{2}+\frac{y^2}{2}\frac{\delta}{2}+\frac{x^2y(1+\frac{\delta}{2})}{2\sqrt2}-\frac{xy^2(1-\frac{\delta}{2})}{2\sqrt2}\right]$$

$$+6\left[-\frac{xy\delta}{4}-\frac{x(x^2+y^2+z^2)}{4\sqrt2}\delta+\frac{x^3\delta}{8\sqrt2}+\frac{xy^2\delta}{8\sqrt2}-\frac{x^2y\delta}{4\sqrt2}+\frac{y(x^2+y^2+z^2)}{4\sqrt2}\delta-\right.$$

$$\left.-\frac{x^2y}{8\sqrt2}\delta-\frac{y^3\delta}{8\sqrt2}+\frac{xy^2}{4\sqrt2}\delta\right]$$

$$r_1+r_2+r_3+r_4 = 4\left\{1+\frac{\delta}{2}-\frac{x}{\sqrt2}\left(1+\frac{\delta}{2}\right)+\frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right)-\frac{x^2}{4}\left(1+\frac{\delta}{2}\right)-\frac{y^2}{8}\left(1-\frac{3\delta}{2}\right)-\frac{z^2}{8}\left(1-\frac{3\delta}{2}\right)\right.$$

$$\left.+\frac{x(x^2+y^2+z^2)}{2\sqrt2}\left(1-\frac{\delta}{2}\right)-\frac{x^3}{4\sqrt2}\left(1+\frac{\delta}{2}\right)-\frac{3xy^2}{8\sqrt2}\left(1-\frac{3\delta}{2}\right)-\frac{3xz^2}{8\sqrt2}\left(1+\frac{3\delta}{2}\right)\right\}$$

$$-\frac{z^2\delta}{8}$$

$$\sum_1^4(r_1-l)^2 = 4\left\{\frac{x^2}{2}(1+\delta)+\frac{y^2}{4}(1-\delta)+\frac{z^2}{4}(1-\delta)-\frac{x\delta}{\sqrt2}+\frac{x^2+y^2+z^2}{2}\delta-\frac{x^2\delta}{4}-\frac{y^2\delta}{8}+\frac{x(x^2+y^2+z^2)}{2\sqrt2}\delta\right.$$

$$-\frac{x^3\delta}{4\sqrt2}-\frac{3xy^2\delta}{8\sqrt2}-\frac{3xz^2\delta}{8\sqrt2}-\frac{x(x^2+y^2+z^2)}{\sqrt2}+\frac{x^3}{2\sqrt2}(1+\delta)+\frac{3xy^2}{4\sqrt2}(1-\delta)$$

$$\left.+\frac{3xz^2}{4\sqrt2}(1-\delta)+\cancel{\frac{xy^2}{2\sqrt2}(1-\delta)}+\cancel{\frac{xz^2}{2\sqrt2}(1-\delta)}\right\}$$

$$\sum_{1\,9}^{4\,12}(r_1-l)^2 = 8\left\{\frac{x^2}{2}(1+\delta)+\frac{y^2}{4}(1-\delta)+\frac{z^2}{4}(1-\delta)+\frac{x^2+y^2+z^2}{2}\delta-\frac{x^2\delta}{4}-\frac{y^2+z^2}{8}\delta\right\}$$

$$\sum_{5678}(r-l)^2 = 4\left\{\quad +\frac{y^2}{2}\quad+\frac{z^2}{2}\right.$$

$$\sum_1^{12}(r-l)^2 = 4(x^2+y^2+z^2)+(6x^2+y^2+z^2)\delta$$

$$\sum_1^4(r_1-l)^3 = 4\left\{-\frac{x^3}{2\sqrt2}\cancel{\left(1+\frac{3\delta}{2}\right)}+\frac{3x^2\delta}{4}+\frac{3y^2\delta}{8}+\frac{3z^2\delta}{8}-\frac{3x(y^2+z^2)(1-\frac{\delta}{2})}{4\sqrt2}-\frac{3x(x^2+y^2+z^2)\delta}{2\sqrt2}\right.$$

$$\left.+\cancel{\frac{3x^3\delta}{4\sqrt2}}+\cancel{\frac{3xy^2\delta}{8\sqrt2}}+\cancel{\frac{3xz^2\delta}{8\sqrt2}}+9\frac{x(y^2+z^2)\delta}{8\sqrt2}\right\}$$

$$=4\left\{\frac{3x^2\delta}{4}+(y^2+z^2)\frac{3\delta}{8}-\frac{3x(y^2+z^2)}{4\sqrt2}-\frac{x^3}{2\sqrt2}-\frac{3x^3\delta}{2\sqrt2}\right\}$$

$$\Phi(l) + (r-l)\varphi + \frac{(r-l)^2}{2}\varphi' + \frac{(r-l)^3}{2\cdot 3}\varphi''$$

$$r_1 = \sqrt{1+\delta}\,\sqrt{1+\alpha} = \left(1+\frac{\delta}{2}\right)\left[1+\frac{\alpha}{2}-\frac{\alpha^2}{8}+\frac{\alpha^3}{16}\right]$$

$$\sqrt{1+\delta}\,\sqrt{1+\alpha+\beta} = \left(1+\frac{\delta}{2}\right)\left[1+\frac{\alpha}{2}+\left(\frac{\beta}{2}-\frac{\alpha^2}{8}\right)-\frac{\alpha\beta}{4}+\frac{\alpha^3}{16}\right]$$

$$\alpha = -x\sqrt{2}+\frac{y\sqrt{2}}{1+\delta} \qquad \beta = \frac{x^2+y^2+z^2}{1+\delta}$$

$$r_1 = \left(1+\frac{\delta}{2}\right)\left[1-\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}(1+\delta)}+\frac{x^2+y^2+z^2}{2(1+\delta)}-\frac{x^2}{4}-\frac{y^2}{4(1+\delta)^2}+\frac{xy}{2(1+\delta)}+\frac{x(x^2+y^2+z^2)}{2\sqrt{2}(1+\delta)}\right.$$
$$\left.-\frac{y(x^2+y^2+z^2)}{2\sqrt{2}(1+\delta)^2}+\frac{2\sqrt{2}}{16}\left(-x^3+\frac{y^3}{(1+\delta)^3}+\frac{3x^2y}{1+\delta}-\frac{3xy^2}{(1+\delta)^2}\right)\right]$$

$$r_1 = 1+\frac{\delta}{2}-\frac{x}{\sqrt{2}}\left(1+\frac{\delta}{2}\right)+\frac{y}{\sqrt{2}}\left(1-\frac{\delta}{2}\right)+\frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right)-\frac{x^2}{4}\left(1+\frac{\delta}{2}\right)-\frac{y^2}{4}\left(1-\frac{3\delta}{2}\right)+\frac{xy}{2}\left(1-\frac{\delta}{2}\right)$$
$$+\frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\left(1-\frac{\delta}{2}\right)-\frac{y(x^2+y^2+z^2)}{2\sqrt{2}}\left(1-\frac{3\delta}{2}\right)-\frac{x^3}{4\sqrt{2}}\left(1+\frac{\delta}{2}\right)+\frac{y^3}{4\sqrt{2}}\left(1-\frac{5\delta}{2}\right)+\frac{3x^2y}{4\sqrt{2}}\left(1-\frac{\delta}{2}\right)-$$
$$-\frac{3xy^2}{4\sqrt{2}}\left(1-\frac{3\delta}{2}\right)$$

$$(r_1-l)^2 = \frac{x^2}{2}(1+\delta)+\frac{y^2}{2}(1-\delta)-\frac{x\delta}{\sqrt{2}}+\frac{y\delta}{\sqrt{2}}+\frac{x^2+y^2+z^2}{2}\delta-\frac{x^2}{4}\delta-\frac{y^2}{4}\delta+\frac{xy}{2}\delta-xy+$$
$$+\frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\delta-\frac{y(x^2+y^2+z^2)}{2\sqrt{2}}\delta-\frac{x^3\delta}{4\sqrt{2}}+\frac{y^3\delta}{4\sqrt{2}}+\frac{3x^2y}{4\sqrt{2}}\delta-\frac{3xy^2}{4\sqrt{2}}\delta$$
$$-\frac{x(x^2+y^2+z^2)}{\sqrt{2}}+\frac{x^3}{2\sqrt{2}}(1+\delta)+3\frac{xy^2}{2\sqrt{2}}(1-\delta)-\frac{3x^2y}{2\sqrt{2}}+\frac{y(x^2+y^2+z^2)}{\sqrt{2}}(1-\delta)-$$
$$-\cancel{\frac{x^2y}{2\sqrt{2}}}-\frac{y^3}{2\sqrt{2}}(1-2\delta)+\cancel{\frac{xy^2}{\sqrt{2}}(1-\delta)}$$

$$r_1^3 + r_2^3 + r_3^3 + r_4^3 = 4\Big\{ 1 + \frac{3\delta}{2} - \frac{3x}{\sqrt2}\left(1+\frac{3\delta}{2}\right) + \frac{3(x^2+y^2+z^2)}{2}\left(1+\frac{\delta}{2}\right) + \frac{3x^2}{4}\left(1+\frac{3\delta}{2}\right) + \frac{3y^2}{8}\left(1-\frac{\delta}{2}\right) + \frac{3z^2}{8}\left(1-\right.$$

$$- \frac{3x(x^2+y^2+z^2)}{2\sqrt2}\left(1+\frac{\delta}{2}\right) + \frac{x^3}{4\sqrt2}\left(1+\frac{3\delta}{2}\right) + \frac{3xy^2}{8\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{3xz^2}{8\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{3z^2}{8}\left(1-\frac{\delta}{2}\right)\Big\}$$

$$+ \frac{3x^3\delta}{16\sqrt2} + \frac{3(y^2+z^2)x\delta}{32\sqrt2}$$

$$\frac{\partial}{\partial x} = 4\Big[\Big\{ -\frac{3}{\sqrt2}\left(1+\frac{3\delta}{2}\right) + 3x\left(1+\frac{\delta}{2}\right) + \frac{3x}{2}\left(1+\frac{3\delta}{2}\right) - \frac{3(3x^2+y^2+z^2)}{2\sqrt2}\left(1+\frac{\delta}{2}\right) + \frac{3x^2}{4\sqrt2}\left(1+\frac{3\delta}{2}\right) +$$

$$+ \frac{3y^2}{8\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{3z^2}{8\sqrt2}\left(1-\frac{\delta}{2}\right)\Big\}$$

$$- 3\Big\{ -(1+\delta)\sqrt2 \quad + 2x \Big\}$$

$$+ 3\Big\{ -\frac{1}{\sqrt2}\left(1+\frac{\delta}{2}\right) + \frac{x}{2l} - \frac{3x\delta}{4l} + \frac{6x^2-y^2-z^2}{8\sqrt2\, l^2} - \frac{18x^2 - [illegible]}{16\sqrt2}\delta \Big\}\Big]$$

$$= 4\Big[\frac{3x\delta}{2} - \frac{3x^2}{2\sqrt2} - [illegible] - \frac{3}{2\sqrt2}y^2 + [illegible] - \frac{3z^2}{2\sqrt2} - \frac{9x^2\delta}{2\sqrt2}$$

$$\bar{X} = -\left\{\left[\varphi - l\varphi' + \frac{l^2\varphi''}{2}\right]\left[-\frac{4}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{2x}{l} - \frac{3x\delta}{l} + \frac{3x^2-y^2+2z^2}{4\sqrt{2}\,l^2} - \frac{9x^2-7y^2+2z^2}{8\sqrt{2}\,l^2}\delta\right]\right.$$

$$+\left[\varphi' - l\varphi''\right] 2l\left[-(1+\delta)\sqrt{2} + \frac{2x}{l}\right] +$$

$$+ \cancel{\quad}$$

$$+ \cancel{\quad}$$

$$\left.+2\frac{\varphi''}{3}l^2\left[-\frac{3}{\sqrt{2}}\left(1+\frac{3\delta}{2}\right) + \frac{9}{2}x + \frac{15}{4}x\delta - \frac{30x^2+9x^2\delta+y^2+\frac{3y^2\delta}{2}+7z^2+\frac{5z^2\delta}{2}}{8\sqrt{2}}\right]\right\}$$

$$\bar{X} = \iiint \frac{X\, e^{-h\left[(\varphi - l\varphi' + l^2\frac{\varphi''}{2})\sum r + \frac{(\varphi' - l\varphi'')}{2}\sum r^2 + \frac{\varphi''}{6}\sum r^3\right]}\, dx\,dy\,dz}{\int e^{-h[\;\cdot\;]}\, dx\,dy\,dz}$$

$$+\frac{3}{2\sqrt{2}}(y\mp z)(x^2+y^2+z^2) - \frac{(y\mp z)^3}{4\sqrt{2}}$$

$$r^3_{5,7} = 1 + \frac{3}{\sqrt{2}}y \mp \frac{3}{\sqrt{2}}z + \frac{3}{2}(x^2+y^2+z^2) + \frac{3}{4}(y^2+z^2) \mp \frac{3}{2}yz + \cancel{\quad}$$

$$\cancel{\quad}$$

$$r_5^3 + r_7^3 + r_6^3 + r_8^3 = 1 + \frac{3}{2}(x^2+y^2+z^2) + \frac{3}{4}(y^2+z^2) = 4\left[1 + \frac{9}{4}(x^2+y^2+z^2) - \frac{3}{4}x^2\right]$$

$$= 4 + 6(x^2+y^2+z^2) + 3(y^2+z^2)$$

$$\sum r^3 = 12 + 12\delta + 24(x^2+y^2+z^2) + \frac{9}{2}\delta(x^2+y^2+z^2) + \frac{21}{2}x^2\delta$$

$$e^{-hU} = e^{-h}\left[(x^2+y^2+z^2)\left\{\left(\frac{\varphi}{\ell}-\varphi'\right)\left(4+\frac{\delta}{2}\right)+6\varphi'\right\} - \frac{5}{2}x^2\delta\left\{\frac{\varphi}{\ell}-\varphi'\right\}\right]$$

$$= \frac{(\varphi-\ell\varphi')}{4\sqrt{2}\,\ell^2}\left\{\frac{\left(3-\frac{9\delta}{2}\right)}{2h^2\left[\left(\frac{\varphi}{\ell}-\varphi'\right)\left(4+\frac{\delta}{2}\right)+6\varphi'-\frac{5}{2}\delta\left(\frac{\varphi}{\ell}-\varphi'\right)\right]^2} + \frac{-\left(1-7\frac{\delta}{2}\right) + (2-\delta)}{2h^2\left[\left(\frac{\varphi}{\ell}-\varphi'\right)\left(4+\frac{\delta}{2}\right)+6\varphi'\right]^2}\right\}$$

$$r^3_{1,2} = 1+\frac{3\delta}{2} - \frac{3x}{\sqrt{2}}\left[1+\frac{3\delta}{2}\right] \pm \frac{3y}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] + \frac{3(x^2+y^2+z^2)}{2}\left[1+\frac{\delta}{2}\right] + \frac{3x^2}{4}\left[1+\frac{3\delta}{2}\right] +$$
$$+\frac{3y^2}{4}\left[1-\frac{\delta}{2}\right] \mp \frac{3xy}{2}\left[1+\frac{\delta}{2}\right] - \frac{3x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1+\frac{\delta}{2}\right] \pm \frac{3y(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^3}{4\sqrt{2}}\left[1+\frac{3\delta}{2}\right]$$
$$\mp\frac{y^3}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right] \mp \frac{3x^2y}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] + \frac{3xy^2}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right] \pm [illegible]$$

$$r^3_{3,4} = 1+\frac{3\delta}{2} - \frac{3x}{\sqrt{2}}\left[1+\frac{3\delta}{2}\right] \mp \frac{3z}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] + \frac{3(x^2+y^2+z^2)}{2}\left[1+\frac{\delta}{2}\right] + \frac{3x^2}{4}\left[1+\frac{3\delta}{2}\right] +$$
$$+\frac{3z^2}{4}\left[1-\frac{\delta}{2}\right] \pm \frac{3xz}{2}\left[1+\frac{\delta}{2}\right] - \frac{3x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1+\frac{\delta}{2}\right] \mp \frac{3z(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^3}{4\sqrt{2}}\left[1+\frac{3\delta}{2}\right] +$$
$$\pm\frac{z^3}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right] \pm \frac{3x^2z}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] - \frac{3xz^2}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right]$$

$$r_1^3+r_2^3+r_3^3+r_4^3 = 8\left\{1+\frac{3\delta}{2} + \frac{3(x^2+y^2+z^2)}{2}\left(1+\frac{\delta}{2}\right) + \frac{3x^2}{4}\left(1+\frac{3\delta}{2}\right) + \frac{3y^2}{8}\left(1-\frac{\delta}{2}\right) + \frac{3z^2}{8}\left(1-\frac{\delta}{2}\right)\right\}$$

$+r_9 \quad 10 \quad 11 \quad 12 \qquad 4 \qquad +9(x^2+y^2+z^2) \qquad -3x^2$

$\cancel{r^3 = 12\,\ell^3(1+\delta) + 2(x^2+y^2+z^2)\,\ell\left(7+3\delta\right) + 6x^2\left(1+\frac{3\delta}{2}\right) + y^2\left(3+\frac{\delta}{2}\right) + z^2\left(3+\frac{\delta}{2}\right)}$

$\cancel{= 12\,\ell^3(1+\delta) + (x^2+y^2+z^2)\,\ell} \; [illegible]$

$\cancel{3\left\{\delta + \frac{3}{2} \ldots + 6\left(3+\frac{21}{2}\delta\right)\right\}}$

$$\frac{\partial}{\partial x}(r_1+r_2+r_3+r_4) = 4\left\{-\frac{1}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{x}{2l} - \frac{3x\delta}{4l} + \frac{3x^2-y^2+2z^2}{4\sqrt{2}\,l^2} - \frac{9x^2-7y^2+2z^2}{8\sqrt{2}\,l^2}\delta\right.$$

$$\frac{6x^2+y^2+z^2}{8\sqrt{2}} - \frac{18x^2-5y^2-5z^2}{16\sqrt{2}}\delta$$

$$\frac{\partial}{\partial x}(r_1^2+r_2^2+r_3^2+r_4^2) = 4l\left[-(1+\delta)\sqrt{2} + \frac{2x}{l}\right]$$

$$X = -\left\{(\varphi - l\varphi')\frac{\partial}{\partial x}(r_1+r_2+\dots) + \frac{\varphi'}{2}\frac{\partial}{\partial x}(r_1^2+r_2^2+\dots)\right\}$$

$$\bar{X} = \frac{\iiint_{-\infty}^{+\infty} X\, e^{-h\left[(\varphi-l\varphi')\sum r + \frac{\varphi'}{2}\sum r^2\right]}\,dx\,dy\,dz}{\iiint_{-\infty}^{+\infty} e^{-h\left[(\varphi-l\varphi')\sum r + \frac{\varphi'}{2}\sum r^2\right]}\,dx\,dy\,dz}$$

$$= 2\sqrt{2}\left[\varphi\left(1+\frac{\delta}{2}\right) + l\varphi'\frac{\delta}{2}\right] +$$

$$-\frac{\iiint(\varphi-l\varphi')\left[\frac{3x^2-y^2+2z^2}{4\sqrt{2}\,l^2} - \frac{9x^2-7y^2+2z^2}{8\sqrt{2}\,l^2}\delta\right]e^{-h[\quad]}\,dx\,dy\,dz}{\iiint e^{-h[\quad]}\,d\dots}$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2}dx = \sqrt{\frac{\pi}{\alpha}} \qquad \frac{\int_{-\infty}^{+\infty} x^2 e^{-\alpha x^2}dx}{\int e^{-\alpha x^2}dx} = \frac{1}{2\alpha^2}$$

$$\int_{-\infty}^{+\infty} x^2 e^{-\alpha x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$r_{5,7}^3 = 1 + \frac{3y}{\sqrt{2}} + \frac{3z}{\sqrt{2}} + 3\frac{x^2+y^2+z^2}{2} \pm \frac{z(\dots)}{2\sqrt{2}} - \frac{y^3}{4\sqrt{2}} \pm \frac{z^3}{4\sqrt{2}} \pm \frac{3y^2z}{4\sqrt{2}} - \frac{3yz^2}{4\sqrt{2}} + \dots$$

$$r_{6,8}^3 = 1 - \frac{y}{\sqrt{2}} \dots \quad + \frac{y^3}{4\sqrt{2}}$$

7.8.7

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$$\frac{r_{1,2}}{l} = 1+\frac{\delta}{2} - \frac{x}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] \pm \frac{y}{\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{y^2}{4}\left[1-\frac{3\delta}{2}\right] +$$

$$\pm \frac{xy}{2}\left[1-\frac{\delta}{2}\right] + \frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] \mp \frac{y(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{3\delta}{2}\right] - \frac{x^3}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] +$$

$$\pm \frac{y^3}{4\sqrt{2}}\left[1-\frac{5\delta}{2}\right] \pm \frac{3x^2y}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right] - \frac{3xy^2}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right]$$

$$\cancel{\frac{r_1+r_2+r_9+r_{10}}{l} = 3+\frac{\delta}{2}}$$

$$\frac{r_1+r_2+r_9+r_{10}}{l} = 4\left\{1+\frac{\delta}{2} + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{y^2}{4}\left[1-\frac{3\delta}{2}\right]\right\}$$

$$\frac{r_{3,4}}{l} = 1+\frac{\delta}{2} - \frac{x}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] \mp \frac{z}{\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{z^2}{4}\left[1-\frac{3\delta}{2}\right] \pm$$

$$\mp \frac{xz}{2}\left[1-\frac{\delta}{2}\right] + \frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] \pm \frac{z(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{3\delta}{2}\right] - \frac{x^3}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] -$$

$$\mp \frac{z^3}{4\sqrt{2}}\left[1-\frac{5\delta}{2}\right] \mp \frac{3x^2z}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right] - \frac{3xz^2}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right]$$

$$\frac{r_3+r_4+r_{11}+r_{12}}{l} = 4\left\{1+\frac{\delta}{2} + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{z^2}{4}\left[1-\frac{3\delta}{2}\right]\right\}.$$

$$\frac{r_{5,7}}{l} = 1 + \frac{y}{\sqrt{2}} \mp \frac{z}{\sqrt{2}} + \frac{x^2+y^2+z^2}{2} - \frac{y^2}{4} - \frac{z^2}{4} \pm \frac{yz}{2} + \frac{y(x^2+y^2+z^2)}{2\sqrt{2}} \pm \frac{z(x^2+y^2+z^2)}{2\sqrt{2}} -$$

$$+ \frac{y^3}{4\sqrt{2}} \mp \frac{z^3}{4\sqrt{2}} \mp \frac{3y^2z}{4\sqrt{2}} + \frac{3yz^2}{4\sqrt{2}}$$

$$r_5+r_7+r_6+r_8 = 4\left\{1 + \frac{x^2+y^2+z^2}{2} - \frac{y^2}{4} - \frac{z^2}{4}\right\}$$

$$\sum r = 12l + 4\delta l + \frac{(x^2+y^2+z^2)}{l}\left(4-\frac{\delta}{2}\right) - \frac{5}{2}\frac{x^2\delta}{l}$$

$$\sum r^2 = 4l^2\left\{3+2\delta+\frac{3(x^2+y^2+z^2)}{l^2}\right\}$$

$$X = -\frac{\partial}{\partial x}\left[\Phi(l) + (r-l)\,\varphi(l) + \frac{(r-l)^2}{2}\varphi'(l) + \frac{(r-l)^3}{2.3}\varphi''(l)\right]$$

$$= -\frac{\partial}{\partial x}\left[\varphi\, r + \frac{r^2 - 2lr}{2}\varphi' + \frac{r^3 - 3r^2 l + 3 r l^2}{2.3}\varphi''\right]$$

$$= -\left[\left(\varphi - l\varphi' + \frac{l^2}{2}\varphi''\right)\frac{\partial r}{\partial x} + \left(\frac{\varphi'}{2} - \frac{\varphi''}{2}\right)\frac{\partial (r^2)}{\partial x} + \frac{\varphi''}{6}\frac{\partial (r^3)}{\partial x}\right]$$

$$\varphi\left[-\frac{1}{\sqrt2}\left(1+\frac{\delta}{2}\right) + x\left(1-\frac{\delta}{2}\right) - \frac{x}{2}\left(1+\frac{\delta}{2}\right) + \cancel{\frac{y}{2}\left(1-\frac{\delta}{2}\right)} + \frac{6x^2+y^2+z^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right) - \cancel{\frac{2xy}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)} - \frac{3x^2}{4\sqrt2}\left(1+\frac{\delta}{2}\right)\right.$$

$$-\frac{3x^2}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)$$

$$\left. + \cancel{\frac{6xy}{4\sqrt2}\left(1-\frac{\delta}{2}\right)} - \frac{3y^2}{8\sqrt2}\left(1-\frac{3\delta}{2}\right)\right] + \frac{\varphi'}{2}\left[2x(1+\delta) - \frac{\delta}{\sqrt2} + \frac{x\delta}{2} - \cancel{y} + \cancel{\frac{y\delta}{2}} + \frac{3x^2+y^2+z^2}{2\sqrt2}\delta\right.$$

$$- \cancel{\frac{xy\delta}{\sqrt2}} - \frac{3x^2\delta}{4\sqrt2} + \cancel{\frac{xy\delta}{2\sqrt2}} - \frac{3y^2\delta}{4\sqrt2} - \cancel{\delta} - \frac{3x^2+y^2+z^2}{\sqrt2} + \frac{3x^2(1+\delta)}{2\sqrt2} + 3y^2\frac{(1-\delta)}{2\sqrt2} - \cancel{\frac{3y}{\sqrt2}}\Big]$$

$$\left. + \cancel{\frac{2yx(1-\delta)}{\sqrt2}} - \cancel{\frac{2xy}{2\sqrt2}} + \cancel{\frac{y^2}{\sqrt2}(1-\delta)}\right]$$

$$+ \frac{\varphi''}{6}\frac{\partial}{\partial x}\left[-\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right) + \cancel{\frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)} + 3\left[\cancel{\frac{x^2y}{2\sqrt2}}\left(1+\frac{\delta}{2}\right) - \frac{xy^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right)\right] + \frac{3x^2\delta}{4} + \frac{3y^2\delta}{8} + \frac{3z^2\delta}{8}\right]$$

$$= \varphi\left[\cancel{-\frac{1}{\sqrt2}}\left(1+\frac{\delta}{2}\right) + \cancel{\frac{x}{2}\left(1+\frac{3\delta}{2}\right)} + \cancel{\frac{3x^2-y^2}{4\sqrt2}} + \frac{3x^2 + \frac{\delta}{2}y^2 + z^2}{4\sqrt2}\delta\right] - \frac{3xz^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right)$$

$$+ \frac{\varphi''}{6}\left[-\frac{3x^2}{2\sqrt2}\left(1+\frac{3\delta}{2}\right) - \frac{3y^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{3x\delta}{2}\right] - \frac{3z^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right)$$

$\int \left(\frac{d\psi}{dn} - \psi \frac{d}{dn} \right) ds = \iint \left(\psi \nabla^2 \psi - \right) dx\,dy$

$\psi = \quad = \qquad \zeta = \nabla^2 \psi =$ vortex

$\iint (\nabla^2 \psi)^2 \, dx\,dy = \iint \psi \, \nabla^4 \psi \, dx\,dy$

$= 0 \qquad \nabla^4 \psi =$

then by [illegible] $\nabla^2 \psi = 0$

$\therefore \psi = 0$ because on contour $\psi = 0$

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... made for the building of
... at Jerusalem. The eager
... people to build a beautiful
... thus helping to replace the mag-
nificent cathedrals of their own country, now deserted and desecrated, was shown by the nature of the gifts, which included rings, bracelets, and promises of cement, gravel, and free labour by masons, carpenters, and others.

Every week a little progress has been made, and the church is already a beautiful building, typically Armenian.

... cycles
13 ... ks, tor-boats, and 445

They police towns and the... prairies; they are to be found ... Arctic outposts of civilisation. When, in 1920, an Inspector made a tour of these outposts he travelled 2000 miles by dog team, visiting most of the islands to the north of Canada and reaching Bache Peninsula, on Ellesmere Island, within 700 miles of the North Pole.

So romance has not yet departed from these Mounties, nor is it likely to while they have the patrolling of these far north wastes.

THE PENNY-IN-THE-SLOT PAPER

Newspapers are being sold at London stations through the agency of penny-in-the-slot machines.

Each machine has three sections, holding 40 newspapers, and each section has a space in which can be seen the front page of the paper.

As in the case of other automatic machines, the idea permits considerable saving of human labour, as bookstalls and shops can be closed as soon as the last deliveries of the evening papers have arrived.

One wonders what effect, if any, this mechanical supply of news of the world's happenings will have on the readers. We have a phrase "hot from the press," but will not news come cold to us from these steel boxes? There has ever been a glamour in snatching our paper from the flying newsboy, and there is something romantic about the news-paper stall or counter with all the day's events set out in a variety of guises before us.

... beat... which ... pa...

It is on ...

Thanks to their handi... Mr Robb, who has constru... sided hive, they will soon kno... the habits of bees.

They have watched the Queen ... from her cell and all the excitement in the hive before the bees swarmed. Now the bees have settled down to everyday domesticity, unperturbed by the small faces constantly on the watch.

SOMETHING WE FORGOT

We forgot something the other day: the fact that the West Kent Staghounds hunt only carted stags.

And so, when we talked of the British stag which lately died at Dunkirk crossing the Channel to escape being torn to pieces by the hounds, we we... wrong. He would not have been killed, but put back into the cart and saved for another day.

Our minds must have been wandering to our old friends down in Somerset.

THE NEW PATIENT

A young blue-tit, found abandoned and nearly dead, was put to bed on some cotton-wool in the Royal Hospital for Incurables at Putney.

Now he is quite well again, but refuses to go. He seems to prefer human beings to his relations outside, and certainly the patients enjoy his company.

an...
little...

The ... like a ... Aucklan... of its ea... conical ... native shr... of the oth... vation ev...

S...

O...
the ... bee... the ... up... nobo... were ... strean...

Th... have ... which ... tables ... upheav... after all.

235

[illegible]

[illegible] 1^a [illegible]

[illegible]

[illegible]

Господинъ Мюллеръ пришёлъ ~~въ~~ дому господина Небохатова и ~~оддалъ~~ отдалъ свою ~~визитное~~ визитную карточку слугѣ. Небогатовъ вышелъ къ нему, попросилъ его садиться и ~~с~~ спросилъ ~~что~~ чѣмъ ему ~~[illegible]~~ послужить. Мюллеръ показалъ рекомандательное письмо отъ ихъ общаго прiятеля Булыгина и попросилъ о ~~м~~ мѣсто приказчика въ торговомъ домѣ Небогатова. ~~[illegible]~~ Этотъ былъ очень радъ з ~~этого~~ этому ~~предложенiя~~ предложенiю, такъ какъ было у него такое мѣсто незанято, и прiнялъ его въ службу. Узнавъ Мюллера имя, отчество и адресъ прощался ~~его~~ съ нимъ ~~[illegible]~~ а тотъ пошёлъ ~~з~~ дома.

$\rho^2 = y^2 + (x-\xi)^2$

$$\mathfrak{F}_2 = 2\pi\left\{\int\limits_{y=0}^{\infty}\int\limits_{\xi=x}^{\infty} y\,dy\,d\xi \cdot n_\xi \cdot f\left(\sqrt{(x-\xi)^2+y^2}\right)\cdot\frac{\xi - x}{\sqrt{(x-\xi)^2+y^2}} - \int\int\limits_{\xi=0}^{x} \cdots \frac{x-\xi}{\sqrt{\cdots}}\right\}$$

$$= 2\pi \int\limits_{\xi=x}^{\infty} d\xi\, n_\xi \int\limits_{\rho=\xi-x}^{\infty} d\rho\; f(\rho)\cdot(\xi - x) - \cdots \int\limits_{\xi=0}^{x} d\xi\, n_\xi \int\limits_{\rho=x-\xi}^{\infty} f(\rho)\, d\rho\, (x-\xi)$$

$$z\int\limits_z^{\infty} f(\rho)\,d\rho = \mathfrak{F}(z)$$

$$\frac{1}{2h}\frac{1}{n}\frac{\partial n}{\partial x} = 2\pi\left\{\int\limits_{\xi=x}^{\infty} n_\xi \cdot \mathfrak{F}(\xi - x)\,d\xi - \int\limits_{\xi=0}^{x} n_\xi\cdot\mathfrak{F}(x-\xi)\,d\xi\right\} = 2\pi\left[\int\limits_0^{\infty} - \int\limits_0^{x} n_\xi\,\mathfrak{F}(x-\xi)\,d\xi\right]$$

$$\mathfrak{F}'(z) = \int\limits_z^{\infty} f(\rho)\,d\rho - z\,f(z)$$

$$\frac{1}{2h}\frac{dn}{dx} = \frac{2\pi A}{\alpha}\, n_x\left[\int\limits_x^{\infty} e^{-\alpha(\xi - x)}\, n_\xi\, d\xi - \int\limits_{\xi=0}^{x} e^{-\alpha(x-\xi)}\, n_\xi\, d\xi\right]$$

$$\frac{1}{2h}\frac{d}{dx}\left(\frac{1}{n}\frac{dn}{dx}\right) = \frac{2\pi A}{\alpha}\left[-2n_x + \alpha\int\limits_x^{\infty} e^{-\alpha(\xi-x)}\, n_\xi\, d\xi + \alpha\int\limits_{\xi=0}^{x} e^{-\alpha(x-\xi)}\, n_\xi\, d\xi\right]$$

$$\frac{1}{2h}\frac{d^2}{dx^2}\left(\frac{1}{n}\frac{dn}{dx}\right) = \frac{2\pi A}{\alpha}\left\{-2\frac{dn}{dx}\right\} + \frac{\alpha^2}{2h}\cdot\left(\frac{1}{n}\frac{dn}{dx}\right)$$

$$\underbrace{\frac{1}{2h}\left\{\frac{d^2}{dx^2}(\lg n)\right\}} - \frac{\alpha^2}{2h}\lg n = -\frac{4\pi A}{\alpha}\, n + \text{const}$$

für Punkte, wo $= 0$, also im ∞ ist $\frac{4\pi A}{\alpha}(n - n_0)\; \frac{\alpha^2}{2h}\lg\frac{n}{n_0}$

also ist $n_0\, e^{\frac{8h\pi A}{\alpha^3}(n-n_0)} = n$

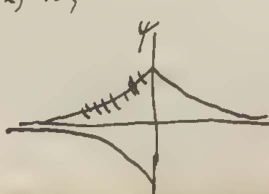
$$\frac{\partial\mathfrak{F}}{\partial x} = 2\pi\Big[2n_x\cdot\overset{=0}{\mathfrak{F}(0)} + \int\limits_{\xi=x}^{\infty} n_\xi\,\mathfrak{F}'(\xi-x)\,d\xi - \int\limits_{\xi=0}^{x} n_\xi\,\mathfrak{F}'(x-\xi)\,d\xi\Big] =$$

$$= \frac{1}{2h}\frac{d}{dx}\left(\frac{1}{n}\frac{dn}{dx}\right)\Big|_x$$

Damit $n_w \frac{mc^2}{3} = \frac{n\,mc^2}{3} - n^2 a$ wird, muss gelten:

$$n_w = n - \frac{2h}{3}n^2 a$$

$$\frac{1}{n}\frac{\partial n}{\partial x} = -\int\limits_0^{\infty} n_\xi\,\Psi(\xi - x)\,d\xi$$



$$\lg\frac{n}{n_0} = \int\limits_0^{\infty} n_\xi\, X(\xi - x)\,d\xi$$

$$X = \int \Psi\,dx$$

$$n - n_w = -\int\limits_{x=0}^{x}\int\limits_{\xi=0}^{\infty} n_x\, n_\xi\,\Psi(\xi - x)\,d\xi\,dx$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - b^2 u$$

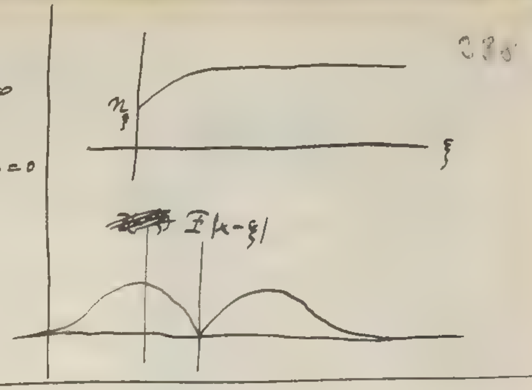
$$u = T \cdot \varphi(x)$$

$$\frac{1}{T}\frac{\partial T}{\partial t} = \frac{1}{\varphi}\left[\frac{\partial^2 \varphi}{\partial x^2} - b^2\varphi\right] = -c$$

$$T = e^{-ct}$$

$$\frac{d^2\varphi}{dx^2} - b^2\varphi = -c\varphi$$

$$\frac{d^2\varphi}{dx^2} + (c - b^2)\varphi = 0$$



$$n_w = n\, e^{-2h \overbrace{\int_0^\infty F(x)\,dx}^{U}}$$

$$\frac{n}{?}\, dx \cdot e^{-2h \int_x^\infty F(x)\,dx}$$

$$p_1 = n_w \frac{mc^2}{3} \qquad ?$$

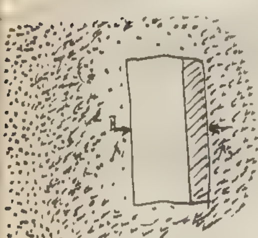
$$p_2 = n\frac{mc^2}{3} - n \underbrace{\int_0^\infty F_1(x)\,dx}_{n^2 a}$$

~~$F(r) \sim \frac{n}{v}$ ist dann nur die Anziehung zu berücksichtigen und die Molekularstöße nicht?~~

$F(r)$ ist Funktion von n und h und zwar komplizierter Gestalt!

$F_1(x) \sim n$

es ist nicht $F(x) = n f_1(h)$!! sondern

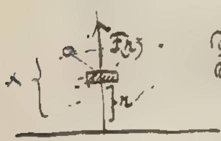


$$\left(\frac{mc^2}{3}\right)\frac{\partial n}{\partial x} = n F(x)$$

$$\cancel{n = n_0 e^{-2h\int}}$$

$$\rho^2 = x^2 + y^2$$

$$n = n_w e$$



$$\frac{\partial p}{\partial x} = \overbrace{\frac{mc^2}{3}}^{=\frac{1}{2h}} \cdot \frac{\partial n}{\partial x} = n F(x)$$

$$n = A\, e^{2h\int_0^x F(x)\,dx}$$

$$n = n_w\, e^{2h\int_0^x F(x)\,dx}$$

$$n_0 = n_w\, e^{2h\int_0^\infty \cdots}$$

$$n = n_0\, e^{-2h\int_x^\infty F(x)\,dx}$$

$$n_w = n_0\, e^{-2h\int_0^\infty F(x)\,dx}$$

$$F(x) = \int n_x \cancel{2\pi\rho^2\, dR\, dx \cos\alpha}\, f(R)$$

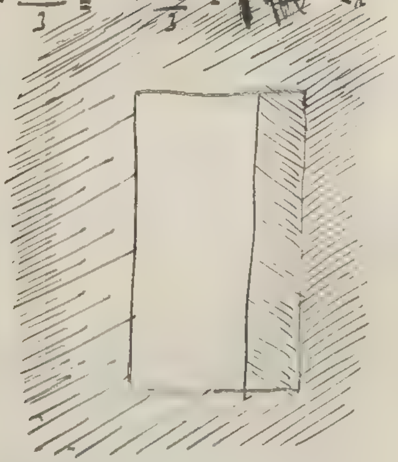
$$= 2\pi \iint_{x=0,\,y=0}^{\infty} y\, dy\, dx \cdot n_x \cdot \underbrace{f(\sqrt{x^2+y^2})\frac{x}{\sqrt{x^2+y^2}}} - \int$$

$$f = A e^{-\alpha\sqrt{x^2+y^2}}$$

$$= 2\pi A \int n_x\, dx \int_x \rho\, \frac{d\rho\, e^{-\alpha\rho}}{\rho} = \cancel{\frac{2\pi A}{\alpha}\int n_x\, dx}$$

$$= \frac{2\pi A}{\alpha}\left[\int_x^\infty e^{-\alpha x}\, n_x\, dx - \int\right.$$

$$n_w \frac{mc^2}{3} = n\frac{mc^2}{3} - \ldots$$



$$\lg \frac{n_0}{n} = 2h\int_x^\infty F(\xi)\,d\xi$$

$$\frac{1}{n}\frac{\partial n}{\partial x} = -2h\ F(x) = -\frac{N}{RT}\ F(x)$$

$$n\frac{RT}{N} = \frac{n\mu}{N}\frac{c^2}{3} = nm\frac{c^2}{3} \quad \Big|\Big|\Big| \quad v = \frac{1}{\rho} = \frac{1}{nm}$$

$$A n^2 = \frac{A}{v^2}\frac{1}{m^2} = A$$

$$e^{-\frac{NA}{RT}} = 1 - \frac{an^2}{\frac{RT}{v}}$$

$$A = -\frac{RT}{N}\lg\left(1 - \frac{an^2}{\frac{RT}{v}}\right)$$

$$p = \frac{n_w mc^2}{3} = n\frac{mc^2}{3} - an^2$$

$$n_w = n_0 - \frac{an^2}{\frac{RT}{v}}$$

$$n_w = n_0 - 2h\,an^2$$

$$a = \int_{x=-\infty}^{x=0}\int_{\xi=0}^{\infty}\psi(\xi - x)\,d\xi\,dx$$

$$\xi - b + \frac{b-x}{\ldots}$$

$$= \int_{z=-\infty}^{0}\int_{\xi=0}^{\infty}\psi(\xi + z)\,d\xi\,dz$$

$$\int_0^X \int_0^\infty n(x)\, n(\xi)\, \Psi(\xi - x)\, dx\, d\xi = \int_0^X n(x)\, dx \left[\int_0^x n(\xi)\, \Psi\, d\xi + \int_x^\infty n(\xi)\, \Psi(\xi - x)\, d\xi \right]$$

$$n_a - n_v \int_0^X \int_X^\infty n(x)\, n(\xi)\, \Psi(\xi - x)\, d\xi\, dx$$

$$\frac{\alpha n^2 \frac{m c^2}{3}}{\frac{RT}{N}} = \frac{\alpha n N}{\mu} \rho = \alpha\, n m \frac{N}{\mu m} \rho$$

$$= \alpha \rho^2 \cdot \frac{N}{\mu m}$$

$$\int_0^x \int_0^x n(x)\, n(\xi)\, \Psi(\xi - x)\, dx\, d\xi =$$

$$p = \rho RT - \rho^2 \underbrace{\alpha \frac{N}{\mu m}}_{\frac{\alpha}{m^2}} \qquad \frac{\mu}{N} = m$$

$$n_0 \frac{m c^2}{3} = \rho\, RT$$

$$\frac{RT}{v} - \frac{a}{v^2} = \frac{RT}{v} e^{-\frac{N}{HT} A}$$

$$e^{-\frac{N}{HT} A} = 1 - \frac{a}{vRT}$$

$$A = -\frac{HT}{N} \lg\left(1 - \frac{a}{vRT}\right)$$

$$\xi_1 = A_1 \sin \alpha_1\left(t - \frac{x}{c}\right) \qquad \left(\xi_1 + \xi_2\right)\Big|_{x = x_0 - vt} = 0$$

$$\xi_2 = A_2 \sin \alpha_2\left(t + \frac{x}{c} + \varepsilon\right)$$

$$A_1 \sin \alpha_1\left(t - \frac{x_0}{c} + \frac{v}{c}t\right) + A_2 \sin \alpha_2\left(t + \frac{x_0}{c} - \frac{v}{c}t + \varepsilon\right) = 0$$

$$\alpha_1\left(1 + \frac{v}{c}\right) = \alpha_2\left(1 - \frac{v}{c}\right) \qquad \varepsilon + \frac{x_0}{c} = -\frac{x_0}{c}$$

$$A_1 = -A_2$$

$$\alpha_2 = \alpha_1 \frac{c+v}{c-v}$$

$$\frac{1}{2}\rho \dot{\xi}_1^2 = \frac{1}{2}\rho A_1^2 \alpha_1^2 \cos^2 \alpha_1(t \dots)$$

$$\frac{1}{2}\rho \overline{\dot{\xi}_1^2} = \frac{1}{4}\rho A_1^2 \alpha_1^2$$

$$c \frac{\rho}{4} A^2(\alpha_2^2 - \alpha_1^2) - v \frac{\rho}{4} A^2(\alpha_1^2 + \alpha_2^2) = P v$$

$$P = \rho \frac{A^2}{4} \alpha_1^2 \frac{\left[\left(\frac{c+v}{c-v}\right)^2 - 1\right] c - \left[\left(\frac{c+v}{c-v}\right)^2 + 1\right] v}{v \quad (c-v)^2}$$

$$\frac{4c^2\cancel{v} - 2(c^2+v^2)\cancel{v}}{4} = \frac{2(c^2-v^2)}{(c-v)^2}$$

$$P = \rho \frac{A^2}{\cancel{4}} \alpha_1^2 \frac{(c^2-v^2)}{(c-v)^2} = \rho A^2 \alpha_1^2 \frac{c+v}{c-v}.$$

$$\overline{E} = \rho \frac{A^2}{2} \alpha_1^2 \left[1 + \left(\frac{c+v}{c-v}\right)^2\right] = \rho \frac{A^2 \alpha_1^2}{\cancel{4}} \cancel{4} \frac{c^2+v^2}{(c-v)^2}$$

$$P = \overline{E} \; \frac{c^2-v^2}{c^2+v^2}$$

$$\frac{4}{3}\pi r \rho = F = -\frac{\partial U}{\partial r}$$

$$U = -\frac{2}{3}\pi\rho r^2 + \text{const} \qquad = 2\pi\rho a^2 - \frac{2}{3}\pi\rho r^2$$

$$U_a = -\frac{2}{3}\pi\rho a^2 + \text{const} = \frac{4}{3}\pi\rho a^2$$

$$\frac{1}{2}\sum U\varphi = \cancel{4}\; 2\pi\sigma^2\int_0^a r^2\,dr\left(2\pi\rho a^2 - \frac{2}{3}\pi\rho r^2\right) = 4\pi^2\sigma^2\left(a^2\cdot\frac{a^3}{3} - \frac{a^5}{15}\right) = \frac{16}{15}a^5\pi^2\sigma^2$$

$$= \left(\frac{4}{3}\pi a^3\sigma\right)^2\cdot\frac{9}{15}\,\frac{1}{a} = \frac{3}{5}\frac{e^2}{a}$$

Energetische Masse eines Elektrons sollte betragen: $\frac{3}{5}\frac{e^2}{a c^2}$
(infolge elektrostatischer Energie)

während scheinbare elektromagnetische Masse ist $m = \frac{e^2}{6\pi a c^2}$

Dasselbe für Schallwellen in einem Gas

$$\frac{\partial\rho}{\partial t} = -\rho\frac{\partial u}{\partial x} \qquad \rho = \rho_0(1+\sigma) \qquad \frac{\partial\sigma}{\partial t} = -\frac{\partial u}{\partial x}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = -\rho\frac{\partial u}{\partial t} \qquad p = p_0(1+k\sigma) \qquad a^2\frac{\partial\sigma}{\partial x} = -\frac{\partial u}{\partial t}$$

$$T = \rho\frac{u^2}{2} \qquad\qquad U = \frac{1}{2}\lambda\sigma^2 = a^2\rho\cdot\frac{\sigma^2}{2}$$

$$\cancel{U = \int\frac{1}{\rho}\,dp = a^2\int d\sigma = a^2\sigma}$$

$$\frac{dp}{d\sigma} = \lambda = p_0 k$$

$$u = A_1\alpha\cos\alpha\left(t - \frac{x}{c}\right)$$

$$\frac{\partial\sigma}{\partial t} \overset{!}{=} -\frac{A_1\alpha^2}{c}\sin\alpha\left(t - \frac{x}{c}\right)$$

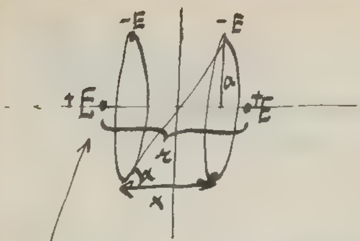
$$\sigma = \frac{A\alpha}{c}\cos\alpha\left(t - \frac{x}{c}\right)$$

$$\overline{T} = \overline{U}$$

Es sollte die ähnliche Formel $P = \overline{E}\,\frac{c \pm v}{c \mp v}$ gültig bleiben! Dagegen Rayleigh?

Ebenso in Flüssigkeit; Reflexion an der freien Oberfläche

Und wie in festen Körpern?



Falls so viele Elektronen auf d. Ring, dass man die Ladung als gleichmässig verteilt ansehen kann.

$$m \frac{d^2 x}{dt^2} = + \frac{E^2}{r^2}$$

$$\sin^2\alpha = \frac{4a^2}{4a^2+x^2} \qquad \sin\alpha = \frac{2a}{\sqrt{4a^2+x^2}}$$

$$\cos\alpha \, d\alpha = - \frac{2a\, x\, dx}{\left(\sqrt{4a^2+x^2}\right)^3} = - \sin^3\alpha \; \frac{x\, dx}{4a^2}$$

$$\frac{2a}{x} = \operatorname{tg}\alpha$$

$$- \frac{2a\, dx}{x^2} = \frac{d\alpha}{\cos^2\alpha} = - \frac{\operatorname{tg}^2\alpha}{2a}\, dx$$

$$d\alpha = - \frac{\sin^2\alpha}{2a}\, dx = - \frac{2a}{4a^2+x^2}\, dx \qquad \Bigg\| \qquad dx = - \frac{d\alpha}{\sin^2\alpha} \frac{1}{2a}$$

$$F_x = + \frac{\partial M}{\partial x} = + \frac{\partial M}{\partial \alpha} \frac{d\alpha}{dx} \left(\frac{E\omega}{c}\right)^2 \qquad \underline{M} = 4\pi a\, y$$

Elektrostatische Abstossung geladener Kreislinien:

$$W = \frac{1}{2} \Sigma \varphi\, e = \frac{E^2}{2} \frac{1}{2\pi a} \int_{\varphi=0}^{2\pi} \frac{a\, d\varphi}{\sqrt{x^2 + a^2(1-\cos\varphi)^2 + a^2 \sin^2\varphi}}$$

$$W = \frac{E^2}{4\pi} \int_0^{2\pi} \frac{d\varphi}{\sqrt{x^2 + 2a^2(1-\cos\varphi)}} = \frac{E^2}{2\pi} \int_0^{\pi} \frac{d\varphi}{\sqrt{x^2 + 4a^2 \sin^2\left(\frac{\varphi}{2}\right)}}$$

$$= \frac{E^2}{\pi x} \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 + \underbrace{\left(\frac{2a}{x}\right)^2}_{k} \sin^2\varphi}} = \underbrace{\frac{E^2}{\pi\sqrt{x^2+4a^2}} \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 + \underbrace{\frac{4a^2}{x^2+4a^2}}_{\sin^2\alpha} \sin^2\varphi}}}_{= K(\sin\alpha)}$$

$$\mathcal{E}_x = - \frac{\partial W}{\partial x}$$

$$= \frac{E^2 \sin\alpha}{2a\pi} K(\sin\alpha)$$

$$m\frac{d^2r}{dt^2} = \frac{E^2}{r^2} - \frac{E^2\cdot\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} - \frac{E^2\cdot\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2+a^2\right]^{3/2}}$$

dr

$$\mathfrak{F}_x = \mathcal{E}_x + \frac{E^2\cdot\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} - \frac{E^2\cdot\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2+a^2\right]^{3/2}}$$

dx

$$\cancel{\frac{m}{2}\left(\frac{dr}{dt}\right)^2 = -\frac{E^2}{r} + \frac{E^2}{\sqrt{\left(\frac{r-x}{2}\right)^2+a^2}} + \frac{E^2}{\sqrt{\left(\frac{r+x}{2}\right)^2+a^2}} + \text{fc}(x)}$$

$\cancel{\frac{m}{2}v_\infty^2}$?

$$\cancel{M\left(\frac{E\omega}{c}\right)^2 + W - \frac{E^2}{\sqrt{\left(\frac{r-x}{2}\right)^2+a^2}} - \frac{E^2}{\sqrt{\left(\frac{r+x}{2}\right)^2+a^2}} = \text{konst} = 0}$$

fc. (r) ?

$$\cancel{\frac{m}{2}v_\infty^2 = \frac{E^2}{r} + M\left(\frac{E\omega}{c}\right)^2 W}$$

8.10^{-11} 5.10^{-15}

$\frac{2}{3}.10^4.4.2.10^{-19}$ 10^{-14}

$$\frac{m}{2}\left(\frac{dr}{dt}\right)^2 + \frac{E^2}{r} - \frac{E^2}{\sqrt{\left(\frac{r+x}{2}\right)^2+a^2}} - \frac{E^2}{\sqrt{\left(\frac{r-x}{2}\right)^2+a^2}} + W - M\left(\frac{E\omega}{c}\right)^2 = \text{konst} = \frac{m}{2}v_\infty^2$$

Falls $v_\infty = 0$ und es werden nur die Ringe berücksichtigt.

Unsinn

Gleichgewicht für:

$$\omega = \frac{h}{2\pi}\frac{1}{\mu a^2}$$

$$E^2\frac{\omega^2}{c^2}M = \frac{E^2}{2a\pi}\sin\alpha\, K$$

$$\cancel{\frac{e}{\mu}} = 1{,}87.10^{+7} \qquad \mu = \frac{4{,}7.10^{-10}}{1{,}87.10^7.c}$$

$$\left(\frac{h}{2\pi}\right)^2\frac{2a\pi}{\mu^2a^4c^2} = \sin\alpha\frac{K}{M}$$

$$\cancel{\frac{1}{\mu c} = \frac{1{,}87.10^7}{4{,}7.10^{-10}} = 0{,}4.10^{+17}}$$

$$\cancel{\frac{h^2}{\mu^2a^3c^2} = 2\pi\sin\alpha\frac{K}{M} = \frac{6{,}27.10^{-27}.0{,}4.10^{17}}{a^3} = \frac{2{,}5.10^{-10}}{a^3}}$$

Falls nur zwei Elektronen $\quad (E = 2e)$

$$i_{(\text{Elektronen})} = \frac{E}{2\pi} \frac{a\omega}{c} = \frac{E\omega}{2\pi c}$$

$$W = \frac{4e^2}{\sqrt{x^2 + 2a^2}}$$

$$\omega = \frac{h}{2\pi} \frac{1}{a^2\mu}$$

$$\mathfrak{F}_x = \left(\frac{E\omega}{2\pi c}\right)^2 \cdot \frac{2a}{4a^2+x^2} \frac{\partial M}{\partial x}$$

$$\left(\frac{\omega}{2\pi c}\right)^2 = \left(\frac{4.2 \,.\, 10^{16}}{2\pi \,.\, 3 \,.\, 10^{10}}\right)^2 = 4 \,.\, 10^{10}$$

$$\frac{\partial W}{\partial x} = \frac{4e^2 x}{\sqrt{x^2+2a^2}^3}$$

<u>Gleichungen entwickeln</u>

$$\frac{1}{r^2} = \frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2 + a^2\right]^{3/2}} + \frac{\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2 + a^2\right]^{3/2}}$$

$$\left(\frac{\omega}{2\pi c}\right)^2 \frac{2a}{4a^2+x^2} \frac{\partial M}{\partial x} = \frac{x}{\sqrt{x^2+2a^2}^3} + \frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} - \frac{\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2+a^2\right]^{3/2}}$$

$$r - x = \delta$$

$$\frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} = \frac{\frac{\delta}{2}}{\left[a^2+\frac{\delta^2}{4}\right]^{3/2}} = \frac{\delta}{2a^3}\left[1+\frac{\delta^2}{4a^2}\right]^{-3/2}$$

$$= \frac{x+\frac{\delta}{2}}{\left[\left(x+\frac{\delta}{2}\right)^2+a^2\right]^{3/2}} = \frac{x}{[x^2+a^2]^{3/2}}\left[1+\frac{\delta}{2x} - \frac{3x\frac{\delta}{2}}{x^2+a^2} \cdot \right]$$

$$\frac{\frac{r+x}{2}}{\cdots}\Big] = \frac{r-\frac{\delta}{2}}{\left[\left(r-\frac{\delta}{2}\right)^2+a^2\right]^{3/2}} = \frac{r}{[r^2+a^2]^{3/2}}\left[1 - \frac{\delta}{2r} + \frac{3r\frac{\delta}{2}}{r^2+a^2} \cdots\right]$$

$$\frac{1}{r^2} - \frac{x}{[r^2+a^2]^{3/2}} = \frac{\delta}{2a^3} - \frac{\delta}{2}\left[\frac{1}{[r^2+a^2]^{3/2}} - \frac{3r^2}{[r^2+a^2]^{5/2}}\right]$$

$$\frac{\delta}{2} = \frac{\frac{1}{x^2} - \frac{2}{[x^2+a^2]^{3/2}}}{\frac{1}{a^3} - \frac{1}{[x^2+a^2]^{3/2}} + \frac{3x^2}{[x^2+a^2]^{5/2}}}$$

$$\frac{1}{x^2}\left[1 - 2\frac{\delta}{x}\right] = \frac{\delta}{2a^3} + \frac{x}{[x^2+a^2]^{3/2}}\left[1 + \frac{\delta}{2x} - \frac{3x\,\frac{\delta}{2}}{x^2+a^2}\right]$$

$$\frac{1}{x^2} - \frac{x}{(x^2+a^2)^{3/2}} = \frac{2\delta}{x^3} + \frac{\delta}{2a^3} + \frac{\delta}{2(x^2+a^2)^{3/2}} - \frac{3\delta}{2}\frac{x^2}{(x^2+a^2)^{5/2}}$$

$$\frac{\omega^2}{(2\pi c)^2}\frac{\partial M}{\partial x} = \frac{\omega^2}{4\pi^2c^2}\cdot 4\pi a\,\frac{\partial y}{\partial x} = \frac{\omega^2 a}{\pi c^2}\frac{\partial y}{\partial x} = \cancel{\frac{h}{2\pi\mu c^2}\frac{\partial y}{\partial \alpha}}$$

$$a^2\omega = \frac{h}{2\pi\mu}$$

$$= \frac{h}{2\pi^2}\left(\frac{h}{\mu}\right)\frac{1}{c^2} = \cancel{\frac{6.5\cdot10^{-27}}{20}\;\frac{1.8\cdot10^{7}}{3\cdot10^{10},\ 4.7\cdot10^{-10}}}$$

$$= \cancel{\quad}$$

$$= \frac{(4.2\cdot10^{16})^2\cdot10^{-8}}{\pi\cdot 9\cdot10^{20}} = \frac{(4.2)^2\cdot10^4}{9\pi} = \frac{17.6}{28.3}\cdot10^4 = \frac{2}{3}\cdot10^4\cdot\frac{\partial y}{\partial x}$$

$$= -\frac{\partial y}{\partial \alpha}\,\frac{2a}{a^2+x^2}$$

$$\cancel{\lim \frac{\partial M}{\partial x} = \lim \frac{\pi}{a}\,\alpha\,\sin\alpha\left[2K - \left(1+\frac{1}{\sin^2\alpha}\right)E\right]}$$

$$y = \frac{2E - (1+\sin^2\alpha)K}{\sin\alpha}$$

$$E = \int_0^{\frac{\pi}{2}} d\varphi\,\sqrt{1-\sin^2\alpha\sin^2\varphi} \qquad K = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1-\sin^2\alpha\sin^2\varphi}}$$

$$\lim E = \frac{\pi}{2} - \frac{\sin^2\alpha}{2}\frac{\pi}{4} \qquad \lim K = \frac{\pi}{2} + \frac{\sin^2\alpha}{2}\frac{\pi}{4}$$

$$\lim_{\alpha\to0} y = \frac{\left(\pi - \sin^2\alpha\frac{\pi}{4}\right) - \left(\frac{\pi}{2} + \sin^2\alpha\frac{\pi}{4}\right)}{\sin\alpha} = -\frac{\pi}{4}\sin\alpha = -\frac{\pi}{4}\frac{2a}{x} = -\frac{a\pi}{2x}$$

Bei Vernachlässigung der Kraft $\frac{\partial \psi}{\partial x}$:

$$r = x + \delta$$

$$\frac{x}{\sqrt{x^2+2a^2}^3} + \frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2 + a^2\right]^{3/2}} - \frac{\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2 + a^2\right]^{3/2}} = 0$$

$$\left.\begin{aligned}
&\frac{x}{\sqrt{x^2+2a^2}^3} + \frac{\delta}{2a^3} - \frac{x}{(x^2+a^2)^{3/2}}\left[1 + \frac{\delta}{2x} - \frac{3}{2}\frac{\delta x}{x^2+a^2}\right] = 0 \\
&\frac{1}{x^2}\left(1 - \frac{2\delta}{x}\right) - \frac{\delta}{2a^3} - \frac{x}{(\quad)^{3/2}}\left[\qquad\qquad\right] = 0
\end{aligned}\right\}$$

$$\frac{x}{\sqrt{x^2+2a^2}^3} - \frac{1}{x^2}\left(1 - \frac{2\delta}{x}\right) + \frac{\delta}{a^3} = 0$$

$$\delta = \frac{\frac{1}{x^2} - \frac{x}{(x^2+2a^2)^{3/2}}}{\frac{1}{a^3} + \frac{2}{x^3}}$$

Elektrostatische Kraft bei grosser Entfernung:

$$\frac{x}{(x^2+2a^2)^{3/2}} + \frac{\delta}{2a^3} = \frac{\left(x+\frac{\delta}{2}\right)}{\left[\left(x+\frac{\delta}{2}\right)^2 + a^2\right]^{3/2}} = \frac{x}{\left[x^2+a^2\right]^{3/2}}\left[1 + \frac{\delta}{2x} - \frac{3}{2}\frac{\delta x}{a^2+x^2}\right]$$

$$\frac{x}{\left[x^2+a^2\right]^{3/2}} - \frac{x}{\left[x^2+2a^2\right]^{3/2}} = \frac{\delta}{2}\left[\frac{1}{a^3} - \frac{1}{\left[x^2+a^2\right]^{3/2}} + \frac{3x^2}{\left[x^2+a^2\right]^{5/2}}\right]$$

$$\frac{1}{x^2}\left\{\underbrace{\left[1 - \frac{3}{2}\frac{a^2}{x^2} + \frac{15}{8}\frac{a^4}{x^4}\right] - \left[1 - 3\frac{a^2}{x^2} + \frac{15}{8}\frac{4a^4}{x^4}\right]}_{\frac{3}{2}\frac{a^2}{x^2} - \frac{45}{8}\frac{a^4}{x^4}}\right\} = \frac{\delta}{2}\left\{\frac{1}{a^3} - \frac{1}{x^3}\left(1 - \frac{3}{2}\frac{a^2}{x^2}\right) + \frac{3}{x^3}\left(1 - \frac{5}{2}\frac{a^2}{x^2}\right)\right\}$$

$$\left(1 - \frac{15}{4}\frac{a^2}{x^2}\right)\frac{3}{2}\frac{a^2}{x^4} = \frac{\delta}{2a^3}\left\{1 + \cancel{\frac{2a^3}{x^3}}\right\}$$

$$\delta = \frac{3a^5}{x^4}\left(1 - \frac{15}{4}\frac{a^2}{x^2}\right)$$

$$\frac{1}{E^2}\mathcal{E}_x = \underbrace{\frac{1}{r^2} - \frac{2}{[r^2+a^2]^{3/2}}}_{} - \frac{\delta}{2a^3} + \frac{\delta}{2}\left[\frac{1}{(r^2+a^2)^{3/2}} - \frac{3r^2}{(r^2+a^2)^{5/2}}\right]$$

$$\frac{1}{x^4} = \frac{1}{(r-\delta)^4} = \frac{1}{r^4}\left(1+4\frac{\delta}{r}\right) = \frac{1}{r^4}\left(1+12\cancel{\frac{a^2}{r^2}}\right)$$

$$\frac{1}{r^2}\left[1-\left(1-\frac{3}{2}\frac{a^2}{r^2}+\frac{15}{8}\frac{a^4}{r^4}\right)\right]$$

$$= \frac{1}{r^2}\left(\frac{3}{2}\frac{a^2}{r^2}-\frac{15}{8}\frac{a^4}{r^4}\right) - \frac{3}{2}\frac{a^2}{r^4}\left(1-\frac{15}{4}\frac{a^2}{r^2}\right)$$

$$= \frac{a^4}{r^6}\,\frac{15}{4}$$

$$\lim \mathcal{E}_x = \frac{15}{4}E^2\frac{a^4}{r^6}$$

Während elektromagnetische Kraft: $\lim F_x = \dfrac{\omega^2 a E^2}{r c^2}\dfrac{a r}{2r^2} = \dfrac{\omega^2 a^2 E^2}{2c^2}\dfrac{1}{r^2}$

Wo werden sie gleich?

$$\frac{15}{4}\frac{a^4}{r^6} = \frac{\omega^2 a^2}{2c^2}\frac{1}{r^2}$$

$$r^4 = \frac{15}{2}\frac{a^2c^2}{\omega^2}$$

$$r = \sqrt[4]{\frac{15}{2}}\sqrt{\frac{ac}{\omega}} = \sqrt[4]{\frac{15}{2}}\cdot\sqrt{\frac{10^{-8}\cdot 3\cdot 10^{10}}{4{,}2\cdot 10^{16}}}$$

$$= \sqrt[4]{\frac{15}{2}}\sqrt{\frac{1}{1{,}4}}\cdot 10^{-7} = 1{,}4\cdot 10^{-7}$$

~~[illegible] besteht, und~~ $\cancel{\lim \mathcal{E}_x \sim \frac{a^{2+2n}}{r^{4+2n}}}$

Arbeit $\displaystyle\int_r^\infty \mathcal{E}_x\,dx = \frac{3}{4}E^2\frac{a^4}{r^5} = \frac{3}{4}\,2\cdot 10^{-19}\cdot\frac{10^{-32}}{(1{,}4)^5\cdot 10^{-35}} = 4\cdot\frac{3}{2}\,\frac{1}{5{,}6}\cdot 10^{-16} = [illegible]\cdot 10^{-16}$

Arbeit $\displaystyle\int_r^\infty F_x\,dx = \frac{\omega^2 a^2 E^2}{2c^2 r} = \left(\frac{4{,}2\cdot 10^{16}\cdot 10^{-8}}{3\cdot 10^{10}}\right)^2 4\cdot\frac{2\cdot 10^{-19}}{2\cdot 1{,}4\cdot 10^{-7}} = 4\cdot 1{,}4\cdot 10^{-4-12} = [illegible]\,10^{-16}$

$c = 0$ / $\overline{W}$

$$D\frac{\partial \overline{W}}{\partial x} = \rho\,\overline{W} f(x) + c \quad \Big|\; W_0$$

$$D\frac{\partial W_0}{\partial x} = \rho\, W_0 f(x) \quad \Big|\; \overline{W}$$

$$D\frac{\left(W_0\frac{\partial \overline{W}}{\partial x} - \overline{W}\frac{\partial W_0}{\partial x}\right)}{W_0^2} = \frac{c}{W_0} = +D\frac{\partial}{\partial x}\left(\frac{\overline{W}}{W_0}\right)$$

$$\overline{W} = W_0\cdot\left(\frac{c}{D}\int\frac{dx}{\overline{W}_0} + c'\right) = \overline{W}_0 c' + \frac{c}{D}\left(x + \overline{W}_0\int x\frac{\overline{W}_0'}{\overline{W}_0^2}dx\right)$$

$$e^{-\frac{\beta}{D}U}\cdot\int e^{+\frac{\beta}{D}U}dx$$

$$= x - e^{-\frac{\beta}{D}U}\int\frac{\beta}{D}U'\cdot x\,dx\; e^{+\frac{\beta}{D}U}$$

lim

$$r\frac{\partial u}{\partial t} = \frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\partial u}{\partial r} + r\frac{\partial^2 u}{\partial r^2}$$

falls für $t=0$ $u=1$ bis $r=a$
und für $r=a$ $u=0$

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

$$u = U e^{-\alpha t} \qquad r\sqrt{\frac{\alpha}{D}} = x$$

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{\alpha}{D}U = 0 \qquad \frac{\partial^2 U}{\partial x^2} + \frac{1}{x}\frac{\partial U}{\partial x} + U = 0$$

$$U = J_0(x)$$

$$u = \sum_\alpha J_0\left(r\sqrt{\frac{\alpha}{D}}\right)e^{-\alpha t}$$

$$= \cancel{\sum_\alpha K_0\left(r\sqrt{\frac{\alpha}{D}}\right)e^{-\alpha t}}$$

$t=0$ $$u = \sum_\alpha J_0\left(r\sqrt{\frac{\alpha}{D}}\right) = \sum_\beta J_0(r\beta)\cdot A_\beta$$

$$J(\beta) = 0 \qquad A_\beta = \frac{2\int_0^1 J(\beta x)\,x\,dx}{J'(\beta)^2}$$

$$\int J\,x\,dx = - J_{=} - \underbrace{\int x \frac{\partial J}{\partial x^2}\,dx}_{}$$

$$= -2J - x\frac{\partial J}{\partial x}$$

$$x\frac{\partial J}{\partial x} - \int \frac{\partial J}{\partial x}\,dx$$

$$\frac{\partial}{\partial x}(Jx^2) = 2xJ + x^2\frac{\partial J}{\partial x}$$

$$\int Jx\,dx = +\frac{Jx^2}{2} - \int$$

$$\int_0^1 J(\beta x)\,x\,dx = -2\frac{J(\beta x)}{\beta^2}\Big|_0^1 - \frac{x}{\beta}\,J'(\beta x)\Big|_0^1$$

$$= -2\cancel{\frac{J(\beta)}{\beta^2}} + \frac{2}{\beta^2} - \frac{J'(\beta)}{\beta}$$

$$\int_0^1 x\,J(\alpha x)\,dx = -\frac{J'(\alpha)}{\alpha} \qquad (\text{p. } 190)$$

$$A_\beta = \frac{4}{[\beta J'(\beta)]^2} - \frac{2}{\beta J'(\beta)}$$

$$\beta = \sqrt{\frac{\alpha}{D}} \qquad \alpha = D\beta^2$$

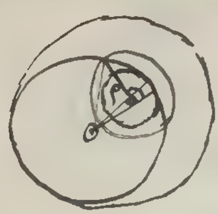
$$u = \sum_{\beta_1, \beta_2 \dots} A_\beta\, J(r\beta)\, e^{-D\beta^2 t}$$

Degegen folls (Riem-Weber I p. 183)

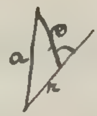
$$f(x) = \int_0^\infty J(\xi x)\, e^{-\frac{\xi^2 D t}{\lambda^2}} \cdot \xi\, d\xi \int_0^\infty f(\lambda)\, J(\lambda\xi)\,\lambda\, d\lambda$$

$$\left.\begin{array}{lll} f(\lambda) = 1 & \text{von } \lambda = 0 & \text{bis } \lambda = 1 \\ f(\lambda) = 0 & \lambda = 1 & \text{bis } \lambda = \infty \end{array}\right\} \qquad \int_0^\infty f(\lambda)\, J(\lambda\xi)\lambda\, d\lambda = \cancel{\frac{2}{\xi}[1 - J(\xi)]} - \frac{J'(\xi)}{\xi}$$

$$f(x) = \int_0^\infty e^{-\xi^2 D t}\left\{\cancel{\frac{2J(\xi x)}{\xi}[1 - J(\xi)]} - J(\xi x)\, J'(\xi)\right\} d\xi = -\int_0^\infty J'(\xi)\, J(\xi x)\, e^{-\xi^2 D t}\, d\xi$$



$$P = \frac{1}{a^2\pi}\int_0^\infty 2r\pi\,dr \cdot \frac{1}{2Dt}\left\{\int_{a-r}^{a+r} e^{-\frac{\rho^2}{4Dt}}\rho\,d\rho \cdot \frac{\arccos}{\pi}\left(\frac{a^2-r^2-\rho^2}{2r\rho}\right) + \int_{a+r}^{\infty} e^{-\frac{\rho^2}{4Dt}}\rho\,d\rho\right\}$$



$$a^2 = r^2+\rho^2+2r\rho\cos\varphi$$

$$\varphi = \arccos\frac{a^2-r^2-\rho^2}{2r\rho}$$

$$\left(\frac{1}{2\sqrt{\pi Dt}}\right)^2\int e^{-\frac{x^2+y^2}{4Dt}}dx\,dy = \frac{2\pi r}{4\pi Dt}e^{-\frac{r^2}{4Dt}}dr = \frac{r\,dr}{2Dt}e^{-\frac{r^2}{4Dt}}$$ = [illegible]

$$\rho^2 = a^2+r^2-2ar\cos\theta$$

$$\rho\sin\varphi = a\sin\theta$$

$$\varphi = \arcsin\left(\frac{a}{\rho}\sin\theta\right)$$

$$+\int_0^\pi e^{-\frac{(a^2+r^2-2ar\cos\theta)}{4Dt}}\, ar\sin\theta\,d\theta \cdot \underbrace{\frac{\arcsin}{\pi}\left(\frac{a\sin\theta}{\sqrt{a^2+r^2-2ar\cos\theta}}\right)}_{\frac{1}{\pi}\left[\frac{\pi}{2}-\arccos\sqrt{\frac{a^2-a^2\cos^2\theta}{a^2+r^2-2ar\cos\theta}}\right]}$$

$$= \int_{-a}^{a} e^{-\frac{a^2+r^2-2r\xi}{4Dt}}\, r\,d\xi\left[\frac{1}{2}-\frac{1}{\pi}\arccos\sqrt{\frac{a^2-\xi^2}{a^2+r^2-2r\xi}}\right]$$

$$\frac{a^2-r^2-\rho^2}{2r\rho} = \cos\varphi$$

$$\rho^2+2r\rho\cos\varphi = a^2-r^2$$

$$\rho = -r\cos\varphi \pm\sqrt{a^2-r^2+r^2\cos^2\varphi}$$

$$= -r\cos\varphi \mp\sqrt{a^2-r^2\sin^2\varphi}$$

$$\cancel{2}\rho\,d\rho + \cancel{2}r\cos\varphi\,d\rho - \cancel{2}r\rho\sin\varphi\,d\varphi = 0$$

$$d\rho = d\varphi\,\frac{r\rho\sin\varphi}{\rho+r\cos\varphi}$$

$$= \int e^{-\frac{\left[-r\cos\varphi+\sqrt{a^2-r^2\sin^2\varphi}\right]^2}{4Dt}}$$

$$\frac{r^2}{4Dt} = z$$

$$\frac{r\,dr}{2} = Dt\cdot dz$$

$$\overline{r^2} = \int_0^\infty \frac{r^3\,dr}{2Dt}e^{-\frac{r^2}{4Dt}} = 4Dt\underbrace{\int_0^\infty z\,e^{-z}dz}$$

$$= 4Dt$$

Dagegen bei Schichte: $\lim P = 1-\frac{2}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{3}+\frac{\beta^5}{10}\right]+\frac{1}{\beta\sqrt{\pi}}\left[\beta^2-\frac{\beta^4}{2}+\cdot\right]$

$$= 1-\frac{\beta}{\sqrt{\pi}}+\frac{1}{6\sqrt{\pi}}\beta^3 = 1-\frac{h}{2\sqrt{\pi Dt}}$$

Wenn man annimmt, dass gleichmässig verbreitet auf $(4Dt)\pi$ so Prozentsatz welcher übrig bleibt: $\frac{a^2\pi}{4Dt\pi} = \frac{a^2}{4Dt}$

$$\frac{1}{2Dt\,\pi}\int_{a-r}^{a+r} e^{-\frac{\rho^2}{4Dt}}\rho\,d\rho\cdot\varphi = \frac{1}{\pi}\left[\varphi\cdot e^{-\frac{\rho^2}{4Dt}}\Big/ - \int e^{-\frac{\rho^2}{4Dt}}\cdot d\varphi\right]$$

$$d\varphi = -\frac{1}{\sqrt{1-\left(\frac{a^2-r^2-\rho^2}{2\rho r}\right)^2}}\left[-\frac{a^2-r^2-\rho^2}{2r\rho^2}-\frac{1}{r}\right]d\rho$$

Im Grenzfall für große t: so dass $e^{-y^2} \neq 1$

$$\int_{a-r}^{a+r}\rho\,d\rho\cdot\frac{\varphi}{\pi} = \frac{1}{\pi}\left[\frac{(a+r)^2}{2}\pi - \frac{a^2\pi}{2}\right] = \frac{2ar+r^2}{2}$$

$$P = \frac{1}{a^2\cancel{\pi}}\int_0^a 2r\cancel{\pi}\,dr\left[\frac{2ar+r^2}{4Dt} + e^{-\frac{(a+r)^2}{4Dt}}\right] = \frac{1}{2a^2Dt}\int_0^a(2ar^2+r^3)\,dr + \underbrace{\frac{2}{a^2}\int_0^a e^{-\frac{(a+r)^2}{4Dt}}r\,dr}_{\frac{2}{a^2}\int_a^{2a}e^{-\frac{\xi^2}{4Dt}}(\xi-a)\,d\xi}$$

$$= \frac{1}{2a^2Dt}\left\{\frac{2a^4}{3}+\frac{a^4}{4}\right\} + \frac{2}{a^2}\left\{-e^{-\frac{\xi^2}{4Dt}}\Big|_a^{2a} 2Dt - a\int_a^{2a}e^{-\frac{\xi^2}{4Dt}}d\xi\right\}$$

$$= \frac{11}{12}\frac{a^2}{2Dt} + 4\frac{Dt}{a^2}e^{-\frac{a^2}{4Dt}} - \frac{4Dt}{a^2}e^{-\frac{a^2}{Dt}} - \frac{4\sqrt{Dt}}{a}\int_{\sqrt{\frac{a^2}{4Dt}}}^{\sqrt{\frac{4a^2}{4Dt}}}e^{-y^2}dy$$

$\frac{a^2}{4Dt} = y^2$ = klein

$$1 - y^2 + \frac{y^4}{2}$$
$$-\left[1-4y^2+\frac{16y^4}{2}\right]$$

$$P = \frac{11}{6}y^2 + \frac{e^{-y^2}-e^{-4y^2}}{y^2} - \frac{2}{y}\int_y^{2y}\underbrace{e^{-y^2}dy}_{(1-y^2)\,dy} = y - \frac{y^3}{3} + \frac{y^5}{10}\Big/ = y - \frac{7y^3}{3} + \cdots$$

$$= \frac{11}{6}y^2 + 3 - \frac{15}{2}y^2 - \left[2 - \frac{14}{3}y^2\right] = 1 + \left(\frac{11}{6}+\frac{14}{3}-\frac{15}{2}\right)y^2$$

$$\lim\left(\underline{P = 1-y^2}\right) = 1 - \frac{a^2}{4Dt}$$

$\left.\begin{matrix}11\\+28\\-45\end{matrix}\right\} = -6$

$$\mathcal{P} = \cancel{\frac{1}{a^2}\int_0^a r\,dr \; \frac{1}{Dt}\left\{\int_{a-r}^{a+r} \rho\, d\rho \cdot \varphi + \int\right.}$$

$$\left.\frac{\partial u}{\partial x}\right| = -\frac{1}{2\sqrt{\pi D t}}$$

$$\int_0^t D \frac{\partial u}{\partial x}\, dt = -\sqrt{\frac{Dt}{\pi}}$$

Für Grenzfall kurzer Zeiten:

$$\mathcal{P} = \frac{2a\pi\sqrt{\frac{Dt}{\pi}}}{a^2\pi} = 2\sqrt{\frac{Dt}{a^2\pi}}$$

$$h\mathcal{P} = 1 - \frac{2}{\sqrt{\pi}}\frac{e^{-\beta^2}}{2\beta} + \frac{1}{\beta\sqrt{\pi}} - \frac{e^{-\beta^2}}{\beta\sqrt{\pi}}$$

$$= \cancel{\lambda} - \cancel{\lambda} + \cancel{\frac{2}{\sqrt{\pi}}\frac{e^{-\beta^2}}{2\beta}} + \frac{1}{\beta\sqrt{\pi}} - \cancel{\frac{e^{-\beta^2}}{\beta\sqrt{\pi}}} = \frac{2}{h}\sqrt{\frac{Dt}{\pi}}$$

$$\iiiint e^{-\frac{(\xi-x)^2+(\eta-y)^2}{4Dt}}\, dx\, dy\, d\xi\, d\eta$$

Einstein

$$u = A\xi - \frac{5}{3} P^3 A \frac{\xi}{\rho^3} - \frac{5}{6} A P^3 \frac{\partial^3 \rho}{\partial \xi^3} - \cdots P^5.$$

$$- \frac{5}{6} B P^3 \frac{\partial^3\rho}{\partial\xi\,\partial\eta^2}$$

$$\frac{\partial\rho}{\partial\xi} = \frac{\xi}{\rho}$$

$$= A\xi - P^3 A\left[\frac{\xi}{\rho^3}\underbrace{\left(\frac{5}{3} - \frac{5}{2}\right)}_{-\frac{5}{6}} + \frac{5}{2}\frac{\xi^3}{\rho^5}\right]$$

$$\frac{\partial^2\rho}{\partial\xi^2} = \frac{1}{\rho} - \frac{\xi^2}{\rho^3} \qquad \frac{\partial^2\rho}{\partial\eta^2} = \frac{1}{\rho} - \frac{\eta^2}{\rho^3}$$

$$\frac{\partial^3\rho}{\partial\xi^3} = -\frac{3\xi}{\rho^3} + \frac{3\xi^3}{\rho^5} \qquad \frac{\partial^3\rho}{\partial\xi\,\partial\eta^2} = -\frac{\xi}{\rho^3} + \frac{3\xi\eta^2}{\rho^5}$$

$$u = A\xi + P^3 A\left[\cancel{\frac{5}{6}\frac{\xi}{\rho^3}} - \frac{5}{2}\frac{\xi^3}{\rho^5}\right]$$

$$+ P^3 B\left[\cancel{\frac{5}{6}\frac{\xi}{\rho^3}} - \frac{5}{2}\frac{\xi\eta^2}{\rho^5}\right] \qquad (6a)$$

$$+ P^3 C\left[\cancel{\frac{5}{6}\frac{\xi}{\rho^3}} - \frac{5}{2}\frac{\xi\zeta^2}{\rho^5}\right]$$

$$\frac{\partial\left(\frac{1}{\rho}\right)}{\partial\xi} = -\frac{\xi}{\rho^3} \qquad \frac{\partial^2\left(\frac{1}{\rho}\right)}{\partial\xi^2} = -\frac{1}{\rho^3} + \frac{3\xi^2}{\rho^5}$$

$$p = [illegible]\,k P^3\left\{\cancel{(A+B+C)\frac{1}{\rho^3}}\left\{\frac{A\xi^2 + B\eta^2 + C\zeta^2}{\rho^5}\right\}\right.$$

$$-2kA + 5kP^3\left\{A\left(\frac{3\xi^2}{\rho^5} - \frac{5\xi^4}{\rho^7}\right) + B\left(\frac{\eta^2}{\rho^5} - \frac{5\xi^2\eta^2}{\rho^7}\right) + C\left(\frac{\zeta^2}{\rho^5} - \frac{5\xi^2\zeta^2}{\rho^7}\right)\right.$$

$$X_\xi = -2kA + 10kP^3\left\{A\frac{\xi^2}{\rho^5}\right\} - 25kP^3(A\xi^2+B\eta^2+C\zeta^2)\frac{\xi^2}{\rho^7}$$

$$X_\eta = +k\frac{5}{2}P^3\left\{(A\xi^2+B\eta^2+C\zeta^2)\left(-\frac{5\xi\eta}{\rho^7} - \frac{5\xi\eta}{\rho^7}\right) + 4B\frac{\eta\xi}{\rho^5}\right\}$$

$$= -kP^3\left[25\frac{\xi\eta\,(A\xi^2+B\eta^2+C\zeta^2)}{\rho^7} - 10B\frac{\eta\xi}{\rho^5}\right\}$$

$$X_n = +2kA\frac{\xi}{\rho} - 10kP^3\frac{(A\xi^3 + B\xi\eta^2 + C\xi\zeta^2)}{\rho^6} + 25kP^3(A\xi^2+B\eta^2+C\zeta^2)\frac{(\xi^3+\xi\eta^2+\xi\zeta^2)}{\rho^8}$$

$$= 2kA\frac{\xi}{\rho} + 15kP^3(A\xi^2+B\eta^2+C\zeta^2)\frac{\xi}{\rho^6}$$

$X_u u + Y_u v + Z_u w =$

$$= \frac{2k(A^2\xi^2 + B^2\eta^2 + C^2\zeta^2)}{\rho} + 15kP^3\,\frac{(A\xi^2 + B\eta^2 + C\zeta^2)^2}{\rho^6} - 5kP^3(A\xi^2 + B\eta^2 + C\zeta^2)^2$$

$$\sum \rho^4 = \sum 4\pi R^6 = \sum 3\,\overline{\xi^4} + 6\,\overline{\xi^2\eta^2} = 3\frac{4}{5}\pi R^6 + 6\frac{4}{15}\pi R^6$$

$$4 = \frac{12}{5} + \frac{8}{5} = \frac{20}{5} \text{ stimmt}$$

$$A^2\xi^4 + B^2\eta^4 + C^2\zeta^4 + 2AB\,\overline{\xi^2\eta^2} + 2AC\,\xi^2\zeta^2 + 2BC\,\eta^2\zeta^2$$

$$= \frac{4}{5}\pi R^6\left\{A^2 + B^2 + C^2 + \frac{2}{3}(AB + AC + BC)\right\}$$

$$A^2 + B^2 + C^2 + 2(AB + AC + BC) = 0$$

$$\sum (A\xi^2 + \cdots C\zeta^2)^2 = \frac{8}{15}\pi R^6\left[A^2 + B^2 + C^2\right]$$

$$A^2\xi^2 + B^2\eta^2 + C^2\zeta^2 = \frac{4}{3}\pi R^4\left[A^2 + B^2 + C^2\right]$$

$$\sum (X_u u + Y_u v + Z_u w) = \frac{2k}{R}\sum (A^2\xi^2 + B^2\eta^2 + C^2\zeta^2) + 10\frac{kP^3}{R^6}\sum (A\xi^2 + B\eta^2 + C\zeta^2)^2$$

$$= \frac{4}{3}\pi R^4\left[A^2 + B^2 + C^2\right]\left\{\frac{2k}{R} + 10\,\frac{kP^3}{R^6}\cdot\frac{2}{5}R^2\right\}$$

$$W = \left\{\frac{8}{3}\pi R^3 k + \frac{16}{3}\pi k\,P^3\right\}\delta^2$$

$$W = 2\delta^2 k\left[V + 2\Phi\right]$$

$$X_\xi \frac{\partial u}{\partial x} + X_\eta \frac{\partial w}{\partial y} + Z_\zeta \frac{\partial w}{\partial z} = \sum \begin{Bmatrix} \left[-2kA + 10kA\,P^3\frac{\xi^2}{\rho^5} - 25kP^3(A\xi^2 + B\eta^2 + C\zeta^2)\frac{\xi^2}{\rho^7}\right] \\ \left[A - 5AP^3\frac{\xi^2}{\rho^5} - \frac{5}{2}\frac{P^3}{\rho^3}(A\xi^2 + B\eta^2 + C\zeta^2)\left(1 - 5\frac{\xi^2}{\rho^2}\right)\right] \end{Bmatrix}$$

$$= -2k(A^2 + B^2 + C^2) + \frac{kP^3}{\rho^5}\left\{10(A^2\xi^2 + \quad) - 25(\quad)^2 + 10(A^2\xi^2 - \quad) - 25(\quad)^2\right\}$$

$$A\,\frac{4}{5}\pi R^6 + \underbrace{(B+C)}_{=-A}\,\frac{4}{15}\pi R^6 = \frac{8}{15}A\pi R^6$$

$\lim_{R\to\infty}\int (u^2+v^2+w^2)\,d\tau = \infty\,?$

$$u^2+v^2+w^2 = A^2\xi^2 + B^2\eta^2 + C^2\zeta^2$$

$$\cancel{\pm} - 5\frac{P^3}{\rho^5}(A\xi^2+B\eta^2+C\zeta^2)^2 - \frac{25}{4}\cdot\frac{P^6}{\rho^8}(A\xi^2+B\eta^2+C\zeta^2)^2$$

$$u = A\xi - \frac{5}{2}P^3(A\xi^2+B\eta^2+C\zeta^2)\frac{\xi}{\rho^5}$$

$$\frac{\partial u}{\partial x} = A - \frac{5}{2}\frac{P^3}{\rho^5}(A\xi^2+B\eta^2+C\zeta^2)\left(1-\frac{5\xi^2}{\rho^2}\right) - 5P^3A\frac{\xi^2}{\rho^5}$$

$$\sum\left(\frac{\partial u}{\partial x}\right)^2 \cancel{\pm} = (A^2+B^2+C^2) + 25\frac{P^3}{\rho^7}(A\xi^2+B\eta^2+C\zeta^2)^2 - 10\frac{P^3}{\rho^5}(A^2\xi^2+B^2\eta^2+C^2\zeta^2)$$

$$\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} = \frac{25}{\cancel{2}}P^3(A\xi^2+B\eta^2+C\zeta^2)\frac{\xi\eta}{\rho^7} - 5P^3\frac{\xi\eta}{\rho^5}(A+B)$$

$$= \frac{5P^3\xi\eta}{\rho^5}\left\{C + \frac{5(A\xi^2+B\eta^2+C\zeta^2)}{\rho^2}\right\}$$

$$(\quad)^2 = 25P^6 \ldots$$

$$\cancel{\Phi = 2\mu\sum(A^2+B^2+C^2)\,4\pi R^2}$$

$$\cancel{\int\frac{\xi^2}{\rho^7}\,ds\,d\rho = \frac{4}{3}\pi\int\frac{\rho^4}{\rho^7}\,d\rho = \frac{2}{3}\pi\frac{1}{\rho^2}}$$

$$\int\frac{\xi^2}{\rho^5}\,ds\,d\rho = \frac{4}{3}\pi\int\frac{d\rho}{\rho^3} = \frac{2}{3}\pi\frac{1}{\rho^2}$$

$$\int(X_n u + Y_n v + Z_n w)\,dS = \int(X_x\cos nx + X_y\cos ny + X_z\cos nz)u\,dS$$
$$+(Y_x\cos nx + Y_y\cos ny + Y_z\cos nz)v\,dS$$
$$+(Z_x\cos nx + Z_y\cos ny + Z_z\cos nz)w\,dS =$$

$$= \int(X_x u + Y_x v + Z_x w)\,dy\,dz + \cdots$$

$$= \iint\left[\frac{\partial}{\partial x}(X_x u + Y_x v + Z_x w) + \frac{\partial}{\partial y}(Y_x u + Y_y v + Y_z w) + \frac{\partial}{\partial z}(X_z u + Z_y v + Z_z w)\right]dx\,dy\,dz$$

$$\iiint \Phi \, dv = \iiint 2k(A^2 + B^2 + C^2) - 10(A^2\xi^2 + B^2\eta^2 + C^2\zeta^2)\frac{kP^3}{\rho^5} + 25(A\xi^2 + B\eta^2 + C\zeta^2)^2 \frac{kP^3}{\rho^7}$$

$$\int dv = \frac{4}{3} R^3 \pi$$

$$\int \frac{\xi^2}{\rho^5} dv = \int \cancel{\frac{4}{3}\pi \rho^3 \ldots} = \int \frac{\cos^2\varphi}{\rho^3} 2\pi \sin\varphi \, d\varphi \, \rho^2 d\rho$$

$$i = \frac{K\Delta\varphi}{4\pi} \frac{p\,\delta}{\mu} \frac{q}{l} \, \delta \qquad W_1 = \left(\frac{K\Delta\varphi}{4\pi}\right)^2 \frac{p^2 \, 6\, q}{\mu^2\, l}$$

$$W_0 = \frac{p^2 R^4 \pi}{8 l \mu}$$

$$\frac{W_1}{W_0} = \left(\frac{K\Delta\varphi}{4\pi}\right)^2 \frac{6\,\xi}{\mu} \cdot \frac{8 \cdot R^2\pi}{R^4\pi} = \left(\frac{K\Delta\varphi}{4\pi}\right)^2 \frac{6}{\mu} \cdot \frac{8}{R^2}$$

$$\alpha = \sqrt{\frac{\beta}{D}}$$

$$u = \sum_\beta A_\beta e^{-\beta t} J\left(r\sqrt{\tfrac{\beta}{D}}\right) = \sum_\alpha \frac{2 e^{-\alpha^2 D t}}{J'(\alpha)^2} \int_0^1 f(\lambda) J(\alpha r) J(\alpha\lambda) \lambda \, d\lambda$$

$$f\left(\tfrac{r}{h}\right) = \varphi(\mu)$$

$$= \sum_\alpha A_\alpha e^{-\alpha^2 D t} J(\alpha r) = \sum \frac{2 e^{-\alpha^2 D t}}{h^2 J'(\alpha)^2} \int_0^a \varphi(\mu) J\left(\frac{\alpha r}{h}\right) J\left(\frac{\alpha \mu}{h}\right) \mu \, d\mu$$

$$= \sum \frac{2\delta\, e^{-h^2 \xi_r^2 D t}}{\pi l \, J'(h\xi_r)^2} \int_0^a \varphi(\mu) J(\xi_r r) J(\xi_r \mu) \mu \, d\mu$$

$$\int_0^1 J(\alpha\lambda) \lambda \, d\lambda = \frac{J_1(\alpha)}{\alpha}$$

$$\sum A_\alpha J(\alpha r) = \begin{matrix} 1 & r < a \\ 0 & r \geqq a \end{matrix}$$

$$u_r = \sum_\alpha \frac{2 e^{-\alpha^2 D t}}{[J'(\alpha)]^2} \int_0^1 J(\alpha r) J(\alpha\lambda) \lambda \, d\lambda = -\sum_{\alpha_1, \alpha_2 \ldots} \frac{2 e^{-\alpha^2 D t} J(\alpha r)}{\alpha J'(\alpha)}$$

$$f(x) = +\int_0^\infty J_1(\xi)\, J(\xi x)\, e^{-\xi^2 Dt}\, d\xi$$

$$= \int \frac{x}{2}\underbrace{\left[1 - \frac{\left(\frac{x}{2}\right)^2}{1.2} + \frac{\left(\frac{x}{2}\right)^4}{2!\,3!} - \frac{\left(\frac{x}{2}\right)^6}{3!\,4!} + \cdots\right]}_{J_1 x}\underbrace{\left[1 - \frac{\left(\frac{x}{2}\right)^2}{(1!)^2} + \frac{\left(\frac{x}{2}\right)^4}{(2!)^2} - \frac{\left(\frac{x}{2}\right)^6}{(3!)^2}\cdots\right]}_{J(x)} d\xi$$

$$= \int_0^\infty \frac{\xi}{2}\left[1 - \frac{\xi^2}{8} + \frac{\xi^4}{16.12}\right]\left[1 - \frac{(x\xi)^2}{4} + \frac{(x\xi)^4}{64}\right] e^{-\xi^2 Dt}\, d\xi$$

$$1 - \frac{\xi^2}{4}\left(x^2 + \frac{1}{2}\right) + \frac{\xi^4}{64}\left(x^4 + 2x^2 + \frac{1}{3}\right)$$

$$= \int\left[\frac{\xi}{2} - \frac{x^2 + \frac{1}{2}}{8}\,\xi^3 + \frac{x^4 + 2x^2 + \frac{1}{3}}{128}\,\xi^5\right] e^{-\xi^2 Dt}\, d\xi$$

$$\int_0^\infty \xi\, e^{-\xi^2 Dt}\, d\xi = \frac{1}{2Dt}\, e^{-\xi^2 Dt}\Big|_0^\infty = \frac{1}{2Dt}$$

$$\int \xi^3\, e^{-\xi^2 Dt}\, d\xi = \frac{1}{2(Dt)^2}$$

$$\int \xi^5\, e^{-\xi^2 Dt}\, d\xi = \frac{1}{(Dt)^3}$$

$$\frac{\partial}{\partial t}\int_0^r r u\, dr = r\frac{\partial u}{\partial r}$$

$$\int_0^a r u\, dr\Big|_t^{t=\infty} = \int_t^\infty dt \cdot a\frac{\partial u}{\partial r}\Big|_a$$

$$\int_0^a r u_t\, dr = -\int_t^\infty a\left(\frac{\partial u}{\partial r}\right)_{a,t} dt$$

$$P = 1 - \frac{2}{a^2}\int_0^a r u\, dr$$

$$\int_0^{a=1} J(\alpha r)\, r\, dr = \frac{J_1(\alpha)}{\alpha} = -\frac{J'(\alpha)}{\alpha}$$

$$P = 1 - 2\sum_{\alpha_1, \alpha_2 \ldots} 2\, \frac{e^{-\alpha^2 Dt}\, J(\alpha r)}{\alpha^2}$$

$$W(n) = \left(\frac{v}{V}\right)^n \left(1-\frac{v}{V}\right)^{N-n} \frac{N!}{n!\,N-n!}$$

$$\lg\sqrt{\frac{2N\pi}{(2\pi)^2\left(\frac{N}{2}\right)^2(1-\delta^2)}} = \lg\sqrt{\frac{2}{N\pi(1-\delta^2)}}$$

$$W_{mm}:\ \frac{v}{V} = \frac{1}{2} \qquad W(n) = \left(\frac{1}{2}\right)^N \frac{N!}{n!\,N-n!}$$

$$\lg W = N\lg\left(\tfrac{1}{2}\right) + N\lg N - \cancel{N} + \tfrac{1}{2}\lg 2N\pi \qquad \cancel{-N+n+(N-n)}$$

$$- n\lg n - (N-n)\lg(N-n) - \tfrac{1}{2}\lg 2n\pi - \tfrac{1}{2}\lg 2(N-n)\pi$$

$$= -N\lg 2 + N\lg N - n\lg n - (N-n)\lg(N-n) + \tfrac{1}{2}\lg \underbrace{\frac{N}{n(N-n)}}_{\frac{4}{N(1-\delta^2)}} - \lg\sqrt{2\pi}$$

$$n = \frac{N}{2}(1+\delta)$$

$$\lg W = -N\lg 2 + N\lg N - \frac{N}{2}(1+\delta)\left[\lg\frac{N}{2} + \delta - \frac{\delta^2}{2}\right] - \lg 2\pi + \lg\sqrt{4}$$

$$- \frac{N}{2}(1-\delta)\left[\lg\frac{N}{2} - \delta - \frac{\delta^2}{2}\right] - \lg\sqrt{N} + \frac{\delta^2}{2} - \frac{\delta^4}{4}$$

$$= N\lg\frac{N}{2} \qquad - N\left[\lg\frac{N}{2} - \frac{\delta^2}{2}\right] - N\delta^2 - \lg\sqrt{N} - \lg\sqrt{2\pi} + \frac{\delta^2}{2} - \frac{\delta^4}{4} + \lg 2$$

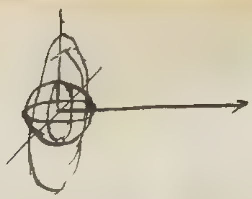
$$= -N\frac{\delta^2}{2} - \lg\sqrt{\frac{N\pi}{2}} + \delta^2 - \frac{\delta^4}{2}$$

$$W = \sqrt{\frac{2}{N\pi}}\, e^{-\frac{N\delta^2}{2}} = \sqrt{\frac{2}{N\pi}}\, e^{-\frac{(n_1-n_2)^2}{2(n_1+n_2)}}$$

$$n_1 - n_2 = N\delta$$

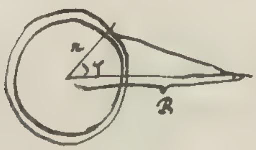
$$\bar{W} =$$

2[illegible]



$$\mu = \mu_0 \left(1 + \alpha n T^{\frac{1}{2}} + \underbrace{\beta n^2 T^2}_{}\right) \qquad T = R^3 \qquad n = \frac{N}{R^3}$$

Einfluss der Zusammenstösse d. Kugeln



$$u_R = \frac{2}{4\pi D t} \int_0^a \int_0^\pi e^{-\frac{R^2 + r^2 - 2Rr\cos\varphi}{4Dt}} \frac{r\, d\varphi\, dr}{a^2 \pi} \qquad R'^2 = \frac{R^2}{4Dt}$$

$\cos\varphi = \xi$

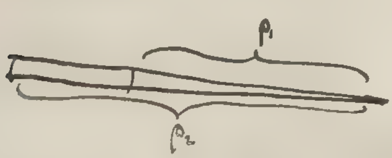
$-\sin\varphi\, d\varphi = d\xi$

$d\varphi = \frac{-d\xi}{\sqrt{1-\xi^2}}$

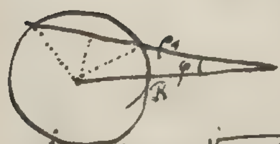
$$= \frac{1}{2\pi^2 a^2 D t} \int_0^a r\, dr \int_{-1}^{+1} \frac{e^{-\frac{R^2+r^2-2Rr\xi}{4Dt}}}{\sqrt{1-\xi^2}}\, d\xi$$

$$P = \frac{1}{2a^2\pi^2 D t} \int_a^\infty 2R\pi\, dR \int_0^a r\, dr \int_{-1}^{+1} \frac{e^{-\frac{R^2+r^2-2Rr\xi}{4Dt}}}{\sqrt{1-\xi^2}}\, d\xi = \frac{16 D t}{a^2 \pi} \int_{\frac{a}{2\sqrt{Dt}}}^\infty R\, dR \int_0^{\frac{a}{2\sqrt{Dt}}} r\, dr \int_{-1}^{+1} \frac{e^{-(R^2+r^2-2Rr\xi)}}{\sqrt{1-\xi^2}}\, d\xi$$

$$\frac{1}{(2\pi\sqrt{Dt})} \int_{x=-a}^{x=+a} 2 \int_{y=0}^{\sqrt{a^2-x^2}} e^{-\frac{(R-x)^2+y^2}{4Dt}}\, dx\, dy = \frac{2}{\pi} \int_{x=-a}^{+a} \frac{e^{-\frac{(R-x)^2}{4Dt}}}{2\sqrt{Dt}}\, dx \cdot \int_0^{z=\sqrt{\frac{a^2-x^2}{4Dt}}} e^{-z^2}\, dz$$



$$\int_{\rho_1}^{\rho_2} e^{-\frac{\rho^2}{4Dt}} \frac{\rho\, d\rho\, d\varphi}{2Dt} = -e^{-\frac{\rho^2}{4Dt}} \Big|_{\rho_1}^{\rho_2} \cdot d\varphi$$



$\cancel{\rho_1, \rho_2 = R\cos\varphi \pm}$ $\quad a^2 = R^2 + \rho_1^2 - 2R\rho_1 \cos\varphi$

$$\rho = R\cos\varphi \pm \sqrt{a^2 - R^2 + (R\cos\varphi)^2}$$

$$2 \int_0^{\arcsin\frac{a}{R}} \left[e^{-\frac{(R\cos\varphi + \sqrt{a^2 - R^2\sin^2\varphi})^2}{4Dt}} - e^{-\frac{(R\cos\varphi - \sqrt{a^2 - R^2\sin^2\varphi})^2}{4Dt}} \right] d\varphi$$

$0 = 2\rho\, d\rho - 2R\cos\varphi\, d\rho + 2R\rho\sin\varphi\, d\varphi$

$d\rho = \frac{R\cos\varphi - \rho}{2R\rho\sin\varphi}\, d\varphi$

=

$$\int_{-1}^{+1} \frac{e^{-\alpha x}}{\sqrt{1-x^2}}\, dx = J(\alpha) = \overset{?}{J_0}(i\alpha)$$

$$\frac{\partial J}{\partial \alpha} = -\int x \frac{e^{-\alpha x}}{\sqrt{1-x^2}}\, dx = \underbrace{+\sqrt{1-x^2}\, e^{-\alpha x}\Big|_{-1}^{+1}}_{0\,?} + \alpha \int \underbrace{e^{-\alpha x}\sqrt{1-x^2}\, dx}$$

$$= \int \frac{e^{-\alpha x}\, dx}{\sqrt{1-x^2}} - \int x^2 \frac{e^{-\alpha x}}{\sqrt{}}\, dx$$

$$+\frac{\partial J}{\partial \alpha} = +\alpha J \mp \alpha \frac{\partial^2 J}{\partial \alpha^2}$$

$$\frac{\partial^2 J}{\partial \alpha^2} + \frac{1}{\alpha}\frac{\partial J}{\partial \alpha} \mp J = 0$$

$$= -\frac{18}{r^8} - 18\frac{x^2}{r^{10}} + 45\left\{\underline{x^4+y^4+z^4+2(x^2y^2+x^2z^2+y^2z^2)}\right\} = -\frac{18}{r^8} - 18\frac{x^2}{r^{10}} + 45\left(x^4 + x^2y^2 + x^2z^2 + y^2z^2\right)$$

$$= -\frac{18}{r^8} - 18\frac{x^2}{r^{10}} + \frac{45x^2}{r^{10}} + 45\frac{y^2z^2}{r^{12}} = -\frac{18}{r^8} + 27\frac{x^2}{r^{10}} + 45\frac{y^2z^2}{r^{12}}$$

$$p = -\frac{5}{3} k P^3 \left\{ A \frac{\partial^2(\frac{1}{r})}{\partial x^2} + B \frac{\partial^2}{\partial y^2}(\tfrac{1}{r}) + C \frac{\partial^2}{\partial z^2}(\tfrac{1}{r}) \right\} *$$

$$\begin{matrix} u \\ v \\ w \end{matrix} = 6 \cdot \frac{K \Delta\varphi}{4\pi\mu} \cdot \nabla p$$

$$\Phi' = \left[6 \cdot \frac{K\Delta\varphi}{4\pi\mu}\right]^2 \left[\frac{5}{3} k P^3\right]^2 \iiint dx\,dy\,dz.$$

$$\left[A \frac{\partial^3}{\partial x^3} + B \frac{\partial^3}{\partial x \partial y^2} + C \frac{\partial^3}{\partial x \partial z^2}\right]^2 + \left[A \frac{\partial^3}{\partial x^2 \partial y} + B \frac{\partial^3}{\partial y^3} + C \frac{\partial^3}{\partial y \partial z^2}\right]^2 + \left[A \frac{\partial^3}{\partial x^2 \partial z} + - \quad\right]^2$$

$$= \sum \left\{ A^2 \left[\left(\frac{\partial^3}{\partial x^3}\right)^2 + \left(\frac{\partial^3}{\partial x^2 \partial y}\right)^2 + \left(\frac{\partial^3}{\partial x^2 \partial z}\right)^2\right] + 2BC\left[\frac{\partial^3}{\partial x \partial y^2}\frac{\partial^3}{\partial x \partial z^2} + \frac{\partial^3}{\partial y^3}\frac{\partial^3}{\partial y \partial z^2} + \frac{\partial^3}{\partial y^2 \partial z}\frac{\partial^3}{\partial z^3}\right]\right\}$$

$$\frac{1}{r} \quad -\frac{x}{r^3} \quad -\frac{1}{r^3} + \frac{3x^2}{r^5} \Big/ \frac{\partial^3}{\partial x^3} = \frac{9x}{r^5} - \frac{15x^3}{r^7} \qquad 81\frac{x^2}{r^{10}} - 9 \cdot 30 \frac{x^4}{r^{12}} + (15)^2 \frac{x^6}{r^{14}}$$

$$\frac{\partial^3}{\partial x^2 \partial y} = \frac{3y}{r^5} - \frac{15x^2y}{r^7} \qquad 9\frac{y^2}{r^{10}} - 3 \cdot 30 \frac{x^2y^2}{r^{12}} + (15)^2 \frac{x^4y^2}{r^{14}}$$

$$\frac{\partial^3}{\partial x^2 \partial z} = \frac{3z}{r^5} - \frac{15x^2z}{r^7} \qquad \frac{9z^2}{r^{10}} - 3 \cdot 30 \frac{x^2z^2}{r^{12}} + (15)^2 \frac{x^4z^2}{r^{14}}$$

$$= \frac{9}{r^8} + 72 \cdot \frac{x^2}{r^{10}} - 180 \cdot \frac{x^4}{r^{12}} - 90 \cdot \frac{x^2}{r^{10}} + 225 \cdot \frac{x^4}{r^{12}}$$

$$= \frac{9}{r^8} - 18 \frac{x^2}{r^{10}} + 45 \cdot \frac{x^4}{r^{12}}$$

$$\cancel{\frac{81x^2}{r^{10}} - 9 \cdot 10 \frac{x^3}{r^{12}} + 9x (15)^2 \frac{x^5}{r^{14}}}$$

$$\left(\frac{3x}{r^5} - \frac{15xy^2}{r^7}\right)\left(\frac{3x}{r^5} - \frac{15xz^2}{r^7}\right) + \left(\frac{9y}{r^5} - \frac{15y^3}{r^7}\right)\left(\frac{3y}{r^5} - \frac{15z^2y}{r^7}\right) + \left(\frac{3z}{r^5} - \frac{15y^2z}{r^7}\right)\left(\frac{9z}{r^5} - \frac{15z^3}{r^7}\right) =$$

$$\left.\begin{aligned} &= 9\frac{x^2}{r^{10}} - \frac{45x^2(y^2+z^2)}{r^{12}} + (15)^2 \frac{x^2y^2z^2}{r^{14}} \\ &+ 27\frac{y^2}{r^{10}} - \underline{45y^4 - 9 \cdot 15 z^2y^2} + (15)^2 y^4z^2 \\ &+ 27 \frac{z^2}{r^{10}} - 45z^4 - \underline{9 \cdot 15 \cdot y^2z^2} + (15)^2 \underline{y^2z^4} \end{aligned}\right\} = \frac{27}{r^8} - 18\frac{x^2}{r^{10}} - \frac{45(x^2y^2 + x^2z^2 + y^4 + z^4) + 9 \cdot 30\, y^2z^2}{r^{12}}$$

$$+ (15)^2 \frac{y^2z^2}{r^{12}} \quad \cdots$$

$$= \frac{27}{r^8} - 18\frac{x^2}{r^{10}} - \frac{45(x^2y^2 + x^2z^2 + y^2z^2 + y^4 + z^4)}{r^{12}}$$

$$\frac{\overline{W}'}{\cancel{\;}} = \cancel{\;}q.M \iiint \Sigma \left\{ A^2\left[\frac{1}{r^8} - \frac{2x^2}{r^{10}} + \frac{5x^4}{r^{12}}\right] + 2BC\left[-\frac{R}{r^8} + \frac{3x^2}{r^{10}} + 5\frac{y^2z^2}{r^{12}}\right]\right\}$$

$$A^2 \underbrace{\frac{\cancel{\;}}{r^8}\left[4r^2\pi - \frac{8}{3}\pi r^2 + \cancel{3}4\pi r^2\right]}_{\frac{16}{3}\pi\frac{r^2}{r^8}} + \cancel{\frac{2BC}{r^8}\left[-2.4r^2\pi + 4r^2\pi + 4r\pi\right]}$$

$$=$$

$$= qM\ (A^2+B^2+C^2)\ \frac{16}{3}\pi \int_P^\infty \frac{dr}{r^6} = \cancel{\;}\ \frac{48\pi\ (A^2+B^2+C^2)}{5\ P^5} \cdot \frac{25}{9}\frac{P^6}{16\pi^2}(6\,K\Delta\varphi)^2$$

$$= \frac{5}{3}\ \frac{(A^2+B^2+C^2)\ (6\,K\,\Delta\varphi)^2 \cdot P}{\pi}$$

$$\overline{W}_0 = 2(A^2+B^2+C^2)\mu \qquad = \frac{5}{4}\left(A^2+B^2+C^2\right)\left(\frac{6\,K\,\Delta\varphi}{\pi}\right)^2 \frac{1}{P^2}\cdot\Phi$$

$$\mu = \mu_0\left[1 + 2{\cdot}5\,\varphi + \frac{5}{8}\left\{\frac{6(K.\Delta\varphi)^2}{\mu\pi^2}\right\}\frac{1}{P^2}\ \varphi\right]$$

$$= \mu_0\left[1 + \left(2{\cdot}5 + 10\,\frac{6}{P^2\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2\right)\varphi\right]$$

$$K\,\Delta\varphi = \frac{2}{300}$$

$$6 = \cancel{\;}\frac{10^6}{9.10^{11}} = \frac{1}{9}.10^{-5}$$

$$10\cdot\frac{10^{-5}}{9.10^{-12}.0{\cdot}02}\cdot\frac{4}{9.10^4.150}$$

$$\mu = 0{\cdot}02$$

$$P = 10^{-6}$$

$$= \cancel{\;}\frac{10^4.4}{81.\ 30} = \frac{400\cancel{0}}{24\cancel{0}} = 16$$

$2{\cdot}5$ für $P = \sqrt{6}.10^{-6}$

$$2P = 5.10^{-6} = \frac{1}{20}\mu$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{u}{4x^2}$$

$$u = \sqrt{x}\int_{-\infty}^{+\infty} e^{-a^2}\,da \int_0^{\pi} \varphi\,(x\cos\omega + 2a\sqrt{t})\,d\omega + \sqrt{x}\int_{-\infty}^{+\infty} e^{-a^2}\,da \int \psi\,(x\cos\omega + 2a\sqrt{t})\,\lg(x\sin^2\omega)\,d\omega$$

$$\sqrt{x}\,\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}\sqrt{x} + \frac{1}{\sqrt{x}}\frac{\partial v}{\partial x} + \cancel{\frac{v}{4x\sqrt{x}}} - \cancel{\frac{v}{4x\sqrt{x}}}$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \frac{1}{x}\frac{\partial v}{\partial x} \qquad v = \frac{u}{\sqrt{x}}$$

$$x\cos\omega + 2a\sqrt{t} = \alpha$$

$$v = \int_{-\infty}^{+\infty} e^{-\left(\frac{\alpha - x\cos\omega}{2\sqrt{t}}\right)^2} \frac{d\alpha}{2\sqrt{t}} \int_0^{\pi} \varphi(\alpha)\,d\omega + \cancel{\iint e^{-\frac{(\alpha - x\cos\omega)^2}{4t}} \frac{d\alpha}{2\sqrt{t}} \lg(x\sin^2\omega)\,d\omega \cdot \psi(\alpha)}$$

$$\int_0^{\infty} e^{-n\xi^2}\cos x\xi\,d\xi = \frac{1}{2}\sqrt{\frac{\pi}{n}}\;e^{-\frac{x^2}{4n}}$$

$$e^{-\frac{(\alpha - x\cos\omega)^2}{4t}} = 2\sqrt{\frac{t}{\pi}}\int_0^{\infty} e^{-t\xi^2}\,\underbrace{\cos(\alpha - x\cos\omega)\xi}_{\cos\alpha\xi\,\cos(x\xi\cos\omega) + \cancel{\sin\alpha\xi\,\sin(x\xi\cos\omega)}}\,d\xi$$

$$v = \cancel{\frac{1}{\sqrt{\pi}}\iiint \varphi(\alpha)\,d\alpha\; e^{-t\xi^2}\cos x\xi\,d\xi}$$

$$= \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} d\alpha \int_{\xi=0}^{\infty} d\xi \int e^{-t\xi^2}\varphi(\alpha)\cos\alpha\xi \int_0^{\pi}\cos(x\xi\cos\omega)\,d\omega$$

$$v = \int_{-\infty}^{+\infty}\int_0^{\pi} e^{-a^2}\,da\;\varphi(x\cos\omega + 2a\sqrt{t})\,d\omega$$

$$\frac{\partial v}{\partial t} = \frac{1}{\sqrt{t}} \iint a\,e^{-a^2}\,da\;\varphi'(x\cos\omega + 2a\sqrt{t})\,d\omega$$

$$\frac{1}{x}\frac{\partial v}{\partial x} = \iint e^{-a^2}\,da\;\frac{\cos\omega\;\varphi'(\qquad)}{x}\,d\omega \quad \Big\| \quad \frac{\partial^2 v}{\partial x^2} = \iint e^{-a^2}\,da\,.\,\cos^2\omega\;\varphi''(\quad)\,d\omega$$

$$\frac{\partial v}{\partial t} = \frac{1}{\sqrt{t}}\int d\omega\left\{-\frac{e^{-a^2}}{2}\,\varphi'(x\cos\omega + 2a\sqrt{t})\Big|_{-\infty}^{+\infty} + \sqrt{t}\,.\,e^{-a^2}\,\varphi''(x\cos\omega + 2a\sqrt{t})\,da\right\}$$

$$= \iint e^{-a^2}\,da\,.\,\varphi''(x\cos\omega + 2a\sqrt{t})\,d\omega$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} - \frac{1}{x}\frac{\partial v}{\partial x} =$$

$$\cancel{\frac{\partial^2 v}{\partial x^2} = \iint e^{-a^2}\,da\,.\,\varphi''\,d\omega - \iint e^{-a^2}\,da\;\varphi'\sin^2\omega\,d\omega}$$

$$\cancel{\frac{\varphi'\sin\omega}{x}}$$

$$\iint \frac{e^{-a^2}\,da}{x}\left[\underbrace{\varphi''(x\cos\omega + 2a\sqrt{t})\,x\sin^2\omega\,d\omega}_{-\varphi'\sin\omega\Big|_0^{\pi} + \int\varphi'\cos\omega\,d\omega} - \varphi'(\ldots)\,\frac{\cos\omega}{x}\,d\omega\right]$$

$= 0$ stimmt.

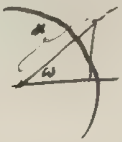
$t = 0$:

$$v = \iint e^{-a^2}\,da\;\varphi(x\cos\omega)\,d\omega = \int e^{-a^2}\,da\int_0^{\pi}\varphi(x\cos\omega)\,d\omega = \sqrt{\pi}\int_0^{\pi}\varphi(x\cos\omega)\,d\omega$$

$$\varphi(\xi) = \sum A_n \sin\frac{n\pi\xi}{a} + \sum B_n \cos\frac{n\pi\xi}{a}$$

$$\varphi(\xi) = 1 \qquad 0<\xi<a \qquad\qquad \varphi(x\cos\omega) = \begin{matrix}1\\0\end{matrix} \;..\; x\cos\omega \begin{matrix}<a\\>a\end{matrix} \;....\; \omega > \arccos\frac{a}{x}$$

$$\varphi(\xi) = 0 \qquad a<\xi$$



$$p = \frac{16Dt}{a^2\pi}\int_{\frac{a}{2\sqrt{Dt}}}^{\infty} e^{-R^2} R\,dR \int_0^{\frac{a}{2\sqrt{Dt}}} e^{-r^2} r\,dr \int_{-1}^{+1} \frac{e^{2Rr\xi}}{\sqrt{1-\xi^2}}\,d\xi$$

$$\int_{-1}^{+1}\frac{\xi^n}{\sqrt{1-\xi^2}}\,d\xi = \int_0^\pi \cos^n\varphi\,d\varphi = \cancel{\frac{1}{2}\int_0^\pi\left[e^{in\varphi}+e^{-in\varphi}\right]d\varphi = \frac{1}{2}\left.\frac{e^{in\varphi}-e^{-in\varphi}}{in}\right| = \left.\frac{\sin n\varphi}{n}\right|_0^\pi}$$

$$\int_0^\pi \cos^{2m}\varphi\,d\varphi = \left(\frac{1}{2}\right)^{2m}\int\left[e^{i\varphi}+e^{-i\varphi}\right]^{2m} = \left(\frac{1}{2}\right)^{2m}\int\left\{e^{2mi\varphi}+e^{-2mi\varphi}+\binom{2m}{1}\left[e^{(2m-2)i\varphi}+e^{-(2m-2)i\varphi}\right]+\cdots\right.$$

$$= \left(\frac{1}{2}\right)^{2m}\int_0^\pi\left[\cancel{\cos 2m\varphi} + \binom{2m}{1}\cancel{\cos 2(m-1)\varphi} + \binom{2m}{2}\cancel{\cos 2(m-2)\varphi} + \cdots \;\; \underline{\binom{2m}{m}\cos 0}\right]d\varphi$$

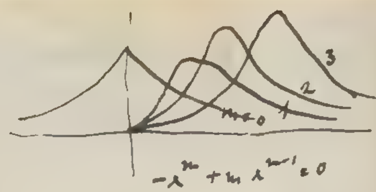
$$= \left(\frac{1}{2}\right)^{2m}\binom{2m}{m}\pi =$$

$$\int_{-1}^{+1}\frac{e^{2Rr\xi}}{\sqrt{1-\xi^2}}\,d\xi = \sum_{n=0}^{\infty}\int\frac{(2Rr\xi)^n}{n!\sqrt{1-\xi^2}}\,d\xi = \sum_{n=0}\frac{(2Rr)^{2m}}{(2m)!}\left(\frac{1}{2}\right)^{2m}\binom{2m}{m}\pi = \underbrace{\sum\frac{(Rr)^{2m}}{(2m)!}\binom{2m}{m}\pi}$$

$$\frac{2m(2m-1)(2m-2)\cdots(m+1)}{1.2.3\cdots\cdots m}\;\frac{}{1.2.3\cdots m(m+1)\cdots(2m-1)2m} = \left(\frac{1}{m!}\right)^2 \qquad = \sum\frac{(Rr)^{2m}}{(m!)^2}\pi$$

$$\int_{\frac{a}{2\sqrt{Dt}}}^{\infty} e^{-R^2}R^{2m+1}\,dR \cdot \int_0^{\frac{a}{2\sqrt{Dt}}} e^{-r^2}r^{2m+1}\,dr = \frac{1}{4}\int_{\frac{a^2}{4Dt}}^{\infty} e^{-x}x^m\,dx \cdot \int_0^{\frac{a^2}{4Dt}} e^{-x}x^m\,dx$$

$$P = \frac{4 \cdot Dt}{a^2} \sum \frac{1}{(m!)^2} \cdot \int_{\frac{a^2}{4Dt}}^{\infty} e^{-x} x^m \, dx \cdot \left(\int_0^{\frac{a^2}{4Dt}} e^{-x} x^m \, dx \right)^2$$



$-x^m + m x^{m-1} = 0$

$x = m$

$n = 0: \quad P_0 = \frac{4 Dt}{a^2} e^{-\frac{a^2}{4Dt}} \left[1 - e^{-\frac{a^2}{4Dt}} \right] = e^{-\frac{a^2}{4Dt}} \left[1 - \left(\frac{a^2}{4Dt} \right) \frac{1}{2!} + \left(\frac{a^2}{4Dt} \right)^2 \frac{1}{3!} - \cdots \right]$

$n = 1: \quad \int e^{-x} x \, dx = -e^{-x} x + \int e^{-x} dx = -e^{-x}(x+1)$

$P_1 = \frac{1}{\alpha} e^{-\alpha}(\alpha+1) \cdot \left[1 - e^{-\alpha}(\alpha+1) \right] = \frac{\alpha+1}{\alpha} e^{-\alpha} \left[1 - (1+\alpha)\left(1 - \alpha + \frac{\alpha^2}{2} - \frac{\alpha^3}{3!} - \right) \right]$

$= 1 - \begin{pmatrix} 1 - \alpha + \frac{\alpha^2}{2} - \frac{\alpha^3}{6} \\ + \alpha - \alpha^2 + \frac{\alpha^3}{2} \end{pmatrix}$

$= \frac{\alpha+1}{\alpha} e^{-\alpha} \left(\frac{\alpha^2}{2} - \frac{\alpha^3}{3} \right) = e^{-\alpha} (\alpha+1) \left(\frac{\alpha}{2} - \frac{\alpha^2}{3} \right) = e^{-\alpha} \left(\frac{\alpha^2}{2} - \frac{\alpha^3}{3} + \frac{\alpha}{2} - \frac{\alpha^2}{3} \right)$

$= \left(\frac{\alpha}{2} + \frac{\alpha^2}{6} \right) e^{-\alpha}$

$P_0 + P_1 = e^{-\alpha} \left[1 - \cancel{\frac{\alpha}{2}} + \frac{\alpha^2}{6} + \cancel{\frac{\alpha}{2}} + \frac{\alpha^2}{6} \right]$

$\neq 1 - \alpha = 1 - \frac{a^2}{4Dt}$

$\frac{\partial}{\partial \alpha} \Big| \quad \int e^{-\alpha x} dx = -\frac{e^{-\alpha x}}{\alpha} \qquad\qquad -e^{-x}$

$-\int x e^{-\alpha x} dx = + \frac{e^{-\alpha x}}{\alpha^2} + \frac{e^{-\alpha x} \cdot x}{\alpha} \qquad\qquad -e^{-x}(1+x)$

$\int x^2 e^{-\alpha x} dx = -2 \frac{e^{-\alpha x}}{\alpha^3} - 2x \frac{e^{-\alpha x}}{\alpha^2} - \frac{e^{-\alpha x} \cdot x^2}{\alpha} \qquad\qquad -e^{-x}(2 + 2x + x^2)$

$-\int x^3 e^{-\alpha x} dx = 6 \frac{e^{-\alpha x}}{\alpha^4} + 6 \frac{x e^{-\alpha x}}{\alpha^3} + \frac{3x^2 e^{-\alpha x}}{\alpha^2} + \frac{x^3 e^{-\alpha x}}{\alpha} \Big\| -e^{-x}(6 + 6x + 3x^2 + x^3)$

$-e^{-x}(24 + 24x + 12x^2 + 4x^3 + x^4)$

$n=0: \quad e^{-x}(1-e^{-x}) \qquad = e^{-x}\left[x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!}\right]$

$n=1: \quad e^{-x}(1+x)\left[1-(1+x)e^{-x}\right] \qquad = e^{-x}\left[\frac{x^2}{2} + \frac{x^3}{6} - \frac{5}{24}x^4\right]$

$n=2: \quad \frac{1}{4}e^{-x}(2+2x+x^2)\left[2-(2+2x+x^2)e^{-x}\right] \qquad = e^{-x}\left[-\frac{x^3}{6} + \frac{x^4}{24}\right]$

$n=3: \quad \frac{1}{36}e^{-x}(6+6x+3x^2+x^3)\left[6-(6+6x+3x^2+x^3)e^{-x}\right] = e^{-x}\left[-\frac{x^4}{8}\right]$

$\left(\frac{1}{24}\right)^2 e^{-x}(24+24x\cdots) \quad = e^{-x}\left[1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}\right]\left[1-(\ \ )e^{-x}\right]$

$\cancel{\Sigma = e^{-x}\left[1+(1+x)+\frac{1}{4}(2+2x+x^2)+\frac{1}{36}(6+6x+3x^2+x^3)\right]}$

$\cancel{-e^{-2x}\left[1+(1+x)^2+\frac{1}{4}(2+2x+x^2)^2+\frac{1}{36}(\quad \frac{\ }{24}-\frac{\ }{24})^2\right]} \quad \frac{8}{24}-\frac{2}{24}$

$$= e^{-x}\left[\begin{array}{l} x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \\ -x + x^2 - \frac{x^3}{2} + \frac{x^4}{6} \end{array}\right\} \quad \left(\frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{8}\right) \left. \begin{array}{l} \\ + \frac{x^3}{2} - \frac{x^4}{3} \end{array}\right\} \frac{x^2}{2} + \frac{x^3}{6} - x^4 \cdot \frac{5}{24}$$

$$\left[1+x+\frac{x^2}{2}\right]\left[1-\underbrace{e^{-x}\left(1+x+\frac{x^2}{2}\right)}\right]$$

$$\left.\begin{array}{l} 1 - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} \\ -1 + x^2 - \frac{x^3}{2} + \frac{x^4}{6} \\ \quad -\frac{x^2}{2} + \frac{x^3}{2} - \frac{x^4}{4} \end{array}\right\} \frac{x^3}{6} + \frac{x^4}{24}(4-6-1) = \left(\frac{x^3}{6} - \frac{x^4}{8}\right)(1+x) = \frac{x^3}{6} + \frac{2 \cdot x^4}{48}$$

$+\frac{x^4}{6}$

$$\left[1+x+\frac{x^2}{2}+\frac{x^3}{6}\right]\left[1-e^{-x}\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\right] \qquad = -\frac{x^4}{8}\left(1+\cdots\right)$$

$$P = e^{-x}\left\{1 + \frac{x^2}{2} - \frac{x^3}{6}\right\} = \left.\begin{array}{l} 1 + \frac{x^2}{2} - \frac{x^3}{6} \\ -x \quad - \frac{x^3}{2} \\ \quad + \frac{x^2}{2} - \frac{x^3}{6} \end{array}\right\} = 1 - x + x^2 - \frac{5}{6}x^3$$

$$\Sigma = e^{-x}\left[1 + (1+x) + \left(1+x+\frac{x^2}{2}\right) + \left(1+x+\frac{x^2}{2}+\frac{x^3}{3!}\right) + \cdots\right.$$

$$-\left(e^{-x}\right)^2\left[1+(1+x)^2 + \left(1+x+\frac{x^2}{2}\right)^2 + (\qquad)^2 + \right.$$

$$= e^{-2x}\left[1\left[e^{x}-1\right] + (1+x)\left[e^{x}-(1+x)\right] + \left(1+x+\frac{x^2}{2}\right)\left[e^{x}-\left(1+x+\frac{x^2}{2}\right)\right] + \cdots\right]$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{6!} + \frac{x^7}{7!}$$

$$x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720} = x + x^2 + x^3 + \frac{1}{2}x^4 + \frac{x^5}{4} + \frac{x^6}{12} + \frac{x^7}{36}$$

$$\frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720}$$

$$+ \frac{x^3}{2} + \frac{x^4}{6} + \frac{x^5}{24} + \frac{x^6}{120} + \frac{x^7}{720}$$

$$\frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720}$$

$$\frac{x^4}{6} + \frac{x^5}{24} + \frac{x^6}{120} + \frac{x^7}{720}$$

$$\frac{x^5}{12} + \frac{x^6}{48} + \frac{x^7}{240}$$

$$\frac{6}{720} + \frac{7}{120} + \frac{2}{48} = \frac{5}{120} + \frac{2}{48} = \frac{1}{12}$$

$$\frac{7}{7.720} + \frac{5}{720} + \frac{2}{240} + \frac{1}{6.24} = \frac{5}{240} + \frac{1}{6.24} = \frac{1}{48} + \frac{1}{3.24} = \frac{1}{36}$$

$$\frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720}$$

$$\frac{x^5}{24} + \frac{x^6}{120} + \frac{x^7}{720}$$

$$\frac{x^6}{48} + \frac{x^7}{240}$$

$$\frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{6.24} + \frac{x^7}{7.720} + \frac{x^7}{720} + \frac{x^7}{240} + \frac{x^6}{720} + \frac{x^7}{7.720} + \frac{x^7}{720} + \frac{x^7}{7.720}$$

$$\frac{x^6}{120}$$

$$\left(1 + x + x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{x^5}{12} + \frac{x^6}{36}\right)\left(1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{6} + \frac{16x^4}{24}\right) - \frac{32x^5}{120} + \frac{64x^6}{720}$$

$$\left(1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \frac{4}{45}x^6\right)$$

$$1 + x + x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{1}{12}x^5 + \frac{1}{36}x^6$$

$$-2x - 2x^2 - 2x^3 - x^4 - \frac{1}{2}x^5 - \frac{1}{6}x^6$$

$$+2x^2 + 2x^3 + 2x^4 + x^5 + \frac{1}{2}x^6$$

$$-\frac{4}{3}x^3 - \frac{4}{3}x^4 - \frac{4}{3}x^5 - \frac{2}{3}x^6$$

$$+\frac{2}{3}x^4 + \frac{2}{3}x^5 + \frac{2}{3}x^6$$

$$-\frac{4}{15}x^5 - \frac{4}{15}x^6$$

$$+\frac{4}{45}x^6$$

$$= 1 - x + x^2 - \frac{5x^3}{6} + \frac{5}{12}x^4$$

$$+\frac{1}{3} + \frac{1}{4} = \frac{7}{12} \quad \frac{1}{12} + \frac{1}{2} - \frac{2}{3} - \frac{4}{15} = \frac{5+30-40-16}{60} = -\frac{21}{60} = -\frac{7}{20}$$

$$\frac{1}{36} + \frac{1}{3} - \frac{4}{15} - \frac{4}{45} = \frac{5+60-48-16}{4.5.9} = \frac{1}{9.20}$$

$$\frac{1}{2} - \frac{4}{3} = \frac{3-8}{6} = \quad \frac{5}{12} \quad \begin{matrix}-64\\+65\end{matrix}$$

$$\boxed{P = 1 - \alpha + \alpha^2 - \frac{5}{6}\alpha^3 + \frac{7}{12}\alpha^4 - \frac{7}{20}\alpha^5 + \frac{1}{180}\alpha^6} \quad \alpha = \frac{a^2}{4Dt}$$

$$P = \frac{1}{a^2\pi Dt}\int_a^\infty R\,dR \int_0^a r\,dr \int_{-1}^{+1} \frac{e^{-\frac{R^2+r^2-2Rr\xi}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi$$

$$\int e^{-\frac{R^2}{4Dt}} R\,dR \iint r \frac{e^{-\frac{(r-R\xi)^2-R^2\xi^2}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi\,dr$$

$$= \int e^{-\frac{R^2}{4Dt}(1-\xi^2)} R\,dR \iint r \frac{e^{-\frac{(r-R\xi)^2}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi\,dr$$

$$\frac{(r-R\xi)\,e^{-\frac{(r-R\xi)^2}{4Dt}} + R\xi\,e^{-\frac{(r-R\xi)^2}{4Dt}}}{\sqrt{1-\xi^2}}$$

$R = a+z$

$r = a-y$

$$P = \frac{1}{a^2\pi Dt}\int_{z=0}^{\infty}(a+z)\,dz \int_{y=0}^{y=a} dy.(a-y)\cdot\int_{-1}^{+1}\frac{e^{-\frac{\quad}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi$$

$a^2+2az+z^2$

$+a^2-2ay+y^2$

$$\underbrace{-2\xi(a^2+a(z-y)-zy)}$$

$$2a^2-2a^2\xi + 2a(z-y)(1-\xi) + z^2+y^2+2zy\xi$$

$a-y = a-z+z-y$

$= a-z+x$

$z-y=x$

$y=z-x$

$$P = \frac{1}{a^2\pi Dt}\int_{z=0}^{\infty}(a+z)\,dz\int_{z-a}^{z}(a-z+x)\,dx\int e^{-2a^2(1-\xi)+2ax(1-\xi)+x^2+(z-x)^2-2z(z-x)\xi \pm 2z(z-x)}$$

$$\underbrace{\pm 2z(z-x)}_{x^2+2z(z-x)(1-\xi)}$$

$$e^{x^2-2(1-\xi)[a^2+ax+z(z-x)]}$$

$$\int dz\; a^2+ax \cancel{\quad} +zx-z^2$$

$+\frac{1}{3.?}$

$\frac{1}{6}$

$-\frac{21}{60}$

$-\frac{7}{20}$

$$\int_0^{\pi} e^{+\frac{Rr\cos\varphi}{2Dt}}\, d\varphi \qquad \lim_{\alpha=\infty} \int_0^{\pi} e^{\alpha\cos\varphi}\, d\varphi = ?$$

$$= \int_{-1}^{+1} e^{+\alpha\xi} \frac{d\xi}{\sqrt{1-\xi^2}}$$

$$(1-\xi^2)^{-1/2} = 1 + \frac{1}{2}\xi^2 + \frac{\frac{1}{2}\frac{3}{2}}{2}\xi^4 + \frac{\frac{1}{2}\frac{3}{2}\frac{5}{2}}{1.2.3}\xi^6$$

$$= 1 + \frac{1}{1}\frac{\xi^2}{2} + \frac{1.3}{1.2}\left(\frac{\xi^2}{2}\right)^2 + \frac{1.3.5}{1.2.3}\left(\frac{\xi^2}{2}\right)^3 +$$

$$\int e^{\alpha\xi}\xi^n\, d\xi = n!\,\frac{e^{\alpha\xi}}{\alpha^{n+1}}\left[1 + \frac{\alpha}{1}\xi + \frac{\alpha^2\xi^2}{2!} + \frac{\alpha^3\xi^3}{3!} + \cdots \frac{\alpha^n\xi^n}{n!}\right]$$

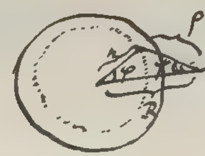
$$P = \frac{1}{a^2\pi Dt}\int e^{-\frac{R^2}{4Dt}} R\, dR \int e^{-\frac{r^2}{4Dt}} r\, dr \sum_{n=0} \underbrace{\frac{1.3.5.\cdots(2n-1)}{1.2.3\cdots n}\left(\frac{1}{2}\right)^n \frac{(2n!)}{}}_{=\left[1.3.5\cdots(2n-1)\right]^2} e^{+\frac{2rR}{4Dt}} \left[\right.$$

$$\left[\left(\frac{2Dt}{rR}\right)^{2n+1} + \frac{}{1!}\left(\frac{2Dt}{rR}\right)^{2n} \cdots + \cdots\right\} \cdots$$

$$\int_{-1}^{+1} e^{\alpha\xi}\xi^n\, d\xi = \frac{n!}{\alpha^{n+1}}\left\{e^{\alpha}\left[1 + \alpha + \frac{\alpha^2}{2!} + \frac{\alpha^3}{3!} + \cdots \frac{\alpha^n}{n!}\right] - e^{-\alpha}\left[1 - \alpha + \frac{\alpha^2}{2} - \frac{\alpha^3}{3!} \cdots \right]\right\}$$

$$\int_a^{\infty}\int_0^a e^{-\frac{R^2+r^2\pm 2rR}{4Dt}} R\, dR\, r\, dr \left(\frac{2Dt}{rR}\right)^m = (2Dt)^m \int dR \int \frac{e^{-\frac{(R\pm r)^2}{4Dt}}}{(rR)^{m-1}}\, dr$$

$$p = \frac{1}{2a^2\pi^2 Dt}\iint R r\, dR\, dr \int e^{-\frac{R^2+r^2-2Rr\cos\varphi}{4Dt}}\, d\varphi$$



$$\int_{\varphi=0}^{\pi} e^{-\frac{\rho^2}{4Dt}}\, d\varphi$$

$\rho : r = \sin\varphi : \sin\psi$

$\sin\psi \cdot \rho = r\sin\varphi$

$\rho^2 = r^2 + R^2 - 2rR\cos\varphi$

$rR\cos\varphi = \frac{r^2+R^2-\rho^2}{2}$

$2\rho\, d\rho = +2rR\sin\varphi\, d\varphi$

$$d\varphi = \frac{\rho\, d\rho}{rR\sin\varphi} = \frac{\rho\, d\rho}{\sqrt{(rR)^2 - \left(\frac{r^2+R^2-\rho^2}{2}\right)^2}}$$

$\rho^2 = v$

$$\int_{\varphi=0}^{\pi} e^{-\frac{\rho^2}{4Dt}}\, d\varphi = \frac{1}{2}\int_{v=(R-r)^2}^{(R+r)^2} \frac{e^{-\frac{v}{4Dt}}\, dv}{\sqrt{(rR)^2 - \frac{(r^2+R^2-v)^2}{4}}} = \frac{1}{2rR}\int \frac{e^{-\frac{v}{4Dt}}\, dv}{\left[1-\left(\frac{r^2+R^2-v}{2rR}\right)^2\right]^{1/2}}$$

u

(r^2+R^2)

$$= \int \frac{e^{-\frac{v}{4Dt}}\, dv}{\sqrt{4(rR)^2 - (r^2+R^2)^2 + 2v(r^2+R^2) - v^2}}$$

$R^2 - r^2 = u$

$$= \frac{1}{(R^2-r^2)}\int \frac{e^{-\frac{v}{4Dt}}\, dv}{\sqrt{-1 + 2v\frac{r^2+R^2}{(R^2-r^2)} - \frac{v^2}{(R^2-r^2)^2}}}$$

$$P = \frac{1}{2a^2\pi^2 Dt} \iint r\,dr\,d\varphi \int_{R=a}^{\infty} e^{-\frac{R^2+r^2-2rR\cos\varphi}{4Dt}} R\,dR$$

$$\frac{\sqrt{\pi}}{2}\left[\cancel{1} + \frac{e^{-x^2}}{x\sqrt{\pi}}\left(1-\frac{1}{2x^2}\right) \cdots\right]$$

$$= e^{-x^2}\left(\frac{1}{2x} - \frac{1}{4x^3}\right)$$

$$\int_a^{\infty} e^{-x^2+\beta x} \cdot x\,dx$$

$$\int_a^{\infty} e^{-\alpha x^2+\beta x}\,dx = \int_a^{\infty} e^{-\left(\sqrt{\alpha}x - \frac{\beta}{2\sqrt{\alpha}}\right)^2 + \frac{\beta^2}{4\alpha}}\,dx = \frac{e^{\frac{\beta^2}{4\alpha}}}{\sqrt{\alpha}} \int_{\sqrt{\alpha}a - \frac{\beta}{2\sqrt{\alpha}}}^{\infty} e^{-z^2}\,dz$$

$$\int_a^{\infty} e^{-x^2+\beta x}\,dx = e^{\frac{\beta^2}{4}} \int_{a-\frac{\beta}{2}}^{\infty} e^{-z^2}\,dz$$

$$\int e^{-x^2+\beta x} x\,dx = \frac{\beta}{2} e^{\frac{\beta^2}{4}} \int_{a-\frac{\beta}{2}}^{\infty} e^{-z^2}\,dz + \frac{1}{2} e^{\frac{\beta^2}{4}} \cdot e^{-\left(a-\frac{\beta}{2}\right)^2}$$

$$= \frac{e^{\frac{\beta^2}{4}}}{2}\left\{\frac{\beta}{2}\left[\frac{1}{2\left(a-\frac{\beta}{2}\right)} - \frac{1}{4\left(a-\frac{\beta}{2}\right)^3}\right] + 1\right\} e^{-\left(a-\frac{\beta}{2}\right)^2}$$

$$\frac{2\beta}{2a-\beta} - \frac{2\beta}{(2a-\beta)^3}$$

$$= e^{-a^2+a\beta}\left[\frac{a}{2a-\beta} - \frac{\beta}{(2a-\beta)^3}\right]$$

$$-x^2 + 2x \cdot \frac{\beta}{2}$$

$$= \frac{1}{2a^2\pi^2 Dt}\int r\,dr\, e^{-\frac{r^2}{4Dt}} \int d\varphi \int_{R=a}^{\infty} e^{-\left(\frac{R}{2\sqrt{Dt}}\right)^2 + 2\frac{R}{2\sqrt{Dt}}\cdot\frac{r\cos\varphi}{\sqrt{Dt}\,2} \pm \frac{r^2\cos^2\varphi}{4Dt}} R\,dR$$

$$= \frac{1}{2a^2\pi^2 Dt}\int r\,dr\, e^{-\frac{r^2}{4Dt}\left[1-\cos^2\varphi\right]}\int d\varphi \int e^{-\left(\frac{R}{2\sqrt{Dt}} - \frac{r\cos\varphi}{2\sqrt{Dt}}\right)^2} R\,dR$$

$$= \frac{2}{a^2\pi^2}\iint e^{-\frac{r^2}{4Dt}}\, r\,dr\, d\varphi\; e^{-\frac{a^2 - 2ar\cos\varphi}{4Dt}}\left[\frac{a}{2(a - r\cos\varphi)} - \cdot\right]$$

$$\int e^{-\beta^2}\beta\, d\beta$$

Ein Sonne: $r^2 \Delta\omega = \varphi$

$\quad \mathcal{S} = r^2\,\Omega$

$$\Delta F_{\odot} = \varepsilon\, \varphi\, \Omega$$

$$\Omega f^2 = 6$$

$$\Delta F_{\odot} = \varepsilon \frac{\varphi\, 6}{f^2}$$

Energieflussverhältnis $\left\{ \frac{\text{Doppelstern}}{\text{Sonne}} \right. = \frac{\varepsilon_B}{\varepsilon_{\odot}} \frac{f^2}{l_b^2}$

$$\varepsilon_{B\text{ Strahlung}} = 11.000$$

$$\varepsilon_{\odot} = 160.000$$

Flächenhelligkeit $\varepsilon = \frac{\Delta F}{\Delta\omega}\frac{1}{\mathcal{S}}$

$$\Delta F = \varepsilon\, \mathcal{S}\, \Delta\omega$$

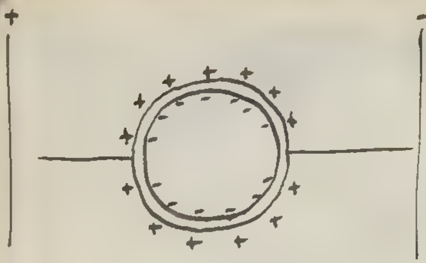
$$\varphi = l_b^2\, \Delta\omega$$

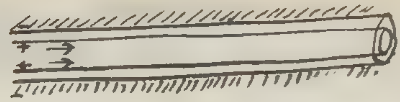
$$\mathcal{S} = 6\frac{\ell_b^2}{\ell_b^2}$$

$$\Delta F = \varepsilon \frac{6\,\varphi}{l_b^2}$$

$$\frac{f}{l_b} = 2$$

$$\frac{4}{3} = \frac{1}{4}$$





Oberflächenleitung pro cm d. Umfanges:

$$J = \int \varepsilon v \, dn \qquad \varepsilon = \frac{1}{4\pi}\frac{\partial^2 U}{\partial n^2}$$

$$X = \varepsilon \frac{\partial \Phi}{\partial x}$$

$$\mu \frac{\partial^2 v}{\partial n^2} = \varepsilon \frac{\partial \Phi}{\partial x} + \left(\frac{\partial p}{\partial x}\right.$$

$$\mu v = \frac{\partial \Phi}{\partial x}\iint \varepsilon \, dn^2 = \frac{1}{4\pi}\frac{\partial \Phi}{\partial x} U + \text{const} = \frac{1}{4\pi}(\varphi_1 - \varphi_0)\frac{\partial \Phi}{\partial x}$$

Elektr. Endosm.: $\left.\begin{matrix} v_\xi \\ v_\eta \end{matrix}\right\} = -K \frac{\varphi_1 - \varphi_0}{4\pi\mu}\left\{\begin{matrix} \frac{\partial \Phi}{\partial \xi} \\ \frac{\partial \Phi}{\partial \eta} \end{matrix}\right.$

dabei Φ angenähert so wie bei unbewegl. Flüssigkeit

Modifikation durch (convection Strömungsstrom entsteh. Oberflächenleitung und) der v_ξ, v_η

Strömungsstrom

$$V_2 - V_1 = \frac{K(\varphi_1 - \varphi_0)}{4\pi\mu}\,\sigma\,(p_2 - p_1)$$

bei Vernachlässig. d. elektrosmot. durch $V_2 - V_1$ hervorgebrachten Druckes (und d. Oberfl. leitg. ε)

$$\frac{J_1}{J_2} = \frac{9}{16}\,\frac{K^2(\varphi_1 - \varphi_0)^2}{16\cdot\delta\cdot\pi^2}\,\frac{\sigma}{\mu}\left(\frac{3\pi}{4}\right)^{2/3}\varphi^{2/3}\,\frac{\varphi}{a}$$

$\varphi = 0.01$ $\quad \sigma = \frac{10^{6}}{9\cdot 10^{11}} \neq 10^{-6}$

$K(\varphi_1 - \varphi_0) = \frac{4}{300}$

$\delta = 4\cdot 10^{-7}$ $\quad \mu = 0.02$ $\quad a = 0.1\,\mu = 10^{-5}$

$$= \frac{9\cdot 16}{16\cdot 16\cdot 10}\,\frac{16}{9\cdot 10^4\cdot 4\cdot 10^{-7}}\,\frac{10^{-6}}{0.02}\,\frac{0.01}{10^{-5}} = \frac{10}{64} \neq \frac{1}{6}$$

also erst bei sehr kleinen Teilchen erheblich (und größer zu Einfluss)

$$\int_0^{\frac{\pi}{2}}\left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right)^4 d\theta = \int_0^{\frac{\pi}{2}}\frac{e^{4i\theta} + e^{-4i\theta} - 4\left(e^{2i\theta} + e^{-2i\theta}\right) + 6}{16}\,d\theta = \frac{3}{8}\,\frac{\pi}{2}$$

$$\frac{1}{16}\left[\cancel{\frac{e^{4i\theta} - e^{-4i\theta}}{4i}} - 4\,\cancel{\frac{e^{2i\theta} - e^{-2i\theta}}{2i}} + \right.$$

Elektr. Änderungen bei Ostwalds d. Oberfl. L $\quad \frac{\partial \Phi}{\partial \xi}$ etc. verändern sich

Es gilt $\operatorname{div} c = 0$

$$(20) \quad \operatorname{div}\left[\frac{\nabla \Phi}{6} + \varepsilon \vartheta\right] = 0$$

$$(21) \quad \therefore \quad \Delta^2 \Phi - 6\left[u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z}\right] \qquad \Delta^2 \Phi = 0$$

$$= -6 v_n \frac{\partial \varepsilon}{\partial n}$$ unter Annahme dass Doppelschicht überall gleich

$$\begin{cases} \frac{\partial P}{\partial \xi} = \mu \Delta^2 v_\xi + \varepsilon \frac{\partial(\Phi + V)}{\partial \xi} \\ \frac{\partial P}{\partial \xi} = \mu \Delta^2 v_\xi \mp \varepsilon \frac{\partial(\Phi + V)}{\partial \xi} = \end{cases}$$

$$\frac{\partial^2 V}{\partial n^2} = -6 v_n \frac{\partial^3 \varphi}{\partial n^3}$$

$$\frac{\partial V}{\partial n} = -6 \int \frac{\partial v_n}{\partial n} \frac{\partial^2 \varphi}{\partial n^2} \, dn$$

$$= -6 \int \frac{\partial^2 v_n}{\partial n^2} \frac{\partial \varphi}{\partial n} \, dn$$

$$\Delta^2 P = - \varepsilon \nabla^2 V = 6 \varepsilon v_n \frac{\partial \varepsilon}{\partial n} = \frac{6}{2} v_n \frac{\partial (\varepsilon^2)}{\partial n} + 6 \frac{\partial \varepsilon}{\partial \xi}\left(\frac{\partial \Phi}{\partial n} + \frac{\partial V}{\partial n}\right)$$

?

Angenäherter Einfluss d. Oberfl. Leitung pro Kugel (bei festgehaltener Kugel)

$$i = \underbrace{\left(\frac{K}{4\pi}\right)^2 \frac{1}{\mu} \int_0^\infty \left(\frac{\partial \varphi}{\partial n}\right)^2 dn}_{A} \cdot \frac{\partial \Phi}{\partial s}$$

$$\Phi = -c \cos\theta \left(r + \frac{a^3}{2r^2}\right)$$

$$\left.\frac{\partial \Phi}{\partial s}\right|_a = \frac{3}{2} c \sin\theta$$

$$u = A \frac{3}{2} c \sin^2\theta \cdot 2 a \pi \sin\theta$$

$$\bar{u} = \int \frac{u \, dx}{a} = \frac{3 A a c \pi}{a} \int_0^{\frac{\pi}{2}} \sin^3\theta \sin\theta \, d\theta = 3 A a c \pi \int_0^{\frac{\pi}{2}} \sin^4\theta \, d\theta = \frac{9}{16} A a c \pi$$

n Kugeln pro Vol. Einheit $= \sqrt[3]{n}$ pro Längeneinheit

$$\varphi = \frac{4}{3} a^3 \pi n \quad \Big\| \quad \Sigma \bar{u} = \frac{9}{16} A a c \pi n^{2/3} = \frac{9}{16} A c \pi a \left(\frac{3\varphi}{4\pi}\right)^{2/3} \frac{1}{a^2} = \frac{9}{16} A c \left(\frac{3\varphi\pi}{4}\right)^{2/3} \frac{1}{a}$$

$$\frac{J_s}{J_\varphi} = \frac{9}{16} A \left(\frac{3\varphi\pi}{4}\right)^{2/3} 6 \frac{1}{a}$$

Dagegen falls Ladungstheorie richtig wäre:

$eX = 6\pi\mu a u$

$$\frac{J}{J_\varphi} = \frac{e\,n\,u}{\lambda X} = \frac{e^2}{\lambda} n \frac{1}{6\pi\mu a} = \frac{6\pi\mu a u^2 n}{\lambda X^2}$$

Kataph. $u = \frac{K(\varphi_i - \varphi_a)}{4\pi} \frac{X}{\mu}$

$$= 6\pi\mu a \frac{6}{\mu} n \left(\frac{K(\varphi_i - \varphi_a)}{4\pi}\right)^2$$

$$= \frac{9}{2}\frac{\varphi}{a^2}\frac{6}{\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2 = 4.10^{-4}\,!!!$$

Für Oberfl. Leitung

~~mittl. Oberflächendichte für den Kugeln (in X Kompon.)~~:

$J =$

Wenn aneinandergereiht in X, so dass Anzahl pro 1 cm ist

$N = \frac{1}{2a}$

dann sind im Querschnitt pro cm²: $2an$ Kugeln vorhanden, also ~~...~~ Oberflächenstrom

$$\frac{J_s}{J_q} = \frac{2an\,\frac{9}{16}\,\Delta\,a\,\varphi\,\pi\;6}{\varphi}$$

$$= \frac{9}{8}a^2\pi\, n\frac{6}{\mu}\left(\frac{K}{4\pi}\right)^2\int\left(\frac{\partial\varphi}{\partial n}\right)^2 dn \;\mp\; \frac{9}{8}\frac{a^2\pi}{\delta} n\frac{6}{\mu}\frac{K^2(\varphi_i-\varphi_a)^2}{(4\pi)^2}$$

$\left(n = \frac{3}{4}\frac{\varphi}{a^3\pi}\right)$

$$= \frac{9}{8}\frac{a^2\pi}{\delta}\frac{3}{4}\frac{\varphi}{a^3\pi}\frac{6}{\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2$$

$$= \frac{27}{32}\frac{\varphi}{a\delta}\frac{6}{\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2$$

z.B. $\varphi = 0{,}001$ $\quad 6 = 10^{-6}$ (abs. elektrost.) $\mp 10^6\,\Omega$

$a = 10^{-5}$ $\quad K(\varphi_i - \varphi_a) = \frac{4}{300}$

$\delta = 4.10^{-7}$

$$= \frac{27}{32}\frac{10^{-3}}{4.10^{-12}}\frac{10^{-6}}{0{,}02}\frac{16}{9.10^4}\frac{1}{16.10}$$

$$= \frac{3}{32.0{,}08}10^{-2} = 10^{-2}\,!!!$$

$\int p\,dv = RT \lg \frac{n}{\nu}$

auf 1 gr. entfallen: $\frac{1}{m}$ Teilchen

also Arbeit pro 1 Teilchen: $m RT \lg\frac{n}{\nu} = m\frac{HT}{\mu}\lg\frac{n}{\nu} = \frac{HT}{N}\lg\frac{n}{\nu}$

$$1-\frac{2}{\sqrt{\pi}}\int_0^\beta\left(1-y^2+\frac{y^4}{2!}-\frac{y^6}{2.3!}\right)dy+\frac{1}{\beta\sqrt{\pi}}\left(\beta^2-\frac{\beta^4}{2!}+\frac{\beta^6}{3!}-\right)$$

$$=1-\frac{2}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{1!3}+\frac{\beta^5}{2!5}-\frac{\beta^7}{3!7}\right]+\frac{1}{\sqrt{\pi}}\left(\beta-\frac{\beta^3}{2!}+\frac{\beta^5}{3!}-\frac{\beta^7}{4!}+\frac{\beta^9}{5!}\right)$$

$$=1-\frac{1}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{6}+\frac{\beta^5}{5.6}-\frac{\beta^7}{3.7.8}+\frac{\beta^9}{12.9.10}\right.$$

$$-\frac{1}{2}+\frac{1}{21}=\frac{+1}{3.7.8}$$

$$+\frac{1}{10.42}-\frac{1}{12.9}=\frac{1}{120.9}$$

$$\frac{\beta^3}{2!\,3}\quad\frac{\beta^5}{3!5}\quad\frac{\beta^7}{4!7}\quad\frac{\beta^9}{5!9}$$

$$=1-\frac{1}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{2!\,3}+\frac{\beta^5}{3!5}-\frac{\beta^7}{4!7}+\frac{\beta^9}{5!9}--\right]$$

$$-2\frac{\beta^n}{\frac{n-1}{2}!\,n}+\frac{\beta^n}{\frac{n+1}{2}!}=$$

$$n=2m+1$$

$$\beta^{2m+1}\left[\frac{-2}{(2m+1)m!}+\frac{1}{m+1!}\right]$$

$$-\frac{2}{2m+1}+\frac{1}{m+1}$$

$$=\frac{-2m-2+2m+1}{(2m+1)\,m+1}$$

$$=\beta^{2m+1}\frac{-2(m+1)+2m+1}{(2m+1)(m+1)!}$$

$$=-\beta^{2m+1}\frac{1}{(2m+1)\,m+1!}=-\frac{\beta^n}{n.\frac{n+1}{2}!}$$

$$\beta=\frac{1}{4}=0{\cdot}25$$

$$\frac{\beta^5}{30}=\frac{1}{16}\,\frac{1}{64}\,\frac{1}{30}=\frac{(0{\cdot}0625)^2}{120}$$

$$\frac{\beta^3}{6}=\frac{0{\cdot}0625}{24}=0{\cdot}02604$$

0·25
0·026
0·2240:

59715
2486

7959
3802
4157

$$\frac{\beta^2}{5}=\frac{1}{800}$$

3502
2486
1016

0·127

$$P=0{\cdot}873\ .\ 31$$

2619
87
2606

$$\mu=m.N$$

$$\frac{1}{W(n)}=e^{\mu}\frac{n!}{\mu^n}=\frac{e^{\mu}}{\mu^n}\left(\frac{n}{e}\right)^n\sqrt{2n\pi}=e^{\mu-n}\left(\frac{n}{\mu}\right)^n\sqrt{2n\pi}\qquad\delta^2=$$

$$p=(n)\frac{HT}{V}$$

$$p v_\rho = R\theta$$

$$p=\frac{HT}{\mu}\rho=\frac{HT}{\mu}\qquad\rho=\frac{(n)}{V}\mu=\frac{\nu.m}{V}$$

$$(n)=\nu.m$$

$$=(n)\frac{m.N}{V}\qquad \nu=(n)N$$

$$=\frac{\nu}{N}\frac{HT}{V}\log\frac{n}{\mu}=\frac{HT}{N}$$

$2.3.4.5.6.7$ $\quad$ $0'4343 . 1'55$ $\quad$ $0'1903 . 7$

$720 . 7$ $\quad$ 21715 $\quad$ $1'3321$

5040 $\quad$ 2172

$0'6732$

$3'7024$ $\quad$ 1105

$4'3756$

$-1'3321$

$3'0435$

$t=0 \qquad u=x$

$$u = c\,\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\,e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\sin\frac{n\pi x}{c} \qquad \frac{\pi n}{c}=y \qquad \frac{\pi}{c}=dy$$

$$\lim_{c=\infty} u = 2\sum(-1)^{n+1}\frac{1}{y}\,e^{-a^2y^2t}\sin yx$$

$$\frac{\partial v}{\partial t} = a^2\frac{\partial v}{\partial r^2} \qquad v = v_1 + v_2$$

$$\left.\begin{matrix} t=0 \\ r>0 \end{matrix}\right\} \begin{matrix} v_1 = R\varepsilon \\ v_2 = r\varepsilon \end{matrix} \qquad \left.\begin{matrix} r=0 \\ 0<t<\infty \end{matrix}\right\} \begin{matrix} v_1 = 0 \\ v_2 = 0 \end{matrix}$$

$$v_2 = r\varepsilon$$

$$v_1 = \frac{2R\varepsilon}{\sqrt{\pi}}\int_0^{\frac{R}{2a\sqrt{t}}} e^{-\beta^2}d\beta \qquad r^2\frac{\partial u}{\partial r} = r^2\frac{\partial \frac{v}{r}}{\partial r} = \underbrace{-v + r\frac{\partial v}{\partial r} = 0}_{\text{für } r=\infty}$$

Verschiebung d. Anfangspunktes nach 0:

$$u = \frac{1}{r}\left[\frac{2R}{\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}} e^{-z^2}dz + (r-R)\right]$$

$$\frac{\partial u}{\partial r} = -\frac{u}{r} + \frac{1}{r}\left\{\frac{2R}{\sqrt{\pi}}\,\frac{1}{2a\sqrt{t}}\,e^{-\frac{(r-R)^2}{4a^2t}} + 1\right\}$$

$$r^2\frac{\partial u}{\partial r} = \frac{R}{a\sqrt{\pi t}}\,e^{-\frac{(r-R)^2}{4a^2t}}\cdot r + r - \frac{2R}{\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}} - r + R\Bigg|^{r=\infty} = 0$$

$$r^2 \frac{\partial u}{\partial r}\bigg|_{r=R} = \frac{R^2}{a\sqrt{\pi t}} + R \cancel{\mu} = R\left[1 + \frac{R}{a\sqrt{\pi t}}\right]$$

$$u = \frac{R u_R}{r}\left[\frac{2}{\sqrt{\pi}} \int_0^{\frac{r-R}{2a\sqrt{t}}} e^{-z^2} dz + \frac{r}{R} - 1\right] = u_R\left[\frac{2}{r\sqrt{\pi}} \int_0^{\frac{r-R}{2a\sqrt{t}}} e^{-z^2} dz + \frac{1}{R} - \frac{1}{r}\right]$$

$$\frac{\partial u}{\partial r} = R u_R \left[\frac{1}{r^2} - \frac{2}{r^2\sqrt{\pi}} \int^{\frac{r-R}{2a\sqrt{t}}} \sim + \frac{1}{r a\sqrt{\pi t}} e^{-\frac{(r-R)^2}{4a^2 t}}\right]$$

$$r^2 \frac{\partial u}{\partial r} = R u_R \left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{r-R}{2a\sqrt{t}}} \sim + \frac{r}{a\sqrt{\pi t}} e^{-\frac{(r-R)^2}{4a^2 t}}\right]$$

$$r^2 \frac{\partial u}{\partial r}\bigg|_{r=R} = R u_R \left[1 + \frac{R}{a\sqrt{\pi t}}\right] = \frac{J}{4\pi K}$$

$$a^2 = \frac{K}{c\rho}$$

$$\int_0^t J\, dt = 4\pi K\, u_R R\left[t + \frac{2R\sqrt{t}}{a\sqrt{\pi}}\right]$$

für lim $t=0$:

$$J = 4R^2\pi \cdot K u_R \cdot \frac{2\sqrt{t}}{a\sqrt{\pi}}$$

Aufgabe: Begrenztes Kugelvolumen von $r = R$ bis $r = \underline{A}$

$t=0$ $\quad u = u_R = c$ $\quad \Big\|\quad r = R:\ u = 0 \quad \Big\|\quad r = A:\ \frac{\partial u}{\partial r} = 0$

$R < r < A$

$v = rc$ $\qquad v = 0 \qquad v = r\frac{\partial v}{\partial r}$

Anstatt dessen Verschiebung $r' = r - R$:

$t=0$ $\quad v = r'c + Rc \quad \Big\|\quad r' = 0 \quad u = 0 \quad \Big\|\quad r' = A' \quad \frac{\partial v}{\partial r'} - \frac{v}{A} = 0$

$0 < r' < A'$

$= r' F(r')$

$F(r') = c\left(1 + \frac{R}{r'}\right)$

$P = 1 - \frac{1}{\sqrt{\pi}}\left[\beta - \frac{\beta^3}{6} + \frac{\beta^5}{30}\right]$, $\beta = 0.25$ $P = 0.873$

$\beta = 0.501$
$- \quad 52$
0.449

$\beta^5 = \frac{1}{32}$

$\frac{\beta^3}{6} = \frac{1}{32.6} = \frac{1}{192}$

$\frac{1}{48} = 20$

$\beta = 0.5$ $P = 0.729$

$\beta = 0.505$ $P = 0.726$

$\frac{\beta_0}{\sqrt{2}} = \beta = 0.357$ $P = 0.803$

6522
−2486
4036

0.2533
0.7467

$\beta \neq \sqrt{\pi}(1-P) + \frac{\beta^3}{6}$ $P = 0.726$
0.274

4378
2486
0.6864 −1
0.0592 −1
−0.7782
0.2810 −2

$\beta = 0.4858$
0.0194

$\bar{\Delta}^2$	Experimentell	
	für 1τ	$P = 0.726$
2.62	2τ	~~0.875~~ 0.845
2.59	3τ	0.835
2.80	4τ	0.903
2.95	5τ	0.951
3.00	6τ	0.968

$\beta = 0.501$
$- \quad 21$
0.480

6812
2486
4326

2707

7033
1505
0.5528 −1
0.6584 −2
0.7640 −3

0.3571
0.0455
0.0058

$\beta = 0.3571$
0.0002
0.3573
−0.0076
0.3497

7033
3010
0.4023 −1
0.2069 −2
0161

2524
−0027
2497

4771
4914
9857

4133
4914
9219

5437
2486
2951

1973
0.8027

168
260
207
128
25
13

3974
2486
1488

1411
0.8589

0	112
1	168
2	130
3	69
4	32
5	5
6	1
7	1

168
520
621
32
192
125
36
49
2031 = 518

3077
7143
5934

8107
9036
7143
1893

8519
7143
1376

801

$\nu = 1.546$

$\frac{1}{\nu} = 0.647$

260

0	1	2	3	4	5	6	7
25	17	19	7	7			~~4~~
18	22	13	5	[illegible]			
19	19	7	5	1			
38	56	~~30+8~~	20	3	—	—	2
25	53	29	11	4	2		
125	187	108	45	15	2		2

$$\frac{\Sigma(n-\nu)^2}{N} = \frac{\Sigma n^2}{N} - \nu^2$$

1903
3806
5721
2402
1.519

432
132
240
57
98
1439

1581
6875
4914
7782

2.955

$\overline{s}^2 =$ [illegible] $1.52 + \nu$

1818
3786
8032
0.636

~~1818~~
~~3806~~
~~8012~~
633

1903
8097
645

772

4378	1803	2175	9868	6902	5051
2486	2486	2486	2486	2486	2486
		1505	2385	3010	3495	3891
6864	4389	4661	2354	9388	7537
·0592-1	3167	3983-2	7062	8164-4	

$\beta = 0{\cdot}486$ ·275 ·292 172 0869 0567

3 4 1 1

$\beta_{corr} = 0{\cdot}505$ 0·278 0·296 0·173 0·087 0·057

4440	4713	2380-1	8395-2	7559-2
1505	2385	3010	3495	3891

7033	5945	7098	5390	2890	1450-1
4314		2041			
505	393	512	346	461	140
4378	1803	2175	9868	6902	5051
1411	7848	9273	5258	9792	6501

27
15
16
10
5
3

1384
6093
8458
3356
953
447
33147 :

5204
8876
6328

6328
·6328-1 3318
8984-2 9954
0·0791-

6328	6328	6328	6328
1505	2385	3495	3891
4823	3943	2833	2437
·4469-1	1829	8599	7311
·280 ·152	099	071	054

$\bar{\beta}_1 = {\cdot}4293$ ~~$\beta_2 = 0{\cdot}2788$~~ ~~$\beta_3 = 0{\cdot}1908$~~ ~~$\beta_4 = 0{\cdot}1283$~~ ~~$\beta_5 = 0{\cdot}0798$~~

$\beta_1 = 0{\cdot}4293$ || $\beta_2 = 0{\cdot}3036$ || $\beta_3 = 0{\cdot}248$ || $\beta_4 = 0{\cdot}2146$ || $\beta_5 = 0{\cdot}192$ || $\beta_6 = 0{\cdot}175$

1/2 1/2

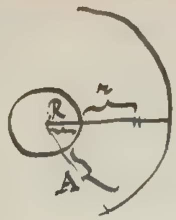
13/2 43/ 2/5 ~~0·216~~ ~~0·193~~ ~~0·176~~

~~0·4125~~ ~~0·308~~ ~~0·250~~ 0·213 0·191 0·174

0·416 0·299 0·243

~~0·404~~ ~~0·701~~ ~~·701~~

6191	4757	3856	3284	2810	2405
2486	2486	2486	2486	2486	2486
3705	2271	1370	0798	0324	9919
235	169	137	120	108	0981
2295	2493	2589	264	2676	2757
0·766	0·831	0·863	0·880	0·892	0·919
726	845	835	903	951	968
~~2188~~	~~2536~~	~~2805~~	~~2709~~	~~2853~~	~~2904~~
237	258	268	273	277	285



$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial r^2}$$

$$\begin{cases} t=0 & v=(r+R)c = \Phi(r) \end{cases}$$

$$\begin{cases} r=0 & v=0 \\ r=A & \dfrac{\partial v}{\partial r} - \dfrac{v}{A+r} = 0 \end{cases}$$

$$u = \frac{v}{R+r}$$

$$\left.\frac{\partial u}{\partial r}\right|_{r=0} = \frac{1}{R+r}\left(\frac{\partial v}{\partial r} - \frac{v}{R+r}\right)_0 = \frac{1}{R}\left(\frac{\partial v}{\partial r} - \frac{v}{R}\right)_{r=0}$$

$$v = V e^{-\gamma^2 t}$$

$$-\gamma^2 V = a^2 \frac{\partial^2 V}{\partial r^2}$$

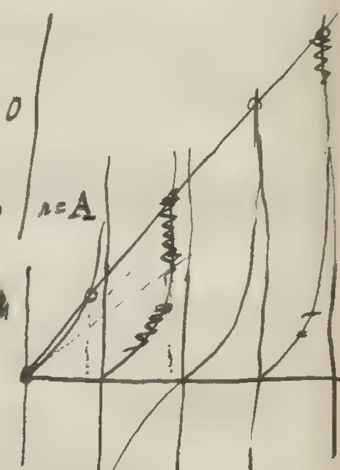
$$V = \sum A \sin\left(\frac{\gamma}{a} r\right)$$

$$\frac{\gamma}{a}\cos\frac{\gamma}{a} r = \frac{\sin\frac{\gamma r}{a}}{A+R}$$

$$\operatorname{tg}\frac{\gamma r}{a} - (A+R)\frac{\gamma}{a} = 0$$

$$\operatorname{tg}\beta r - (A+R)\beta = 0 \quad \Big| \; r=A$$

$$\operatorname{tg}\beta A = (A+R)\beta$$



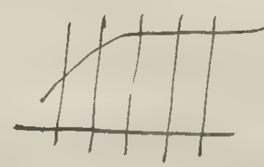
$$V = A\sin\beta r$$

$$v = \sum A_n e^{-a^2\beta_n^2 t}\sin\beta_n r$$

$$\lim_{A=\infty}\beta_1 A = 0$$

$$\lim_{A=\infty}\beta_r \neq \frac{3\pi}{2A}$$

$$\lim_{A=\infty} v = \sum A_n e^{-a^2\left[(2n+1)\frac{\pi}{2}\right]^2\frac{t}{A^2}} \quad (2n+1)\frac{\pi}{2}\frac{r}{A}$$



$$v = \sum A_n e^{-\frac{(2n+1)^2\pi^2}{4}\frac{a^2 t}{A^2}}\sin\left[\frac{(2n+1)\pi}{2}\frac{r}{A}\right] \cdot \frac{\pi}{A}r, \frac{A}{\pi r} \;\Big\|\; \frac{n\pi r}{A} = x$$

$$= \frac{A}{\pi r}\int f\left(\frac{Ax}{\pi r}\right) e^{-\frac{x^2 a^2 t}{r^2}}\sin x\,dx \qquad \frac{x}{r} = \xi$$

$$= A\int_0^\infty f(A\xi)\, e^{-a^2\xi^2 t}\sin(r\xi)\,d\xi$$

$$f(r) = \frac{2}{\pi}\int_0^\infty\int_0^\infty f(\xi)\sin\alpha r\,\sin\alpha\xi\,d\alpha\,d\xi = \frac{2}{\pi}\iint f(\lambda)\sin r\xi\,\sin\lambda\xi\,d\xi\,d\lambda$$

$$v = \iint f(\lambda)\, e^{-a^2\xi^2 t} \sin r\xi \sin \lambda\xi \; d\xi\, d\lambda$$

$$\frac{\partial v}{\partial r} = \iint f(\lambda)\, d\lambda \quad \xi e^{-a^2\xi^2 t} \cos r\xi \sin\lambda\xi \, d\xi$$

$$\xi e^{-a^2\xi^2 t}$$

$$v = \sum_n \cancel{}\, B_n\, e^{-a^2\beta_n^2 t} \sin\beta_n r$$

$$J = -4\pi k R^2 \frac{\partial u}{\partial r}\Big|_{r=0} = 4\pi k R \left\{ \frac{v}{R} - \frac{\partial v}{\partial r} \right\}\Big|_{r=0} = 4\pi k \left\{ v - R\frac{\partial v}{\partial r} \right\}_{r=0}$$

$$= -4\pi k R \sum B_n\, e^{-a^2\beta_n^2 t}\; \beta_n$$

$$Q_t = 4\pi \sum B_n \int_{r=0}^{A} (R+r) \sin\beta_n r \; dr$$

$$\int \cancel{\cos\beta r\, dr = \frac{\sin\beta r}{\beta}}$$

$$\int r\cos\beta r\, dr = \frac{\sin\beta r}{\beta^2} + \frac{r\cos\beta r}{\beta}$$

$$= 4\pi \sum B_n \underbrace{\left\{ -R\frac{(\cos\beta_n A - 1)}{\beta_n} - A\frac{\cos\beta A}{\beta_n} + \frac{\sin\beta A}{\beta_n^2} \right\}}$$

$$\sin\beta A = (A+R)\beta\cos\beta A$$

$$\frac{1}{\beta}\left\{ -R\cos\beta A + R - A\cos\beta A + (A+R)\cos\beta A \right\} = \frac{R}{\beta}$$

$$\boxed{\begin{aligned} Q_t &= 4\pi R \sum{}' \frac{B_n}{\beta_n} e^{-a^2\beta_n^2 t} \\ J &= -\frac{\partial Q}{\partial t} = R\, 4\pi a^2 \sum{}' B_n \beta_n e^{-a^2\beta_n^2 t} \end{aligned}}$$

$$R + r = 2\sum{}' \frac{\sin\beta_n r}{\beta_n}$$

$t=0$: $\quad 0<r<A$

$$\sum_n B_n \sin\beta_n r = \Phi(r) \qquad \operatorname{tg}\beta A = (A+R)\beta$$

$$\sum B_n \underbrace{\int_0^A \sin\beta_n r \cos\beta_m r\, dr}_{} = \int_0^A \Phi(r) \cos\beta_m r\, dr$$

$$\frac{\sin(\lambda_m-\lambda_n)A}{2(\lambda_m-\lambda_n)} - \frac{\sin(\lambda_m+\lambda_n)A}{2(\lambda_m+\lambda_n)} = \cos\beta_m A \cos\beta_n A \underbrace{\frac{\beta_n \operatorname{tg}\beta_n A - \beta_m \operatorname{tg}\beta_m A}{\beta_m^2-\beta_n^2}}_{\cancel{(A+R)\beta A}\,=0} = 0$$

$$\int_0^A \sin^2\beta_n r\, dr = \frac{A}{2} - \frac{\sin 2\beta_n A}{4\beta_n} = \frac{A}{2} - \frac{\operatorname{tg}\beta_n A}{2(1+\operatorname{tg}^2\beta_n A)\beta_n}$$

$$= \frac{A}{2} - \frac{(A+R)\cancel{\beta_n}}{2\cancel{\beta_n}\left[1+(A+R)^2\beta_n^2\right]} = \frac{1}{2}\,\frac{A(A+R)^2\beta_n^2 - R}{(A+R)^2\beta_n^2+1}$$

$$B_n = 2\,\frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2-R}\int_0^A \Phi(r)\sin\beta_n r\, dr \qquad \Bigg\| \quad \lim_{\frac{R}{A}=0} B_n = \frac{2}{A}\left(1+\frac{1}{(A+R)^2\beta_n^2}\right)\int_0^A \Phi(r)\sin\beta_n r\, dr$$

$$\Phi(r) = (r+R)\,c$$

$$\int_0^A (R+r)\sin\beta_n r\, dr = \frac{R}{\beta_n} \qquad \int_0^A \Phi(r)\cos\beta_n r\, dr = \frac{cR}{\beta_n}$$

$$J = 4\pi a^2 R \sum B_n \beta_n e^{-a^2\beta_n^2 t}$$

$$= 8\pi a^2 R^2 c \sum \frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2-R}\cdot e^{-a^2\beta_n^2 t}$$

(mit Ausnahme der ersten Wurzel! Siehe später!)!!

$$\beta_n \neq (2n+1)\frac{\pi}{2}\frac{1}{A}$$

Nach genügend langer Zeit ist angenähert: $J = 8\frac{R^2\pi a^2 c}{A} \cdot \frac{(A+R)^2 \frac{9\pi^2}{4A^2}+1}{(A+R)^2\frac{9\pi^2}{4A^2} + \frac{R}{A}} e^{-a^2\frac{9\pi^2 t}{4A^2}}$

für große $\frac{A}{R}$: $J = 8\frac{R^2\pi a^2 c}{A}\left(1+\frac{4}{9\pi^2}\right) e^{-\frac{9\pi^2}{4A^2}a^2 t}$

Falls nun Anzahl der Kondensationskerne die gleiche, so ist: $A^3 c = \text{const}$

Koeffizienten B_n: $\frac{1}{A}\frac{\frac{(A+R)^2}{A^2}\frac{(2n+1)^2\pi^2}{4}+1}{\left(\frac{A+R}{A}\right)^2\frac{(2n+1)^2\pi^2}{4} - \frac{R}{A}} \neq \frac{1}{A}$ (sehr angenähert)

also ist Größenordnung der Zeit wo obige Annäherung erlaubt ist: $\frac{9\pi^2 a^2 \tau}{4A^2} = 1$

$$\tau = \frac{4}{9\pi^2}\frac{A^2}{a^2}$$

also wenn $a^2 = D = 10^{-10}$

für $A = 10^{-4}$ ist $\tau = 5$ sek.

$= 1\mu$

$$\int_{t_1}^{\infty} J\,dt = 8\frac{R^2\pi a^2 c}{A}\frac{4A^2}{9\pi^2 a^2} e^{-\frac{9\pi^2}{4A^2}a^2 t_1} = \frac{8}{9\pi} R^2 A c\, e^{-\frac{9\pi^2 a^2}{4A^2}t_1}$$

also wird dadurch nur ein minimaler Bruchteil kondensiert

$$u = \frac{2cR^2}{r}\sum_{n=0}^{\infty}\left[\frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R}\right]\frac{1}{\beta_n} e^{-D\beta_n^2 t}\sin\beta_n r$$

Verschiedenes

Zwei Grenzfälle:

I). Gegebene Anzahl von Kondensationskernen, an welchen sich (durch Diffusion) Alles anlagert. Theorie umstehend.

II). Kondensation tritt nur relativ selten und jedesmal nur in bestimmten geringen Beträge ein, so dass Alles durch Diffusion wieder ausgeglichen wird.

Dann gilt chem.-kinetisches Gesetz für Reaktion d. zweiten Ordnung

$$\frac{d\varphi}{dt} = -a\varphi^2 \qquad \varphi = \text{Konz. d. unkondensierten Gases}$$

$$\left.\begin{aligned}\frac{1}{\varphi} &= at + \text{konst}\\ \frac{1}{\varphi_0} &= \qquad \text{konst}\end{aligned}\right\} \quad \frac{1}{\varphi} = \frac{1}{\varphi_0} + at \qquad \varphi = \frac{\varphi_0}{1+\varphi_0 a t}$$

Dabei muss kondensiertes Produkt unwirksam werden.

III) Falls angenommen wird, dass die (Weiter-)Anlagerung an einem kondensierten Doppel-Molekül so vor sich geht wie an den Einzel-Bestandteilen, kann (I) auch vielleicht auch zur Theorie der Fällung verwendet werden, wenn Anzahl d. Kond. Kerne = Gesamtzahl d. Teilchen, $A = \frac{1}{2}$ Distanz bis zum nächsten Teilchen und wenn Resultat halbiert wird.

IV) Andere Auffassung: Von einem bestimmten Moment an werden alle Zusammenstöße angekreidet und es wird die Zahl der Teilchen erwogen, welche <u>noch keinen Zusammenstoss</u> erlitten haben. Wie bestimmt sich diese?

In Gasen analoge Aufgabe: $N = N_0 e^{-\frac{x}{\lambda}} = N_0 e^{-\frac{ct}{\lambda}}$ || $\lambda = \frac{1}{\sqrt{2}\pi N \sigma^2}$

Das wäre hier unrichtig, viel zu groß, denn Wahrsch. eines Zusam. stosses ist auf Brown'schen

Zick-zack-weg viel geringer als wie wenn der Weg ausgebreitet wird.

Also vielleicht so:

$$\overline{\Delta s^2} = 3\overline{\Delta x^2} = 6Dt$$

$$s = \sqrt{6Dt}$$

$$\frac{ds}{dt} = \sqrt{\frac{3D}{2t}}$$

$$dN = -N\frac{ds}{\lambda_0} = -\frac{N}{\lambda_0}\sqrt{\frac{3D}{2t}}\,dt$$

$$\frac{dN}{N} = -\sqrt{\frac{3D}{2t}}\,\frac{dt}{\lambda}$$

$$\lg N = -\frac{\sqrt{6Dt}}{\lambda_0}$$

$$N = N_0\, e^{-\frac{\sqrt{6Dt}}{\lambda_0}} \qquad N = N_0\left[1 - e^{-\frac{\sqrt{6Dt}}{\lambda_0}}\right]$$

~~Das ist aber auch unrichtig~~ denn

$$6Dt = \lambda_0^2 = \frac{1}{2\pi^2 N^2 \sigma^4}$$ = Zeit, wenn nicht zusammengestoßen mehr ~~als~~ $\frac{1}{e}$ der Anfangszahl bleibt

$$t = \frac{1}{12\, D\pi^2 N^2\sigma^4} = \frac{\sigma^2}{\pi D\alpha^2 \cdot 36.}$$

$\frac{\pi N\sigma^3}{6} = \alpha =$ Volumkonzentration

$$= \frac{1}{3\cdot(12)^2}\,\frac{\sigma^2}{\alpha^2 D}$$

z.B. $\sigma = 38\,\mu\mu = 38\cdot 10^{-7}$

$D = 10^{-7}$

$\alpha = 10^{-4}$ $\quad \| \; t = \frac{(38)^2\cdot 10^{-6}}{3\cdot(12)^2\cdot 10^{-7}} = 30$ sek.

$1\,cm^3$ wenn gerade undurchsichtig:

$$\frac{\pi N\sigma^2}{4} = 1$$

$$\therefore\ \alpha = \frac{2\sigma}{3}$$

$$t = \frac{1}{3\cdot(12)^2}\,\frac{4}{9D} = \frac{1}{4\cdot 3^5}\,\frac{1}{D}$$

$$\neq \frac{10^3}{D} = 10^{10}$$

Auch in der Form:

Teilchenabstand δ $\qquad N = \frac{1}{\delta^3}$

$$t = \frac{1}{12\pi^2}\cdot\frac{\delta^6}{D\sigma^4} = \frac{1}{12\pi^2}\cdot\frac{\delta^2}{D}\left(\frac{\delta}{\sigma}\right)^4$$

~~$\frac{mc^2}{2} = \frac{3RT}{2N} = \frac{\pi}{6}R^3\rho\,\frac{c^2}{2}$~~ $\qquad D = \frac{RT}{N}\,\frac{1}{6\pi R\mu}$

~~$c = \sqrt{\frac{RT}{N}\cdot\frac{18}{\pi\rho R^3}}$~~

~~$\frac{ct}{\sqrt{6Dt}} = \sqrt{t}\sqrt{\frac{RT}{N}\,\frac{9}{4\pi\rho R^3}\cdot\frac{N}{RT}\,6\pi R\mu} = \sqrt{t}\sqrt{\frac{\mu}{\rho R^2}}\,\frac{3}{2}$~~

~~$= \sqrt{t}\,\frac{1}{R}\sqrt{\frac{\mu}{\rho}}$~~

$$\frac{t\sqrt{D\pi}}{2R\sqrt{t}}$$

$$\sqrt{6Dt}\left[1 + \frac{\sqrt{\pi Dt}}{2R}\right]$$

$$= \left[\frac{Dt}{R}\sqrt{\frac{3\pi}{2}} + \sqrt{6Dt}\right]$$

Kondensations-Aufgabe ist eigentlich direkt zu behandeln:

1. Diffusionsaufgabe

Kugel R, umgeben von Kugelfläche A, erfüllt mit Medium von d. Dichte c

$$\int_0^t J(t)\,dt = \varphi_t = \text{~~[illegible]~~ in die Kugel eingeströmtes Quantum}$$

Wahrsch., dass ein in A befindl. Teilchen an die Kugel bis zur Zeit t zum ersten Mal gestorben sei $= \dfrac{\varphi}{\frac{4}{3}\pi(A^3-R^3)c} = U_t$

Wahrsch. dass es nicht daran gest. sei $= 1-U_t$

Falls N Teilchen drin vorhanden sind: $W = (1-U_t)^N = e^{-NU}$

∴ Wahrsch. dass innerhalb unendlich ausgedehnten Mediums mit Teilchenzahl n pro cm³ bis t kein Teilchen angelagert sei $= \lim\limits_{A=\infty} W \Big|_{N = n\cdot\frac{4\pi}{3}(A^3-R^3)}$ mittl. Abstand $= \delta$ $N = \frac{4\pi}{3}\left(\frac{A}{\delta}\right)^3$

Wahrsch., dass ein Teilchen (angelagert) angelagert sei $= 1-e^{-NU} = \dfrac{\text{Anzahl d. Teilchen welche Kondensationskerne sind}}{\text{Anzahl aller Teilchen}}$

Für Grenzfall langer Zeiten:

$$\cancel{\lim e^{-NU} = \lim e^{\ldots}} \qquad U = 1 - \int_t^\infty \frac{8\pi a^2 R^2 c}{\frac{4\pi}{3} c A^3}\frac{1}{A}\, e^{-\frac{9\pi^2}{4A^2}a^2 t}\,dt$$

$$= 1 - \frac{8\pi a^2 R^2}{\frac{4\pi}{3} A^4}\,\frac{4A^2}{9a^2\pi^2}\, e^{-\frac{9\pi^2}{4A^2}a^2 t} = 1 - \frac{8}{3\pi^3}\frac{R^2}{A^2}\, e^{-\frac{9\pi^2}{4A^2}a^2 t}$$

Anwendbar nur wenn Exponent eine sehr kleine Grösse

Dann ist näherungsweise: $U =$

$$\frac{\partial v}{\partial t} = D\,\Delta v \quad \Big| \quad v = e^{-D\lambda^2 t}\,u$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \lambda^2 u = 0$$

$$u = \frac{y}{\sqrt{r}}$$

$$\frac{du}{dr} = \frac{1}{\sqrt{r}}\frac{dy}{dr} - \frac{1}{2\sqrt{r^3}}\,y$$

$$\cancel{\frac{d^2u}{dr^2}} \quad \frac{1}{\sqrt{r}}\frac{d^2y}{dr^2} - \frac{1}{\sqrt{r^3}}\frac{dy}{dr} + \frac{3}{4\sqrt{r^5}}\,y$$

$$+ \frac{2}{\sqrt{r^3}}\frac{dy}{dr} - \frac{1}{\sqrt{r^5}}\,y + \lambda^2\frac{y}{\sqrt{r}} = 0$$

$$\frac{d^2y}{dr^2} + \frac{1}{r}\frac{dy}{dr} - \frac{1}{4r^2}\,y + \lambda^2 y = 0$$

$$y'' + \frac{1}{r}y' + \left(\lambda^2 - \frac{1}{4r^2}\right)y = 0 \qquad \Big\| \quad J_m'' + \frac{1}{x}J_m' + \left(1 - \frac{m^2}{x^2}\right)J_m = 0$$

$$y = J_{\frac{1}{2}}(r\sqrt{\lambda}) = \sqrt{\frac{2}{\pi r\sqrt{\lambda}}}\,\sin(r\sqrt{\lambda})$$

$$u = \sqrt{\frac{2}{\pi\sqrt{\lambda}}}\,\frac{1}{r}\sin(r\sqrt{\lambda})$$

$r = R \quad u = 0$ also $R\lambda = n\pi$

$$\lambda = \frac{n\pi}{R}$$

$$u = \frac{1}{r}\sum_n B_n\, e^{-D\lambda^2 t}\sin\left(\frac{n\pi r}{R}\right) = \frac{1}{r}\sum_n B_n\, e^{-\frac{n^2 D\pi^2 t}{R^2}}\sin\left(\frac{n\pi r}{R}\right)$$

$$B_n = -\frac{2cR}{n\pi}\cos n\pi = (-1)^{n+1}\cdot\frac{2cR}{n\pi}$$

$t = 0 \quad u = c$

$$cr = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi r}{R}$$

$$B_n = \frac{2c}{R}\int_0^R r\sin\frac{n\pi r}{R}\,dr$$

$$= \frac{2c}{R}\left[\cancel{\left(\frac{R}{n\pi}\right)^2\sin\frac{n\pi r}{R}} - \frac{R}{n\pi}r\cos\frac{n\pi r}{R}\right]_0^R =$$

$$\int_0^R \sin^2\frac{n\pi r}{R}\,dr = \frac{1}{2}\int_0^R\left[1 - \cos\frac{2n\pi r}{R}\right]dr = \frac{R}{2}$$

$$\int_0^R \sin\frac{n\pi r}{R}\sin\frac{m\pi r}{R}\,dr = \frac{1}{2}\int_0^R\left[\cos(n-m)\frac{\pi r}{R} - \cos(n+m)\frac{\pi r}{R}\right]dr = \frac{1}{2}\,\frac{\sin(n-m)\frac{\pi r}{R}}{\cdots}\Bigg|_0^R - \frac{\sin}{\cdots}\Bigg|_0^R = 0$$

$$u = \frac{2cR}{\pi r}\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n}\, e^{-n^2\frac{D\pi^2 t}{R^2}} \cdot \sin\frac{n\pi r}{R}$$

$$\frac{\partial u}{\partial r}\Big|_{r=R} = -\frac{u}{r}\Big|_{R} + \frac{2c}{\cancel{\pi R}}\cdot\frac{\cancel{\pi}}{R}\sum_{n=1}^{\infty} (-1)^{n+1} e^{-\frac{n^2 D\pi^2 t}{R^2}} \underbrace{\cos n\pi}_{(-1)^n}$$

$$= \cancel{\ldots} - \frac{2c}{R^2}\sum_{n=1}^{\infty} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

$$J = -D\cdot 4\pi R^2 \frac{\partial u}{\partial r}\Big|_{r=R} = 4\pi R^2 D\, \frac{2c}{R}\sum_{1}^{\infty} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

$$= 8\pi c R D \sum_{n=1}^{\infty} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

Exponent Größenordnung: $D = \cancel{\ldots}\, 10^{-7}$

[illegible] $R = 19\cdot 10^{-7}$ $\qquad \frac{D\pi^2}{R^2} = \frac{10^{-7}\cdot 10}{381\cdot 10^{-14}} = \frac{10^7}{381} \approx \frac{1}{4}10^6$

Somit bleibt nach sehr kurzer Zeit nur das erste Glied übrig:

$$\int J\, dt = \frac{8\pi c R^3 D}{D\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

~~[illegible]~~ ~~[illegible]~~!

Diese ganze Berechnung ~~gilt~~ entspricht der Aufgabe: Diffusion einer Kugel (an die äussere Kugelfläche)

Dagegen unsere Aufgabe: $\sum_{n=1}^{\infty} D_n \sin\frac{n\pi r}{R} = c\,r$ für: $R<r<\infty$

Das ist unmöglich

$u = \frac{1}{r}\sum_n D_n e^{-\frac{n^2 D \pi^2 t}{R^2}} \sin\frac{n\pi r}{R} = \frac{v}{r}$ Dies gibt also im ∞ eine endliche Gesamtströmung!

$\frac{1}{4\pi D} J_A = -\frac{v}{A} + \frac{\pi A}{R}\sum_n D_n e \cos\frac{n\pi r}{R}$

Allgemeine Lösung:

$u = \frac{1}{r}\left[A \sin\lambda r + B\cos\lambda r\right]$ | $r = R$ $u=0$

$= \frac{1}{r}B_\lambda\left[\sin\lambda r - \operatorname{tg}\lambda a . \cos\lambda r\right]$ $r^2\frac{\partial u}{\partial r}\Big|_{r=A} = 0 = r^2\left(-\frac{v}{r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right)$

$v - r\frac{\partial v}{\partial r} = 0$

Vielleicht besser so:

$u = \frac{v}{r} = \frac{1}{r}\left[M\sin\lambda r + N\cos\lambda r\right]$

$\begin{cases} M\sin\lambda R + N\cos\lambda R = 0 \\ M\sin\lambda A + N\cos\lambda A - A\lambda\left[M\cos\lambda A - N\sin\lambda A\right] = 0 \end{cases}$

$M\left[\sin\lambda A - \lambda A\cos\lambda A\right] + N\left[\cos\lambda A + \lambda A\sin\lambda A\right] = 0$

Oder auch:

$u = \frac{1}{r}C\sin\lambda(r-a)$

$\sin\lambda(r_1-a) - r\lambda\cos\lambda(r_1-a) = 0$

$\operatorname{tg}\lambda(\underbrace{r_\infty - R}_{A}) = r_\infty\lambda$

~~$\operatorname{tg}\lambda(A-a) = A\lambda$~~ ~~Gleichung finden~~ $\operatorname{tg}\lambda A = (A+R)\lambda$

Für große A angenähert: $\lambda A = (2n+1)\frac{\pi}{2}$

Ausserdem erste Wurzel:

$$\operatorname{tg} AR = \frac{AR - \frac{(AR)^3}{6}}{1 - \frac{(AR)^2}{2}} = AR\left[1 + \frac{(AR)^2}{2} - \frac{(AR)^2}{6}\right] = AR\left[1 + \frac{(AR)^2}{3}\right] = AR$$

$$\operatorname{tg} \lambda A = A\lambda\left[1 + \frac{(A\lambda)^2}{3}\right] = A\lambda + R\lambda$$

$$\frac{(A\lambda)^2}{3} = R\lambda$$

$$\lambda = \sqrt{\frac{3R}{A^3}} = \frac{1}{A}\sqrt{\frac{3R}{A}}$$

$$\mathfrak{B}_n = 2\,\frac{(A+R)^2\lambda_n^2 + 1}{A(A+R)^2\lambda_n^2 - R}\int_0^A (r+R)c \sin\lambda_n r \, dr \qquad (A+R)\lambda_n = \operatorname{tg}(A\lambda_n)$$

$$\mathfrak{B}_0 = 2\,\frac{\operatorname{tg}^2(A\lambda_0) + 1}{A\operatorname{tg}^2(A\lambda_0) - R}\int \cdots \qquad = \lim \frac{2}{\underbrace{A\sin^2(A\lambda_0)}_{3R} - \underbrace{R\cos^2(A\lambda_0)}_{-R}}\int \cdots$$

$$\frac{1}{R}\int_0^A (r+R)c \sin\lambda_0 r \, dr = \frac{1}{R}\,\frac{cR}{\lambda_0} = \frac{c}{\lambda_0} = cA\sqrt{\frac{A}{3R}}$$

$$J = 4\pi DR\left[\underbrace{\mathfrak{B}_0\lambda_0}_{c} e^{-D\lambda_0^2 t} + \mathfrak{B}_1\lambda_1 e^{-D\lambda_1^2 t} + \mathfrak{B}_2\lambda_2 e^{-D\lambda_2^2 t} + \cdots\right]$$

$$= 4\pi DRc\left[e^{-\frac{D \cdot 3R}{A^3}t} + 2R\sum_{n=1}^{\infty} \underbrace{\frac{(A+R)^2\lambda_n^2 + 1}{A(A+R)^2\lambda_n^2 - R}}_{\left(= \frac{1}{A\sin^2(A\lambda_n) - R\cos^2(A\lambda_n)}\right)} e^{-D\lambda_n^2 t}\right]$$

$$J = 4\pi DRc \left[e^{-\frac{3DRt}{A^3}} + 8\pi DRc \sum_{n=1}^{\infty} \left[\frac{1}{A \sin^2(A\lambda_n) - R\cos^2(A\lambda_n)}\, e^{-\left[\frac{(2n+1)\pi}{2}\right]^2 \frac{Dt}{A^2}} \right] \right.$$

falls dann zuerst lim $A = \infty$

so $\sum \frac{\pi}{A} = \infty$

$\frac{4\pi A^3}{3} n = 1$ falls für jedes Teilchen Kugelraum

$$= \frac{4\pi A^3}{3} c\, e^{-4\pi n DRt}$$

$$\boxed{\int_t^{\infty} J\,dt = \frac{4\pi A^3}{3} c\, e^{-\frac{3DRt}{A^3}} + \sum \cdots}$$

falls dagegen zuerst lim $n = \infty$ so ist dies $\frac{1}{A}\left[\vartheta_3 0 - \vartheta 0\right]_{q=1}$

$$v = 2cR \sum_{n=}^{\infty} \frac{1}{A \sin^2 \lambda_n A - R\cos^2 \lambda_n A}\, \frac{1}{\lambda_n}\, e^{-D\lambda_n^2 t} \sin \lambda_n r$$

$$= 2cR \sum \underbrace{\frac{1}{A\lambda \sin^2 A\lambda - \cos^2 \lambda A\left[\operatorname{tg} A\lambda - A\lambda\right]}}\, e^{--}$$

$$\frac{1}{A\lambda_n \cos 2A\lambda_n - \frac{1}{2}\sin 2A\lambda_n} = \frac{2}{2A\lambda_n \cos 2A\lambda_n - \sin 2A\lambda_n}$$

$$v = 2cR \left\{ \frac{1}{2R}\, \frac{1}{\lambda_0}\, e^{-D\lambda_0^2 t} \sin \lambda_0 r + \underbrace{\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n+1}\, e^{-D\frac{(2n+1)^2\pi^2}{4}\frac{t}{A^2}} \sin \frac{(2n+1)\pi}{2}\frac{r}{A}} \right\}$$

$$\frac{(2n+1)\pi}{2A} = x$$

$$\Delta x = \frac{\pi}{A}$$

$$= \underbrace{\frac{1}{A} \sum_{\Delta x = \frac{\pi}{A}} \frac{1}{x}\, e^{-Dx^2 t} \sin rx}$$

$$= \frac{1}{\pi} \int_0^{\infty} e^{-Dx^2 t}\, \frac{\sin rx}{x}\, dx$$

$$u = \frac{2cR}{\pi(n+4)} \int_0^{\infty} e^{-Dx^2 t}\, \frac{\sin rx}{x}\, dx + e^{-D\lambda_0^2 t} c.\, \frac{\sin \lambda_0 r}{\lambda_0 r}$$

$$u = \frac{2cR}{\pi r} \int_0^{\infty} e^{-Dx^2 t}\, \frac{\sin (r-R)x}{x}\, dx$$

$$\Phi_{(r)} = \sum_0 D_n \sin\beta_n r = c(R+r)$$

$$= 2\sum \frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R} \int_0^A \Phi(r)\sin\beta_n r\,dr \cdot \sin\beta_n r$$

$$\operatorname{tg}(A\beta_n) = (A+R)\beta_n$$

$$c(R+r) = 2cR \sum_{n=1}^{\infty} \frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R} \cdot \frac{\sin\beta_n r}{\beta_n}$$

$\sin A\beta_n = \cancel{(A+R)\beta_n \cos A\beta_n}$

$\cancel{(\sin V\beta_n \cos VR - \cos V\beta \sin VR)}$

$= V\beta$

$\cancel{r+R = y}$

$\cancel{A+R = V}$

$$\cancel{cy = 2cR \sum \frac{V^2\beta_n^2+1}{AV^2\beta_n^2 - R} \frac{1}{\beta_n}\left[\sin\beta_n y \cos\beta_n R - \cos\beta_n y \sin\beta_n R\right]}$$

$$= 2cR \sum_{n=1}^{\infty} \underbrace{\frac{1}{A\sin^2 A\beta_n - R\cos^2 A\beta_n}} \frac{\sin\beta_n r}{\beta_n}$$

$$\frac{1}{A - (A+R)\cos^2 A\beta_n} = \frac{1}{A - \frac{\sin A\beta_n \cos A\beta_n}{\beta_n}}$$

$$c(R+r) = \cancel{2cR \sum \frac{\sin\beta_n r}{A\beta_n}} \quad \boxed{\lim_{A=\infty} \beta_1 = \frac{1}{A}\sqrt{\frac{3R}{A}}}$$

$$= \lim_{A=\infty} 2cR \left\{ \frac{\sin\beta_1 r}{A\beta_1 \sin^2 A\beta_1 - R\beta_1 \cos^2 A\beta_1} + \sum_{n=2}^{\infty} \right.$$

$$= \underbrace{\lim \frac{\sin\beta_1 r}{\beta_1} c}_{= cr} + 2cR\Big[\lim \underbrace{\sum_{n=2}^{\infty}\qquad}\Big] = \frac{1}{\pi}\overbrace{\int_0^\infty \frac{\sin rx}{x}\,dx}^{\frac{\pi}{2}} = c(r+R)$$

$$v = c r e^{-\beta_1^2 D t} + \frac{2cR}{\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx$$

$$u = \frac{c}{r+R}\left[r + \frac{2R}{\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx\right] = c\left[1 - \frac{R}{R+r}\left(1 \mp \frac{2}{\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx\right)\right]$$

$$\frac{\partial u}{\partial t} = -\frac{2cR}{\pi(r+R)}\, D\int_0^\infty \underbrace{e^{-Dx^2 t}\, x \sin rx\,dx}$$

$$-\cancel{e^{-x^2 Dt}\frac{\sin rx}{2Dt}}\Bigg|_0^\infty + \int \frac{r}{2Dt}\, e^{-Dx^2 t}\cos rx\,dx$$

$$= -\frac{cRr}{\pi(r+R)\,t}\int_0^\infty e^{-Dx^2 t}\cos rx\,dx$$

$$\frac{\partial u}{\partial r} = +\frac{cR}{(R+r)^2} - \frac{2cR}{(r+R)^2\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx + \frac{2cR}{\pi(r+R)}\int_0^\infty e^{-Dx^2 t}\cos rx\,dx$$

$$\frac{\partial^2 u}{\partial r^2} = -\frac{2cR}{(R+r)^3} + \frac{4cR}{(r+R)^3\pi}\int e^{-Dx^2 t}\frac{\sin rx}{x}\,dx - \frac{4cR}{(r+R)^2\pi}\int e^{-Dx^2 t}\cdot\cos rx\,dx$$

$$\mp \frac{2cR}{\pi(r+R)}\int e^{-Dx^2 t}\, x \sin rx\,.dx$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - D\frac{\partial u}{\partial t} = \frac{cR}{(R+r)^2}\left[\frac{2}{r} - \frac{2}{R+r}\right] + \frac{2cR}{(r+R)^3\pi}\int e^{-Dx^2 t}\frac{\sin rx}{x}\left[2 - 2\frac{r+R}{r+R}\right]dx$$

$$-\frac{2cR}{(r+R)^2\pi}\int e^{-Dx^2 t}\cos rx\,[2-2] = 0$$ stimmt

$$\cancel{= \frac{cR}{(R+r)^3} + \frac{2cR}{(R+r)^3\pi}\int e^{-Dx^2 t}\frac{\sin rx}{x}\,dx - \frac{2cR}{(r+R)^2\pi}\int \underbrace{e^{-Dx^2 t}\cos rx\,dx}_{\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\,e^{-\frac{r^2}{4Dt}}}}$$

$$\int_0^\infty e^{-p\alpha^2}\cos q\alpha \, d\alpha = \frac{1}{2}\sqrt{\frac{\pi}{p}}\, e^{-\frac{q^2}{4p}}$$

$$\int_0^\infty e^{-p\alpha^2}\frac{\sin q\alpha}{\alpha}\, d\alpha = \frac{1}{2}\sqrt{\frac{\pi}{p}}\int_0^q e^{-\frac{q^2}{4p}}\, dq = \frac{\sqrt{\pi}}{2}\int_0^{\frac{q}{2\sqrt{p}}} e^{-z^2} dz$$

$$u = \frac{c}{r+R}\left[r + \frac{2R}{\sqrt{\pi}}\int_0^{\frac{r}{2\sqrt{Dt}}} e^{-z^2} dz\right]$$

Bei Einführung der vom Mittelpunkt aus gerechneten r:

$$u = \frac{c}{r}\left[r - R + \frac{2R}{\sqrt{\pi}}\int_0^{\frac{r-R}{2\sqrt{Dt}}} e^{-z^2} dz\right] = c\left[1 - \frac{R}{r} + \frac{2R}{\sqrt{\pi}\,r}\int_0^{\frac{r-R}{2\sqrt{Dt}}} e^{-z^2} dz\right]$$

$$\frac{\partial u}{\partial r} = c\left[\frac{R}{r^2} + \frac{2R}{\sqrt{\pi}}\frac{1}{2\sqrt{Dt}}\, e^{-\left(\frac{r-R}{2\sqrt{Dt}}\right)^2} - \frac{2R}{r^2\sqrt{\pi}}\int_0^{\frac{r-R}{2\sqrt{Dt}}} e^{-z^2} dz\right]$$

$$J_R = 4\pi D R^2 \frac{\partial u}{\partial r}\Big|_R = 4\pi D R^2 c\left[\frac{1}{R} + \frac{R}{\sqrt{\pi D t}}\right] = 4\pi D c R\left[1 + \frac{R}{\sqrt{\pi D t}}\right]$$

$$Q = \int_0^t J\, dt = 4\pi D c R\left[t + \frac{2R\sqrt{t}}{\sqrt{\pi D}}\right]$$

Das gilt aber

$$J_A = 4\pi D R c\left[1 + \frac{A^2}{\sqrt{\pi D t}}\, e^{-\frac{(A-R)^2}{4Dt}} - \frac{2}{\sqrt{\pi}}\int_0^{\frac{A-R}{2\sqrt{Dt}}} \underbrace{e^{-z^2} dz}_{\frac{\sqrt{\pi}}{2}}\right]$$

~~also entspricht es einer fortwährenden Zufuhr in jenem Abstand von außen so dass $\lim_{r=\infty} u =$ dort erhalten wird.~~

$$\lim_{A=\infty} J_A = 0$$

also ist wirklich die Zufuhr von Aussen $= 0$

Zeitdauer wann der stationäre Zustand merklich erreicht ist:

$$\sqrt{t} = \frac{2R}{\sqrt{\pi D}} \qquad t = \frac{4R^2}{\pi D} = \frac{4R^3 \cdot 6\pi\mu}{\pi \frac{HT}{N} \cdot} =$$

von derselben Grössenordnung wie Stationärwerdung innerhalb Kugel die ganz verschwindend

$$U_t = \frac{3}{4\pi}\frac{1}{A^3_c} 4\pi D_c R t = \frac{3DRt}{A^3}$$

Wahrsch. dass noch kein Teilchen angelagert sei:

$$\lim_{A=\infty} W \Big|_{N = n\frac{4\pi}{3}A^3} = \lim e^{-NU_t} = \lim e^{-\frac{4\pi}{3}n\, 3DRt} = e^{-4\pi n DRt}$$

Mittelwert für Anlagerungszeit: $\tau_1 = \frac{1}{4\pi n DR} = \frac{\delta^3}{4\pi DR} \Bigg\| \frac{(38)^2 \cdot 10^{-12}}{4\pi \cdot 10^{-7}\, 38 \cdot 10^{-6}} = \frac{38 \cdot 10^{-6}}{4\pi \cdot 10^{-7}} = 2 \text{ sek.}$

Im ersten Stadium liegen, für äusserst kurze t:

$$\lim W = \cancel{e^{-\frac{4\pi}{3}n}} \lim_{t=0} e^{-4\pi n DR\left[t + \frac{2R\sqrt{t}}{\sqrt{\pi D}}\right]} = e^{-8\pi n \sqrt{D} R^2 \sqrt{t}}$$

$$= e^{-8 n R^2 \sqrt{\pi D t}}$$

Anlagerungszeit wäre in diesem Falle: $\tau_2 = \frac{\delta^6}{64\pi D R^4}$ also fast derselbe Schritt wie nach „mittl. Weglängen-Ketten"

$$= \tau_1 \cdot \frac{\delta^3}{16 R^3}$$

Was wirklich zustande kommt, hängt ab von:

$\frac{\tau_1}{\tau_2} = 16\frac{R^3}{\delta^3}$ also in Praxi immer $\tau_1 \ll \tau_2$ falls R = Teilchenradius

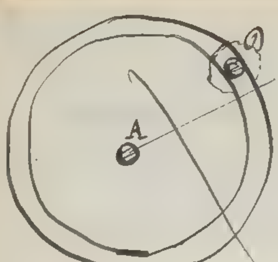
also überwiegt der Prozess (1)

$n = 10^{10}$

$R = 10^{-9}$

$$\cancel{}\, \frac{4\pi\sqrt{n}R}{\sqrt{\pi D}} = 4\sqrt{\left(\frac{R}{\delta}\right)^3}$$

$4\pi n DRt\,[1 + 10^{-8}]$! also ist das zweite Glied ganz verschwindend



Wahrsch. dass ein Teilchen B zur Zeit t im Entf. r – dr sich befinde
(n Teilchen pro cm³ und gestellt)

$$W_1 = 4\pi r^2 dr \cdot n$$

Wahrsch., dass das Teilchen A in der Zeit t (bis in Entf. r gekommen sei: in der sich befinde

$$W_2 = \frac{1}{(\sqrt{\pi D t})^3} e^{-\frac{r^2}{4Dt}} \int dx\,dy\,dz = \frac{4\pi r^2}{(\sqrt{\pi D t})^3} e^{-\frac{r^2}{4Dt}} dr$$

~~Wahrsch. eines Zusammenstoss in der Kugelschale~~

Der Anzug kann ersetzt werden durch Verschiebungen im Betrage von δ in Intervallen τ (in X, Y Z)

$$D = \frac{\delta^2}{2\tau}$$

~~Wahrsch. dass Teilchen A sich zur Zeit~~

Somit „Teilchendichte" A zur Zeit $t = \frac{1}{\sqrt{\pi D t}^{\,3}} e^{-\frac{r^2}{4Dt}}$

Also Wahrsch. eines Zusammenstosses zwischen t .. $t+\tau$

$$\int_{r=0}^{\infty} 4\pi r^2 dr\, n \frac{1}{(\sqrt{\pi D t})^3} e^{-\frac{r^2}{4Dt}} \Big| \cdot 4R^2\pi \cdot \delta \cdot 3 = 12 R^2 \pi n \delta = \varepsilon$$

Wahrsch. dass zwischen t .. $t+\tau$ kein Zusammenstoss stattfinde: $1 - 12R^2\pi n\delta$

" " " 0 t $= (1-\varepsilon)^{\frac{t}{\tau}} = e^{-\frac{\varepsilon t}{\tau}} = e^{-12R^2\pi\frac{n\delta t}{\tau}}$

Wahrsch. dass der erste Zusam. zwischen t – $t+\tau$ $= \varepsilon e^{-\varepsilon\frac{t}{\tau}}$

.. .. t .. $t+dt = \frac{\varepsilon}{\tau} e^{-\varepsilon\frac{t}{\tau}} dt$

" " " " " t .. ∞ : $\int_t^\infty = e^{-\frac{\varepsilon t}{\tau}} =$

Dabei Grundfehler: Voraussetzung d. Unabhängigkeit d. Wahrsch. d. Z. ‖ $e^{-12R^2\pi D\frac{t n}{\delta}}$

Richtiger vielleicht so:

Falls O in r..dr : Wahrsch., dass zwischen 0..t kein Zusammenstoss stattgefunden habe:

$$= e^{-\frac{t}{\tau}\cdot\frac{12R^2\pi\delta}{(\sqrt{\pi D t})^3}}\, e^{-\frac{r^2}{4Dt}}$$

das Allgemeine, wenn Zeit von Bewirkt:

$$\frac{dN}{dt}\Big|_{t=0} = -N_0^2\beta$$

Verschiebung um 2R:

$$(2R)^2 = 6D\tau$$

$$\frac{ds}{dt} = \frac{2R}{\tau} = \frac{6D}{2R} = \frac{3D}{R}$$

$$dN = -N\frac{ds}{\lambda_0} = -\frac{N}{\lambda_0}\frac{3D}{R}dt = -N\frac{3D}{R}n\cdots dt$$

als konstant betrachtet!?

$$\lg N = -\frac{3D}{R\lambda_0}t$$

$$N = N_0 e^{-\frac{3D}{R\lambda_0}t} = N_0 e^{-\frac{3D}{R}t\cdot\sqrt{2}\pi n\sigma^2} = N_0 e^{-n\beta t}$$

$$= N_0 e^{-12\sqrt{2}\pi n R t D}$$

Dann: pro Zeiteinheit durchstrichenes ~~zurückgelegtes~~ Volumen wird der Grössenordnung nach entsprechen der Grösse: $\sigma^2\pi\cdot\frac{2R}{\tau}$

|| Ansatz d. Kondensationszeit $n\beta t = 1$

Tatsächlich stimmt das Resultat der Form nach überein mit dem früher abgeleiteten

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Wenn man die allmähliche Abnahme der Zahl der noch freien O und damit Zusammenstösse in Betracht zieht:

$$dN = -\overbrace{\boxed{\frac{3D}{R}\sqrt{2}\,\pi\sigma^2}}^{=\beta} N^2\, dt$$

$$\frac{1}{N} = \beta t + \text{konst}$$

$$\frac{1}{N} = \frac{1}{N_0} + \beta t$$

$$N_0 = N + N N_0\beta t$$

$$N = \frac{N_0}{1+N_0\beta t} =$$

$$\frac{dN}{dt}\Big|_{t=0} = -\beta N_0^2$$

Ansatz der Kondensationszeit: $n\beta t = 1$

Allerdings ist dann diesen Einfluss übertrieben, denn die Abnahme hat vor Allem in dem schon durchstrichenen Raum stattgefunden, aber weniger ~~nicht~~ in dem noch zu durchstreichenden

Wenn wir zur Lösung mit begrenztem Kugelraum zurückkehren und annehmen:

1). Kugelraum so gross, dass er dem für ein Teilchenpaar entfallenden Raum entspricht

$\therefore \quad \frac{4\pi}{3}R^3 \cdot 2n = 1$

2). Da der Aggregationskern nicht unbeweglich ist, sondern sich ebenso bewegt, wie die gegenwärtigen Teilchen selbst, kann diese durch Vergrösserung des D berücksichtigt werden

Wenn man 2D setzt, so bleibt die frühere Formel unverändert

Einfluss des ersten Stadiums ; Unterschied, welcher durch Vernachlässigung desselben hervorgebracht wird β_2

$$W_{1+2} - W_1 = e^{-4\pi n R D t}\underbrace{\left[e^{-\overbrace{8\sqrt{\pi n}\sqrt{D}\,R^2\sqrt{t}}^{\beta_1}} - 1\right]}$$

Falls $\beta_1 \ll \beta_2$: $\qquad \doteq -8\sqrt{\pi D}\, n R^2 \sqrt{t}$

$z = x^2$
$dz = 2x\,dx$

$$\int_0^\infty \sqrt{t}\, e^{-\alpha t}\, dt = \int_0^\infty \frac{1}{\alpha^{3/2}} \sqrt{z}\, e^{-z}\, dz = \frac{2}{\alpha^{3/2}}\int_0^\infty \underbrace{x^2 e^{-x^2}\, dx}_{\frac{\sqrt{\pi}}{4}} = \frac{\sqrt{\pi}}{2\alpha^{3/2}}$$

$$\Delta W = \frac{\sqrt{\pi}}{2}\,\frac{8\sqrt{\pi D}\, n R^2}{(4\pi n R D)^{3/2}} = \cancel{\frac{1}{16\pi^2 n^2 D^{3/2} R}} = \frac{\sqrt{R}}{2\sqrt{\pi}\, D\sqrt{n}}$$

$n = \frac{1}{\delta^3} = (3\cdot 5)^{\frac{1}{2}} 10^{-12}$

z.B. $\frac{\sqrt{3{,}8\cdot 10^{-6}}\cdot 3{,}8\cdot 10^{-6}}{2\sqrt{\pi}\; 10^{-7}} = 2\cdot 10^{-2} = 2\%$ wird aber in verdünnterer Lösung recht merklich sein können

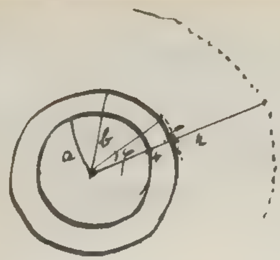
ΔW für Halbzeit t_1 $= \frac{8\sqrt{\pi D}\; nR^2}{2\sqrt{\pi}\sqrt{nRD}} = 4\sqrt{nR^3} = 4\sqrt{\left(\frac{R}{\delta}\right)^3}$

$= 4\sqrt{\text{Volumkonzentration}}$

Nachdem [Wenn] alle Teilchen sich in Teilchenpaare vereinigt haben ist

1). $n' = \frac{n}{2}$

2). $D' \neq \frac{D}{2}$ aber so dass $DR = \text{const}$ also würde weiterer Prozess mit Hilfe zweier Stücke fortschreiten



Wenn ein + Elektron auf inneren Kugelfläche, so wird das entsprechende $-e$ auf der äusseren ~~mit~~ nach d. Wahrscheinlichkeitsgesetz

$$W_{(\varphi)}\,d\varphi = A\, e^{-\frac{N}{RT}\chi}\ \cancel{\varphi}\ \sin\varphi\, d\varphi$$

$$\chi = \frac{e^2}{K\sqrt{a^2+b^2-2ab\cos\varphi}} = \frac{e^2}{Kr}$$

$$2ab\sin\varphi\, d\varphi = 2r\,dr$$

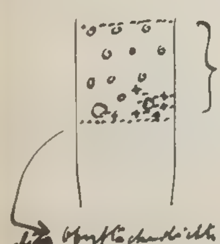
$$\int_0^{\pi} \sin\varphi\, e^{-\frac{N}{RT}\chi}\, d\varphi = \int_{r=b-a}^{a+b} \frac{e^{-\frac{\varepsilon}{r}}\, r\,dr}{ab}$$

$$\frac{\varepsilon}{r} = u$$

$$-\frac{\varepsilon}{r^2}\,dr = du = -\frac{u^2\,dr}{\varepsilon}$$

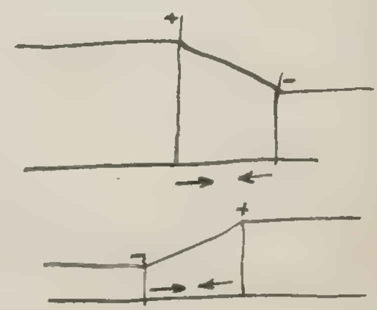
$$dr = -\varepsilon\frac{du}{u^2}$$

$$= -\int_{\frac{\varepsilon}{b-a}}^{\frac{\varepsilon}{b+a}} \frac{\varepsilon^2\, e^{-u}\, u^{-3}\, du}{ab}$$



Gewicht pro 1 cm³

$$\frac{\Delta E}{\Delta x} = \underbrace{\varphi\,\frac{\delta'-\delta_0}{\delta'}\, g}_{\frac{\Delta p}{\Delta x}}\ \frac{K(\varphi_1-\varphi_0)}{4\pi}\ \frac{6}{\mu}$$



~~Die~~ Oberflächendichte $= -\frac{K}{4\pi}\frac{\Delta E}{\Delta x}$

(Kataphoret. Sedim. infolge Einzelkraft:

$$u = \frac{K(\varphi_1-\varphi_0)}{4\pi\mu}\,\frac{\Delta E}{\Delta x} = \varphi\,\frac{\delta'-\delta_0}{\delta'}\left(\frac{K(\varphi_1-\varphi_0)}{4\pi\mu}\right)^2 6\, g$$

Sedim. infolge Stokes: $v = \frac{2}{9}\,\frac{a^2\,(\delta'-\delta_0)}{\mu}\, g$

$$u = \frac{1}{4}\,\frac{1{\cdot}4}{2{\cdot}6}\left(\frac{1}{300\cdot 4\pi\cdot 0{\cdot}02}\right)^2 \frac{10^7}{9\cdot 10^{11}}\, g = \frac{0{\cdot}7}{1{\cdot}3\cdot 4\cdot 10\cdot 36\cdot 9}\,\frac{10^{-4}}{9}\, g = \frac{10^{-7}}{2\cdot(3{\cdot}6)^2} = \cancel{\tfrac{1}{2}\cdot 10^{-8}} = \tfrac{1}{2}\cdot 10^{-5}$$

$$v = \frac{2}{9}\cdot\frac{10^{-8}\cdot 1{\cdot}4}{0{\cdot}02}\cdot 980 = 1{\cdot}5\cdot 10^{-4}$$

während Stock findet für das schärfste Grenze:

(?) $v = \frac{5}{60}\frac{cm}{sek}$ bis $0{\cdot}05\ \frac{cm}{sek}$!!!

elektr. Oberflächendichte $\frac{K}{4\pi}\frac{K(\varphi_1-\varphi_0)}{4\pi\mu}\varphi.\frac{\delta^1 \delta_0}{\delta^1}6 = \rho.\frac{K}{300.K\pi.0{\cdot}02}.\frac{1}{4}\frac{1}{7}\frac{10^7}{9.10^{11}}$

$= \frac{10^{-4}}{36.\pi.6.} = \frac{1}{7}10^{-6}$ (elektrost. abs.)

$$P = \frac{16\,Dt}{\sigma\pi}\int_{\frac{a}{2\sqrt{Dt}}}^{\infty} e^{-R^2} R\,dR \int_0^{\frac{a}{2\sqrt{Dt}}} e^{-r^2} r\,dr \int_{-1}^{+1}\frac{e^{2Rr\xi}}{\sqrt{1-\xi^2}}\,d\xi$$

$R = \beta + u$

$r = \beta - v$

$$= \frac{16\,Dt}{\sigma\pi}\int_0^{\infty} e^{-(\beta+u)^2}(\beta+u)\,du \int_{\beta}^{0} e^{-(\beta-v)^2}(\beta-v)\,dv \int_{-1}^{+1}\frac{e^{2(\beta^2+(u-v)\beta-uv)\xi}}{\sqrt{1-\xi^2}}\,d\xi$$

$$\int_0^{\infty} e^{-(\beta+u)^2}(\beta+u)\,e^{+2u(\beta-v\xi)}\,du = e^{-\beta^2}\int_0^{\infty} e^{-u^2-2uv\xi}(\beta+u)\,du$$

$$= e^{-\beta^2+v^2\xi^2}\int_0^{\infty} e^{-(u+v\xi)^2}(\beta+u)\,du$$

$$\int_0^{\infty} e^{-(u+v\xi)^2}(\beta+u)\,du = \int_{v\xi}^{\infty} e^{-z^2}(z+\beta-v\xi)\,dz = \frac{e^{-z^2}}{2}\Big| + (\beta-v\xi)\int_{v\xi}^{\infty} e^{-z^2}\,dz$$

$$= \frac{e^{-v^2\xi^2}}{2} + (\beta-v\xi)\left[\frac{\sqrt{\pi}}{2} - \int_0^{v\xi}\left[1-z^2+\frac{z^4}{2}-\frac{z^6}{6}\cdots\right]dz\right]$$

$$= \frac{e^{-\beta^2}}{2} + (\beta-v\xi)\,e^{-\beta^2+v^2\xi^2}\left[\frac{\sqrt{\pi}}{2} - v\xi + \frac{(v\xi)^3}{1!\,3} - \frac{(v\xi)^5}{2!\,5} + \frac{(v\xi)^7}{3!\,7} - \cdots\right]$$

$$\frac{\partial}{\partial t}(2\pi r u) = +D\,\frac{\partial}{\partial r}\left(2\pi r\,\frac{\partial u}{\partial r}\right)$$

$$\frac{\partial}{\partial t}(r u) = +D\,\frac{\partial}{\partial r}\left(r\,\frac{\partial u}{\partial r}\right)$$

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\,\frac{\partial u}{\partial r}\right)$$

$$u = \sum v_n\, e^{-\mu_n t}$$

$$D\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\,\frac{\partial v}{\partial r}\right) + \mu\, v = 0$$

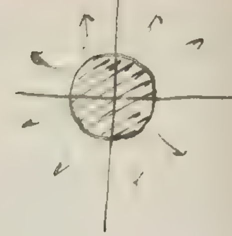
$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\,\frac{\partial v}{\partial r} + \frac{\mu}{D}\, v = 0$$

$$\int_0^\infty r u\, dr = \text{const}$$

$$\int_0^\infty r\,\frac{\partial u}{\partial t}\, dr = 0$$

$$\sum e^{-\mu_n t}\,\mu_n \int_0^\infty r\, v_n\, dr = 0$$

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho}\,\frac{\partial v}{\partial \rho} + \frac{a^2\mu}{D}\, v = 0$$



$\mu > 0$ $\quad \displaystyle\int_0^\infty r\, v_n\, dr = 0$

$$\int_0^\infty r\, J_0(r)\, dr = 0 \quad (?)(?)$$

$$\int_0^\infty r\, e^{-p^2 r^2}\, J_\nu(\alpha r)\, dr = \frac{1}{2p^2}\, e^{-\frac{\alpha^2}{4p^2}}$$

Mathews p. 77

$t = 0$:

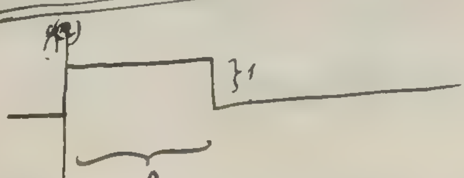
$$\sum A_n\, J_0\!\left(r\sqrt{\tfrac{\mu}{D}}\right) = 1 \qquad r < a$$

$$\cancel{\sum A_n\, J_0\!\left(r\sqrt{\tfrac{\mu_n}{D}}\right) = 0 \qquad r > a}$$

$$\int_0^\infty d\lambda \int_0^a \lambda\,\rho\,\varphi(\rho)\, J_0(\lambda\rho)\, J_0(\lambda r)\, d\rho = \varphi(r) \qquad 0 < r < a$$

$$= 0 \qquad r > a$$

$$r < 0$$



$$\int_0^\infty d\lambda \int_0^a \lambda\,\rho\, J_0(\lambda\rho)\, J_0(\lambda r)\, d\rho = 1 \quad \| \quad 0 < r < a$$

$$= 0 \quad \| \quad r > a$$

$$\cancel{\lambda^2 \frac{\partial^2 J_0}{\partial r^2} + \frac{\lambda}{r}\,\frac{\partial J_0}{\partial r} + \frac{\mu}{D} J_0}$$

$$\frac{\partial^2 J_0(\lambda r)}{\partial r^2} + \frac{1}{r}\,\frac{\partial}{\partial r} J_0(\lambda r) + \lambda^2 J_0 = 0$$

$$\lambda = \sqrt{\frac{\mu}{D}}$$

$$u = \int_0^\infty e^{-D\lambda^2 t}\, d\lambda \int_0^a \lambda\, J_0(\lambda\rho)\, J_0(\lambda r)\, \rho\, d\rho$$

$$I_n(t) = i^{-n} J_n(it)$$

$$\int_0^\infty r e^{-p^2 r^2} J_n(\lambda r)\, J_n(\mu r)\, dr = \frac{1}{2p^2} e^{-\frac{\lambda^2+\mu^2}{4p^2}} I_n\left(\frac{\lambda\mu}{2p^2}\right)$$

Gray & Mathews p. 78

$$\cancel{u = \sum e^{-\gamma_n t} J_0\left(r\sqrt{\frac{\gamma_n}{D}}\right)}$$

$$\alpha \frac{J'(\alpha)}{J(\alpha)} = \beta \frac{J'(\beta)}{J(\beta)}$$

$t=0$

$$\cancel{f = u\, J_0}$$

$$\cancel{\frac{1}{2}\left[J(\alpha)^2 + J'(\alpha)^2\right]}$$

$$\frac{1}{2}\, J(\alpha_n)^2 + J'(\alpha_n)^2 = \int_0^1 J(\alpha_n x)\, dx$$

$$\frac{\partial^2 v}{\partial\left(\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}}\right)^2} + \frac{1}{\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}}} \frac{\partial v}{\partial\left(\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}}\right)} + v = 0 \qquad \frac{r}{a}\sqrt{\frac{\gamma a^2}{D}} = x'$$

$$\cancel{\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho}} \qquad \frac{\partial^2 v}{\partial x'^2} + \frac{1}{x'} \frac{\partial v}{\partial x'} + v = 0 \qquad \frac{r}{a} = x$$

$$\sqrt{\frac{a^2\gamma}{D}} = \xi$$

$$v = J_0(x') = J_0\left(\frac{r}{a}\sqrt{\frac{a^2\gamma}{D}}\right) = J_0(x\,\xi)$$

$$u = v e^{-\gamma t} = \sum v_n e^{-\frac{D\xi_n^2 t}{a^2}}$$

$t=0$

$f = 1; \quad 0 < x < 1$

$f = 0 \quad 1 < x < \infty$

$$f(x) = \int_0^\infty J(\xi x)\, \xi\, d\xi \underbrace{\int_0^\infty f(\lambda)\, J(\xi\lambda)\, \lambda\, d\lambda}$$

$$\int_0^1 \lambda\, J(\xi\lambda)\, d\lambda = \frac{J_1(\xi)}{\xi}$$

$$f(x) = \int_0^\infty \left[J_1(\xi)\, J(\xi x)\, d\xi \cdot e^{-\frac{D\xi^2 t}{a^2}}\right]$$

$$\frac{\partial f}{\partial r} = \frac{1}{a}\frac{\partial f}{\partial x}\Big| = \frac{1}{a}\int_0^\infty J_1(\xi)\, J_1'(x\xi)\, \xi\, d\xi \,.\, e^{-\frac{D\xi^2 t}{a^2}}$$

$$P = \frac{D}{a^2 r}\int_t^\infty 2\pi r \frac{\partial f}{\partial r}\,dt = \frac{D}{\cancel{a^2 r}}\,\frac{\cancel{2\pi r}}{\cancel{a}}\int_t^\infty dt \int_0^\infty J_1(\xi)\, J_1'(\xi)\, \xi\, d\xi\; e^{-\frac{D\xi^2 t}{a^2}}$$

$$= 2\int_0^\infty \frac{J_1(\xi)\, J_1'(\xi)\, d\xi}{\xi}\,.\,e^{-\frac{D\xi^2 t}{a^2}} = 2\underbrace{\int_0^\infty [J_1(\xi)]^2 \frac{d\xi}{\xi}}\,.\,e^{-\frac{D\xi^2 t}{a^2}}$$

$$J_0 = \frac{dJ_1}{dx} + \frac{J_1}{x}$$

$t=0$

$$= 2\int_0^\infty J_1(\xi)\, J_0(\xi)\, d\xi - 2\int_0^\infty J_1 \frac{dJ_1}{d\xi}\, d\xi$$

$$\downarrow$$

$$-J_0'(\xi)$$

$$= -[J_0(\xi)]^2 - [J_1(\xi)]^2 \Big|_0^\infty = 1$$

$$2\underbrace{\int_0^\infty J_0(\xi)\, J_1(\xi)\, e^{-\frac{D\xi^2 t}{a^2}}\, d\xi} - 2\underbrace{\int_0^\infty J_1 \frac{dJ_1}{d\xi}\, e^{-\frac{D\xi^2 t}{a^2}}\, d\xi}$$

$$\underbrace{-[J_0(\xi)]^2 e^{-\frac{D\xi^2 t}{a^2}}\Big|_0^\infty}_{=1} - \frac{2Dt}{a^2}\int_0^\infty J_0^2\, \xi\, e^{-\frac{D\xi^2 t}{a^2}}\, d\xi \quad = \cancel{-J_1^2\, e^{-\frac{D\xi^2 t}{a^2}}\Big|_0^\infty} - \int_0^\infty \frac{2Dt}{a^2}\, J_1^2\, \xi\, e^{-\frac{D\xi^2 t}{a^2}}\, d\xi$$

$$P = \frac{2Dt}{a^2}\left\{\underbrace{\int_0^\infty J_0^2\, \xi\, e^{-\frac{Dt\xi^2}{a^2}}\, d\xi} + \int_0^\infty J_1^2\, \xi\, e^{-\frac{Dt\xi^2}{a^2}}\, d\xi\right\}$$

$$\frac{a^2}{2Dt}\, e^{-\frac{a^2}{2Dt}}\left[I_0\!\left(\frac{a^2}{2Dt}\right) + I_1\!\left(\frac{a^2}{2Dt}\right)\right] \quad = e^{-\frac{a^2}{2Dt}}\left[I_0\!\left(\frac{a^2}{2Dt}\right) + I_1\!\left(\frac{a^2}{2Dt}\right)\right]$$

$$I_0(\beta) = 1 + \frac{\beta^2}{(2)^2} + \frac{\beta^4}{(2.4)^2} + \frac{\beta^6}{(2.4.6)^2}$$

$$I_1(\beta) = \frac{\beta}{2} + \frac{\beta^3}{2.2.4} + \frac{\beta^5}{2.2.4.4.6} \cdots \quad = I_0'(\beta)$$

$$= 1 - \frac{a^2}{4Dt} + \frac{a^4}{16(Dt)^2}$$

$$\left(1-\beta+\frac{\beta^2}{2}\right)\left(1+\frac{\beta}{2}+\frac{\beta^2}{4}\right) = 1 - \frac{\beta}{2} + \cancel{\frac{\beta^2}{2}} - \cancel{\frac{\beta^2}{2}} + \frac{\beta^2}{4}$$

$$\frac{1}{6} - \frac{1}{16} = \frac{8-3}{48}$$

$$\left(1-\beta+\frac{\beta^2}{2}-\frac{\beta^3}{6}+\frac{\beta^4}{24}\right)\left(1+\frac{\beta}{2}+\frac{\beta^2}{4}+\frac{\beta^3}{16}+\frac{\beta^4}{64}\right)$$

$$-\frac{1}{24} + \frac{1}{8} - \frac{1}{16} + \frac{1}{64}$$

$$+\frac{\beta}{2} - \frac{\beta^2}{2} + \frac{\beta^3}{4} - \frac{\beta^4}{12}$$

$$\frac{-8+24-12+3}{3.64}$$

$$+\frac{\beta^2}{4} - \frac{\beta^3}{4} + \frac{\beta^4}{8} \qquad = 1 - \frac{\beta}{2} + \frac{\beta^2}{4} - \frac{5}{48}\beta^3 + \frac{7}{3.64}\beta^4$$

$$+\frac{\beta^3}{16} - \frac{\beta^4}{16}$$

$$\beta = 2\alpha$$

$$+\frac{\beta^4}{64} \qquad = 1 - \alpha + \alpha^2 - \frac{5}{6}\alpha^3 + \frac{7}{12}\alpha^4$$

stimmt mit früherem Resultat

$\cancel{I_0(\beta) = J_0(i\beta)}$

Gray & Mathews p. 227: $\quad \mathcal{I}$

$$I_n(t) = e^t \frac{t^n}{2^n . n!}\left\{1 - t + \frac{2n+3}{2(2n+2)}t^2 - \frac{2n+5}{2.3.(2n+2)}t^3 + \frac{(2n+5)(2n+7)}{2.3.4.(2n+2)(2n+4)}t^4 \right.$$

$$\left. - \frac{(2n+7)(2n+9)}{2.3.4.5\,(2n+2)(2n+4)}t^5 + \cdots \right\}$$

p. 68

for large t:

$$I_n(t) = \frac{1}{\sqrt{2\pi t}}\, e^t \left\{1 - \frac{4n^2-1}{8t} + \frac{(4n^2-1)(4n^2-9)}{2!\,(8t)^2} - \frac{(4n^2-1)(4n^2-9)(4n^2-25)}{3!\,(8t)^3} + \cdots \right\}$$

$$\therefore\ e^{-\beta}\left[I_0(\beta)+I_1(\beta)\right] = \frac{1}{\sqrt{2\pi\beta}}\left\{1 + \frac{1}{8\beta} + \frac{9}{2!(8\beta)^2} + \frac{9.25}{3!(8\beta)^3}\right.$$

$$\left. + 1 - \frac{3}{8\beta} - \frac{3.5}{2!(8\beta)^2} - \frac{3.5.21}{3!(8\beta)^3} \cdots \right\}$$

$$P = \frac{1}{\sqrt{2\pi\beta}}\left\{2 - \frac{2}{8\beta} - \frac{\cancel{8}\,3}{8(8\beta)^2} - \frac{15\overbrace{(21-15)}^{6}}{6.(8\beta)^3}\right.$$

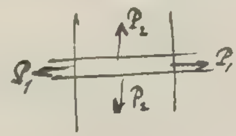
$$= \sqrt{\frac{2}{\pi\beta}}\left\{1 - \frac{1}{8\beta} - \frac{3}{128.\beta^2} - \frac{15}{2.8^3\beta^3}\right.$$

$$= \sqrt{\frac{1}{\alpha\pi}}\left\{1 - \frac{1}{16\alpha} - \frac{3}{4.128.\alpha^2} - \frac{15}{2.8^4.\alpha^3}\right.$$

stimmt

$$= \sqrt{\frac{1}{\alpha\pi}}\left\{1 - \frac{1}{16\alpha} - \frac{3}{2}\left(\frac{1}{16\alpha}\right)^2 - \frac{15}{2}\left(\frac{1}{16\alpha}\right)^3 \cdots\right\} \qquad \alpha = \frac{a^2}{4Dt}$$

$$1 - P = (1-P_1)(1-P_2)$$



$$P = P_1 + P_2 - P_1 P_2$$

$$u = \left(\frac{K(\varphi_1-\varphi_0)}{4\pi}\right)^2 6\pi$$

 $$\frac{\partial V}{\partial s} = \frac{K(\varphi_1-\varphi_0)}{4\pi}\ \frac{3}{2}\, c\, \frac{6\cos\theta}{A^2}$$

$$c = \frac{9}{2}\,\frac{A^2\Delta S\, g}{u}$$

$$v = \frac{K(\varphi_1-\varphi_0)}{4\pi}\,\frac{1}{\mu}\,\frac{\partial V}{\partial s} = \left[\frac{K(\varphi_1-\varphi_0)}{4\pi}\right]^2 \frac{3}{2}\,\frac{6}{A^2}\,\frac{9}{2}\,\frac{A^2\Delta S\, g}{\mu^2}$$

$$= \left[\frac{K(\varphi_1-\varphi_0)}{4\pi}\right]^2 \frac{6}{\mu^2}\cdot g\cdot\Delta S\cdot\frac{27}{4}$$

$$\left[\frac{2}{4\pi\,300}\right]^2 \frac{10^7\ .\ 10^3}{9.10^{11}(0{\cdot}02)^2}\cdot 1{\cdot}2\cdot\frac{27}{4} = \frac{10^4\ .\ 8}{9.10^7.4} = 2.10^{-4}$$

$$V = \alpha\frac{\cos\theta}{r^2} = \alpha\frac{x}{r^3} \qquad X = \alpha\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right) \qquad F = \alpha\left(\frac{1}{r^6} + \frac{3x^2}{r^8}\right)$$

$$\sqrt{Y^2+Z^2} = -\frac{3\alpha\, x\omega}{r^5} \qquad = \frac{\alpha}{r^6}\left(1+3\cos^2\theta\right)$$

$$V = \frac{\alpha}{a^2}\left(x + \frac{a^3 x}{2r^3}\right) \qquad X = \frac{\alpha}{a^2}\left[1 + \frac{a^3}{2r^3} - \frac{3a^3x^2}{2r^5}\right]_{x=a} = 0$$

$$\Omega = -\frac{3\alpha\, x\omega}{2a^4}$$

$$\int\frac{\partial^2}{\partial x^2}(\varphi\, v)\,dx = \int v\frac{\partial^2\varphi}{\partial x^2}dx + 2\underbrace{\int\frac{\partial v}{\partial x}\frac{\partial\varphi}{\partial x}dx}_{\cancel{v\frac{\partial\varphi}{\partial x}} - \int v\frac{\partial^2\varphi}{\partial x^2}dx} + \int\varphi\frac{\partial^2 v}{\partial x^2}dx = 0$$

$$-\int v\,\frac{\partial^2\varphi}{\partial x^2}\,dx + \int\varphi\,\frac{\partial^2 v}{\partial x^2}\,dx = 0$$

$$\frac{1}{4\pi}\int v\,\frac{\partial^2\varphi}{\partial x^2}\,dx = \frac{1}{4\pi}\int\varphi\,\frac{\partial^2 v}{\partial x^2}\,dx$$

$$\int v\,\varepsilon\,dx = -\frac{1}{4\pi\mu}\underbrace{\int\varphi\,\varepsilon\,dx}\cdot\frac{\partial\Phi}{\partial x}$$

$$= +\frac{\frac{\partial\Phi}{\partial x}}{16\pi^2\mu}\underbrace{\int\varphi\frac{\partial^2\varphi}{\partial x^2}dx}_{\varphi\frac{\partial\varphi}{\partial x} - \int\left(\frac{\partial\varphi}{\partial x}\right)^2 dx}$$

$$\mp\left[\frac{K}{4\pi}(\varphi_1-\varphi_0)\right]^2\frac{1}{\delta}\,\frac{1}{\mu}\,\frac{\partial\Phi}{\partial x}$$

$a = 3\mu\mu = 3.10^{-7}$ // $\frac{A}{a} = 300$

$A = 10^{-4}$

 $$\frac{I_6}{I_3} = \frac{2\pi}{\cancel{a}}\left[\frac{K}{4\pi}(\varphi_1-\varphi_0)\right]^2\frac{6}{\mu}\,\frac{1}{\delta}$$

$$\bar{u} = \frac{\beta_2 u_1 + \beta_1 u_2}{\beta_1+\beta_2}$$

~~$n_1 v_1 + n_2 v_2$~~

$$(\beta_2+\beta_1)\bar{u} = \beta_2 u_1 + \beta_1 u_2$$

$$\mathrm{div}\,(n_1 u + n_2 v) = 0$$

$$\cancel{u_1 \frac{\partial n_1}{\partial x} + v_1 \frac{\partial n_1}{\partial y} + w_1 \frac{\partial n_1}{\partial z} + \left(u_2 \frac{\partial n_2}{\partial x} + v_2 \frac{\partial n_2}{\partial y} + w_2 \frac{\partial n_2}{\partial z}\right) + n_1 \,\mathrm{div}\, v_1 + n_2 \,\mathrm{div}\, v_2 = 0}$$

$$u_1 - \bar{u} = -\beta_1 \frac{\partial U}{\partial x}$$

$$u_2 - \bar{u} = \beta_2 \frac{\partial U}{\partial x}$$

$$\mathrm{div}\, v_1 = \mathrm{div}\, \bar{v} + \beta_1 \nabla^2 U$$

$$\mathrm{div}\, v_2 = \mathrm{div}\, \bar{v} + \beta_2 \nabla^2 U$$

$$\rho = (n_1 - n_2)\varepsilon = -\frac{1}{4\pi} \nabla^2 U$$

$$-\rho \nabla U = \mathfrak{f}$$

$$\varepsilon(n_1 v_1 - n_2 v_2) = \underbrace{\varepsilon(n_1 - n_2)\bar{v}}_{\rho\bar{v}} + \underbrace{\varepsilon n_1 (v_1 - \bar{v})}_{-\varepsilon n_1 \beta_1 \nabla U} + \underbrace{\varepsilon n_2 (\bar{v} - v_2)}_{-\varepsilon n_2 \beta_2 \nabla U}$$

$$\mathrm{div}\left[\rho\bar{v} - \underbrace{\varepsilon(n_1\beta_1 + n_2\beta_2)}_{\lambda} \nabla U\right] = 0$$

$$\mathrm{div}[\rho\bar{v} + \bar{c}]$$

wenn überall $\rho = 0$

oder $\bar{v} = 0$

so gilt $\mathrm{div}(\lambda \frac{\partial U}{\partial x}) = 0$

$$\mathrm{curl}\,(\nabla U \,\nabla^2 U)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial U}{\partial x}\cdot \nabla^2 U\right) - \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial y}\cdot \nabla^2 U\right) = \frac{\partial U}{\partial x}\cdot \nabla^2\left(\frac{\partial U}{\partial y}\right) - \frac{\partial U}{\partial y}\cdot \nabla^2\left(\frac{\partial U}{\partial x}\right) \neq 0$$

$$\mu \nabla^2 \bar{v} = \mathfrak{f} - \nabla p$$

$$= -\rho \nabla U - \nabla p$$

$$\mathrm{div}\, \bar{v} = 0$$

$$\mathrm{div}\left[\rho\bar{v} - \varepsilon(n_1\beta_1 + n_2\beta_2)\nabla U\right] = 0$$

~~die φ~~ Modification in der Doppelschicht:

$$\cancel{u_1 - u = -\beta_1 \frac{\partial(\Phi - \Phi)}{\partial x}}$$

$$\left.\begin{aligned} v_1 - \bar{v} &= -\beta_1 \nabla(U - \Phi) \\ \bar{v} - v_2 &= -\beta_2 \nabla(U - \Phi) \end{aligned}\right\} \quad \bar{v} = \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1 + \beta_2}$$

$$\rho = (n_1 - n_2)\, e = -\frac{1}{4\pi} \nabla^2 U$$

$$\cancel{\rho \nabla U = \nabla p}$$

$$\left\{\begin{aligned} &\mu \nabla^2 \bar{v} = -\rho \nabla U - \nabla p \\ &\mathrm{div}\left[\rho \bar{v} - \varepsilon (n_1 \beta_1 + n_2 \beta_2) \nabla(U - \Phi)\right] = 0 \\ &\qquad \mathrm{div}\, \bar{v} = 0 \end{aligned}\right.$$

vielleicht: $= -\rho \nabla(U - \Phi) - \nabla p$ (?)

~~Stat~~ Bewegungslosigkeit nur falls:

$$\rho \nabla U = -\nabla p$$

$$\therefore \mathrm{curl}(\rho \nabla U) = 0$$

$$\frac{\partial U}{\partial x} \frac{\partial (n_1 - n_2)}{\partial y} = \frac{\partial U}{\partial y} \frac{\partial (n_1 - n_2)}{\partial x}$$

d.h. also falls $(n_1 - n_2) =$ const-Flächenschar ~~zu~~ zusammenfällt mit Äquipotentialflächen.

~~Eigentlich ist Alles zu machen durch Betrachtung d. vorstehd. Zustands~~

Falls geschlossenes Gefäss, in welches die Elektr. durch (unauflösliche) metallische Elektr. oder ein- oder austritt, wird n_1 oder n_2 mit der Zeit ganz erschöpft werden, falls nicht Neu-Erzeugung von Ionen. Gleichgewichtszustand erfordert Berücksichtigung der letzteren (sowie der Recombination).

Pro Zeiteinheit werden pro Vol. in ruhendem H_2O γ ± Ionen erzeugt:

Rekombination:

$$\left.\begin{aligned} \frac{dn_1}{dt} &= \gamma - \alpha n_1 n_2 \\ \frac{dn_2}{dt} &= \gamma - \alpha n_1 n_2 \end{aligned}\right\} \text{ohne Feld}$$

Zunahme in Zeit Δt:

$$\Delta t\left[n_{1x}\, u_{1x} - (n_1 u_1)_{x+\Delta x}\right] + (\gamma - \alpha n_1 n_2)\Delta x$$

$$\Delta t \iint n_1 v_{1n}\, dS \;+\; \iiint \gamma \ldots$$

Im stationären Zustand:

$$\operatorname{div}(n_1 v_1) = \gamma - \alpha n_1 n_2 = -\operatorname{div}(n_2 v_2)$$

$$\cancel{v_1 - \bar{v} = v_1 - \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1+\beta_2} = \frac{\beta_1(v_1 - v_2)}{\beta_1+\beta_2} = -\beta_1 \nabla(U - \Phi)}$$

$$v_1 - v_2 = -(\beta_1+\beta_2)\nabla(U - \cancel{\Phi})$$

$$\bar{v} = \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1+\beta_2}$$

$$\rho = (n_1 - n_2)\varepsilon = -\frac{1}{4\pi}\nabla^2(U - \Phi)$$

$$\mu \nabla^2 \bar{v} = -(n_1 - n_2)\varepsilon \cdot \nabla(U - \ldots) \; \nabla p$$

$$\operatorname{div} \bar{v} = 0$$ $\beta_2 \operatorname{div} v_1 = -\beta_1 \operatorname{div} v_2$

$$\cancel{\operatorname{div}\left[\rho\bar{v} - \varepsilon(n_1\beta_1 + n_2\beta_2)\nabla(U - \Phi)\right] = 0}$$ (schon in $\operatorname{div}(n_1 v_1)$ .. enthalten)

$$\operatorname{div}\left[\frac{(n_1 - n_2)(\beta_2 v_1 + \beta_1 v_2)}{\beta_1+\beta_2}\right] = \operatorname{div}\left[(n_1\beta_1 + n_2\beta_2)\nabla(U \ldots)\right]$$

$$\left\{\begin{aligned}
&\operatorname{div}(n_1 v_1) = -\operatorname{div}(n_2 v_2) = p - \alpha\, n_1 n_2 \\
&v_1 - v_2 = -(\beta_1+\beta_2)\,\nabla(U- \\
&n_1 - n_2 = -\frac{1}{4\pi\varepsilon}\,\nabla^2(U-\Phi) \\
&\beta_2 \operatorname{div} v_1 = -\beta_1 \operatorname{div} v_2 \\
&\cancel{\operatorname{div}\left[(n_1-n_2)(\beta_2 v_1+\beta_1 v_2) - (\beta_1+\beta_2)(n_1\beta_1+n_2\beta_2)\nabla(U-\Phi)\right] = 0} \\
&\frac{\mu}{\beta_1+\beta_2}\left[\beta_2\nabla^2 v_1 + \beta_1 \nabla^2 v_2\right] = -\nabla p - (n_1-n_2)\,\varepsilon\cdot\nabla(U-\Phi)
\end{aligned}\right.$$

$$\therefore \operatorname{div} v = \operatorname{div} v_1 + (\beta_1+\beta_2)\nabla^2(U-\Phi) = -\operatorname{div} v_1 \cdot \frac{\beta_2}{\beta_1}$$

$$\therefore \operatorname{div} v_1 = -\beta_1 \nabla^2(U-\Phi)$$

$$\therefore \operatorname{div} v_2 = \beta_2 \nabla^2(U-\Phi)$$

$$\therefore \cancel{\operatorname{div}\left[(n_1-n_2)(\beta_2 v_1+\beta_1 v_2) + (n_1\beta_1+n_2\beta_2)(v_1-v_2)\right]}$$

$$n_1 v_1(\beta_2-\beta_1) + n_1 v_2(\beta_2-\beta_1) + \cancel{n_2 v_1(\beta_2-\beta_1)} + n_2 v_2(-2\beta_2)$$

$$= n_1(v_1+v_2)(\beta_2-\beta_1) - 2 n_2 v_2 \beta_2 \qquad = n_1(\beta_2-\beta_1)v_1$$

$$= n_1 v_1(\beta_2+\beta_1) - n_2 v_2(\beta_1+\beta_2) + n_1 \cancel{v_2(\beta_1-\beta_2) + n_2 v_1(-\beta_2+\beta_1)}$$

Wenn Erzeugung und Rekombination vernachlässigt wird (z.B. $CuSO_4$ Ionen zwischen Cu Elektroden)

$$\left.\begin{aligned}
&\operatorname{div}(n_1 v_1) = 0 = \operatorname{div}(n_2 v_2) = 0 \\
&\operatorname{div} v_1 = -\beta_1 \nabla^2(U- \\
&\operatorname{div} v_2 = \beta_2 \nabla^2(U- \\
&n_1 - n_2 = -\frac{1}{4\pi\varepsilon}\nabla^2(U-\Phi)
\end{aligned}\right\} \quad v_1 - v_2 = -(\beta_1+\beta_2)\nabla U \quad \text{(nicht erfüllbar!)}$$

$$\nabla p = (n_2-n_1)\,\varepsilon\cdot\nabla U - \frac{\mu}{\beta_1+\beta_2}\left[\beta_1 \nabla^2 v_2 + \beta_2 \nabla^2 v_1\right]$$

Unter vereinfachender Annahme dass $\mathfrak{I}_1 = \mathfrak{I}_2$:

$$n_1 \operatorname{div} v_1 + u_1 \frac{\partial n_1}{\partial x} + v_1 \frac{\partial n_1}{\partial y} + w_1 \frac{\partial n_1}{\partial z} = 0$$

$\operatorname{div} v_1 = - \operatorname{div} v_2 =$

$$n_2 \operatorname{div} v_2 + u_2 \frac{\partial n_2}{\partial x} + v_2 \frac{\partial n_2}{\partial y} + w_2 \frac{\partial n_2}{\partial z} = 0$$

$(n_1 - n_2)\, d$ An Stellen wo $\bar{\Phi} = 0$, falls: $v_1 = - v_2$

$$\operatorname{div} v_1 = - \operatorname{div} v_2 = 4\pi\varepsilon\beta(n_2 - n_1)$$

$$(n_1 - n_2) \operatorname{div} v_1 + u_1 \frac{\partial(n_1 - n_2)}{\partial x} + v_1 \frac{\partial(n_1 - n_2)}{\partial y} + w_1 \frac{\partial}{\partial z}{}' = 0$$

$$4\pi\varepsilon\beta (n_1 - n_2)^2 = \left(u_1 \frac{\partial}{\partial x} + v_1 \frac{\partial}{\partial y} + w_1 \frac{\partial}{\partial z}\right)(n_1 - n_2)$$

$n_1 = n_2$

$$\left(u_1 \frac{\partial}{\partial x} + v_1 \frac{\partial}{\partial y} + w_1 \frac{\partial}{\partial z}\right)\left(\frac{1}{n_1 - n_2}\right) = - 4\pi\varepsilon\beta = \text{const}$$

An Stellen wo $\bar{\Phi} = 0$ gibt es eine mögliche Lösung:

$n_1 = n_2 = \text{const}$ $\qquad \operatorname{div} v_1 = \operatorname{div} v_2 = 0 \qquad \nabla p = 0$

vorausgesetzt, dass diese Bedingungen an den Grenzen des betrachteten Raumes erfüllt sind!

Dabei braucht aber $v_1 \gtrless v_2$ sein

$$\frac{\cancel{x}}{\cancel{\beta_1+\beta_2}}\left[\beta_1 \alpha_2 c - \beta_2 \alpha_1 c\right] = - \frac{1}{4\pi}\frac{(\varphi_i - \varphi_a)}{\mu} \frac{\alpha_1 + \alpha_2}{\cancel{\beta_1+\beta_2}}$$

$v_{1\infty} = - \alpha_1 c + \gamma$
$v_{2\infty} = \alpha_2 c + \gamma$

$$\nabla_\infty U = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} c = \frac{v_{2\infty} - v_{1\infty}}{\beta_1 + \beta_2} c$$

Durch Integration über Grenzschichte:

$$\frac{\mu}{\beta_1+\beta_2}\left[\beta_1 v_{2s}+\beta_2 v_{1s}\right]=-\frac{1}{4\pi}(\varphi_1-\varphi_2)\frac{\partial U}{\partial s}$$ | für Kugeloberfläche

Im Aussenraum: $\nabla^2 U=0$ $\qquad n_1=n_2$ $\qquad$ || $v_1=v_2=c$ für $r=\infty$

$$\operatorname{div} v_1=\operatorname{div} v_2=0$$

$$\nabla p=-\frac{\mu}{\beta_1+\beta_2}\left[\beta_1\nabla^2 v_2+\beta_2\nabla^2 v_1\right]$$

$$v_1-v_2=-(\beta_1+\beta_2)\nabla U$$

$$\therefore \nabla^2 p=0$$

$$v_n\,\Delta x-\int(v_s-v_s')\,dy=0$$

$$v_{1n}=v_{2n}=0 \text{ für Kugeloberfläche?}$$

$$v_n=\frac{\partial}{\partial x}\int v_s\,dy$$

Annahme:

Stokes

$$v_1=-\nabla\left[\alpha_1\, cx\left(1+\frac{a^3}{2r^3}\right)\right]+\gamma_1 v_0$$

$$v_2=\nabla\left[\alpha_2 \qquad \text{''} \qquad\right]+\gamma_2 v_0$$

$$u_1=-\alpha_1 c\left(1+\frac{a^3}{2r^3}-\frac{3x^2a^3}{2r^5}\right)$$

$$v_1=-\alpha_1 c\left(-\frac{3xya^3}{2r^5}\right)$$

$$w_1=-\alpha_1 c\left(-\frac{3xza^3}{2r^5}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) \qquad \frac{\partial^2}{\partial x\partial y} \qquad \frac{\partial^2}{\partial x\partial z}$$

$$\nabla^2 v_1=\gamma_1\nabla^2 v_0=\ldots$$

$$p=-\frac{\mu}{\beta_1+\beta_2}\,\frac{3}{2}\,\frac{ax}{r^3}\left[\beta_2\gamma_1+\beta_1\gamma_2\right] \longleftarrow =-\frac{3}{2}\mu\frac{ax}{r^3}\gamma$$

$$-\nabla\left[(\alpha_1+\alpha_2)\,cx\left(1+\frac{a^3}{2r^3}\right)\right]+(\gamma_1-\gamma_2)v_0=-(\beta_1+\beta_2)\nabla U \qquad \| \ \gamma_1=\gamma_2$$

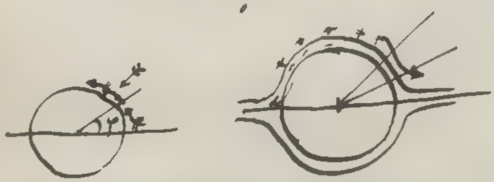
$$U=\frac{\alpha_1+\alpha_2}{\beta_1+\beta_2}\,cx\left(1+\frac{a^3}{2r^3}\right)$$

$\mu \frac{\partial^2 v_\xi}{\partial \xi^2} = \frac{K}{4\pi}\frac{\partial \Phi}{\partial \xi}\cdot\frac{\partial^2\varphi}{\partial\xi^2}$

$$v_\xi = \frac{K}{4\pi\mu}\frac{\partial\Phi}{\partial\xi}(\varphi-\varphi_a)$$

Oberflächen-
Konvektionsstrom: $i_\xi = \int_0^\infty \varepsilon\, v_\xi\, d\xi = -\left(\frac{K}{4\pi}\right)^2 \frac{1}{\mu}\frac{\partial\Phi}{\partial\xi}\underbrace{\int_0^\infty \frac{\partial^2\varphi}{\partial\xi^2}(\varphi-\varphi_a)\,d\xi}$

$$= \int_0^\infty \left(\frac{\partial\varphi}{\partial\xi}\right)^2 d\xi \qquad = A\,\frac{\partial\Phi}{\partial\xi}$$



$$A = \left(\frac{K}{4\pi}\right)^2\frac{1}{\mu}\int_0^\infty\left(\frac{\partial\varphi}{\partial\xi}\right)^2 d\xi$$

Kontinuitätsgleichung: $2\pi a^2\sin\varphi\,d\varphi\cdot i_n = d\left(2\pi a\sin\varphi\; i_\xi\right)$

$$i_n = \frac{1}{a\sin\varphi}\frac{\partial}{\partial\varphi}\left(i_\xi\sin\varphi\right)$$

$$= \frac{A}{a\sin\varphi}\,\frac{3}{2}c\,\frac{\partial}{\partial\varphi}\left(\sin^2\varphi\right)$$

$$i_n = \frac{3Ac}{a}\cos\varphi$$

$$\Phi = -c\cos\varphi\left(r+\frac{a^3}{2r^2}\right)$$

$$\left.\frac{\partial\Phi}{\partial\xi}\right|_{r=a} = \frac{3}{2}c\sin\varphi$$

Es ist dann noch zu untersuchen ob die übrig bleibenden v_ξ nicht einen Konvektionsstrom begleitenden Strömungen ergeben!

Das entspricht einer ~~Potent~~ superponierten Potentialverteilung:

$$\Phi'_{\varphi} = \cdot\, b\cos\varphi\,\frac{a^3}{2r^2}$$

$$i_n = -\frac{1}{\sigma}\left.\frac{\partial\Phi'}{\partial n}\right|_\varphi = +\frac{3b}{\sigma}\cos\varphi = \frac{3Ac}{a}\cos\varphi$$

$$\therefore\; b = \frac{Ac\sigma}{a} = \left(\frac{K}{4\pi}\right)^2\frac{\sigma c}{\mu a}\int_0^\infty\left(\frac{\partial\varphi}{\partial\xi}\right)^2 d\xi$$

$\Phi' =$

Ausrechnen der ~~...~~ radiale Komponente der Geschw.:

$$v_\xi = -\frac{K(\varphi_i-\varphi_a)}{4\pi\mu}\, c \cos\varphi \underbrace{\left(1-\frac{a^3}{r^3}\right)}_{\left[1-\frac{1}{(1+\frac{\xi}{a})^3}\right] \neq \frac{3\xi}{a}} \qquad r = a+\xi$$

$\mathrm{div}(\varepsilon v) = v_\xi \frac{\partial\varepsilon}{\partial\xi}$

pro dS: $\lambda\frac{\partial\psi}{\partial n} = \int_0^\infty v_\xi \frac{\partial\varepsilon}{\partial\xi}\,d\xi = \int v_\xi \frac{\partial^3\varphi}{\partial\xi^3}\,d\xi =$

$= \int \frac{\partial v}{\partial\xi}\frac{\partial^2\varphi}{\partial\xi^2}\,d\xi = \int\frac{\partial^2 v}{\partial\xi^2}\frac{\partial\varphi}{\partial\xi}$

Dieselbe trägt aber zum konvektiven Radialstrom nicht bei, falls Fläche ausserhalb Doppelschicht, wo $\varepsilon = 0$.

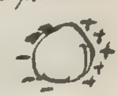
Somit im äusseren Raum:

$$\Sigma\Phi = -c\cos\varphi\left(r+\frac{a^3}{2r^2}\right) + \underbrace{\frac{a^2 c\cos\varphi}{2r^2}\cdot\left(\frac{K}{4\pi}\right)^2\frac{6}{\mu}\int_0^\infty\left(\frac{\partial\varphi}{\partial\xi}\right)^2 d\xi}$$

kann aufgefasst werden als Potentialänderung infolge Oberflächenleitung

desto grössere je grösser σ!

also so als ob noch Superposition einer Oberflächenladung



Kräfte solcher Dipole aufeinander

$-+ \qquad -+$

$$W = \frac{E^2}{4}\left(\frac{x}{r^3}-\frac{x'}{r'^3}\right) = -E^2\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = -E^2\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right)$$

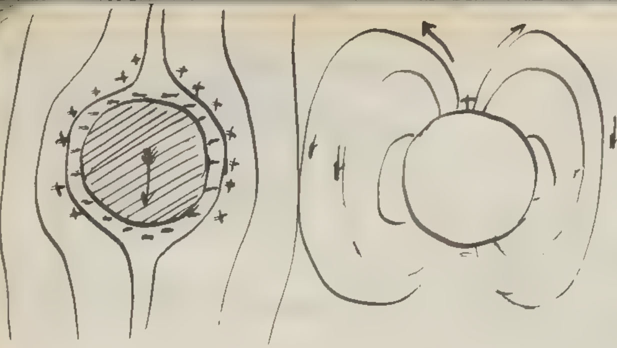
also bei Bewegung in X-Axe: $W = \frac{2E^2}{x^3}$

$F_x = -6\frac{E^2}{x^4}$

Anziehung bei Berührung:

$$6\left[\left(\frac{K}{4\pi}\right)^2\frac{6}{\mu}\right]^2 c^2 \frac{\frac{a_1^2}{2}\cdot\frac{a_2^2}{2}}{(a_1+a_2)^4}\left[\int_0^\infty - \right]^2$$

$$\left(\frac{K}{4\pi}\right)^2\frac{6}{\mu}\frac{(\varphi_i-\varphi_a)^2}{d} = \left(\frac{4}{300\cdot 4\pi}\right)^2\frac{10^7}{9\cdot10^{11}\cdot 0.02\cdot 5\cdot10^{-7}} = \frac{10^{-6}\cdot10^{14}}{9\cdot10^{10}} = \frac{1}{900}$$



In erster Annäherung $\varepsilon = -\frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\xi^2}$

aber es wird eben durch die Ladung geändert!

$$\frac{1}{1+x} = 1 - x + x^2 \ldots$$

$$\frac{1}{(1+x)^3} = 1 - 3x + \underbrace{\frac{-3\cdot -4}{1\cdot 2}}_{+6x^2}$$

Einfluss der Tangentialströmung:

Strömende Flüssigk. in der Nähe der Kugel: (gegenüber ruhender Kugel)

$$u = c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right) - \frac{3}{4}\frac{c a x^2}{r^3}\left(1 - \frac{a^2}{r^2}\right)$$

$$\left.\begin{aligned} v &= -\frac{3}{4}\frac{c a x y}{r^3}\left(1 - \frac{a^2}{r^2}\right) \\ w &= \end{aligned}\right\} v_\omega = -\frac{3}{4}\frac{c a x \omega}{r^3}\left(1 - \frac{a^2}{r^2}\right)$$

$$v_r = \frac{ux + v\omega}{r} = c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)\cos\varphi - \frac{3}{4}\frac{ca}{r}\left(1 - \frac{a^2}{r^2}\right)\cos\varphi = c\left(1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)\cos\varphi$$

$$v_{r\varphi} = -\frac{u\omega + vx}{r} = -c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)\sin\varphi + \cancel{\frac{3}{4}\frac{ca}{r}\left(1 - \frac{a^2}{r^2}\right)\cos^2\varphi\sin\varphi}$$

$$= -c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)\sin\varphi$$

In Entfernung ξ: $r = a + \xi$

$$v_r = c\cos\varphi\left(1 - \frac{3}{2}\left(1 + \frac{\xi}{a}\right)^{-1} + \frac{1}{2}\left(1 + \frac{\xi}{a}\right)^{-3}\right) = c\cos\varphi\left[\frac{3}{2}\left(\frac{\xi}{a} - \frac{\xi^2}{a^2}\right) - \frac{1}{2}\left(3\frac{\xi}{a} - 6\frac{\xi^2}{a^2}\right)\right]$$

$$= +\frac{3}{2}c\cos\varphi\cdot\frac{\xi^2}{a^2}$$

$$v_{r\varphi} = -c\sin\varphi\left(1 - \frac{3}{4}\left(1 + \frac{\xi}{a}\right)^{-1} - \frac{1}{4}\left(1 + \frac{\xi}{a}\right)^{-3}\right) = -c\sin\varphi\left[-\frac{3}{4}\left(\frac{\xi}{a} - \frac{\xi^2}{a^2}\right) - \frac{1}{4}\left(3\frac{\xi}{a} - 6\frac{\xi^2}{a^2}\right)\right] =$$

$$= \frac{3}{2}c\frac{\xi}{a}\sin\varphi$$

$$p = \frac{3}{2}\frac{c\mu a\cos\varphi}{r^2}$$

$$\left.\frac{\partial p}{\partial r}\right|_{r=a} = -3\frac{c\mu\cos\varphi}{a^2}$$

$$\operatorname{div}(\varepsilon\mathfrak{v}) = -\frac{1}{2\pi r^2 \sin\varphi}\frac{\partial}{\partial\varphi}\,2\pi\left[r\sin\varphi\,\varepsilon\cdot v_\varphi\right] + \underbrace{\frac{1}{r^2}\frac{\partial (v_r\, r^2)}{\partial r}}_{\frac{\partial v_r}{\partial r}+\frac{2}{r}v_r}$$

$$= -\frac{1}{r}\left(\frac{\partial v_\varphi}{\partial\varphi} + \frac{\cos\varphi}{\sin\varphi}v_\varphi\right) + \frac{\partial v_r}{\partial r} + \frac{2}{r}v_r$$

$$= \underbrace{-\frac{3c\psi}{2r}\left[\frac{\xi}{a}\cos\varphi + \frac{\cos\varphi}{\sin\varphi}\sin\varphi\,\frac{\xi}{a}\right]}_{-\frac{3c}{a}\frac{\xi}{a}\cos\varphi} + \frac{3c}{a}\cos\varphi\frac{\xi}{a^2} = 0$$

Solange ε nur als Funktion ξ angesehen wird, solange also die Dicke der Doppelschicht nicht ungleichförmig wird, kommt für div (ε𝔳) nur die Normalkomponente v_r in Betracht

$$\operatorname{div}(\varepsilon\mathfrak{v}) = u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

$$= v_r\frac{\partial\varepsilon}{\partial r}$$

also bleibt frühere Theorie ganz streng, solange Dichte d. Doppelsch. konzentrisch geschichtet, also Rückwirkung d. Bewegung auf ε unberücksichtigt

---

$U = V - \varphi$ ← eingeprägtes Potential

$$\varepsilon = -\frac{1}{4\pi}\nabla^2 V = -\frac{1}{4\pi}\nabla^2(U+\varphi)$$

$$U = \frac{\varepsilon}{4\pi}\iiint\frac{1}{r}\left(u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}\right)d\omega$$

$$\frac{\partial\varepsilon}{\partial\xi} = -\frac{1}{4\pi}\nabla^2\left[\frac{\partial(\varphi+U)}{\partial\xi}\right]$$

$$\frac{\partial\varepsilon}{\partial\eta} = -\frac{1}{4\pi}\nabla^2\left[\frac{\partial U}{\partial\eta}\right]$$

$$U+\varphi = \iiint\frac{\varepsilon}{r}\,d\omega$$

Korrekte Behandlung, wenn tangentiale Variabilität des ε berücksichtigt

$\nabla\varphi$ = eingeprägte Kraft, welche die gestörte Doppelschicht wiederherstellt

V = ~~[illegible]~~ Gesamt Potential; im Normalzustand $V = \varphi$

$U = V - \varphi$ = Pot. der Zusatzkräfte, welche Leitungsstrom erzeugen

$$\operatorname{div}\left[\frac{1}{\sigma}\nabla U + \varepsilon v\right] = 0$$

(Vernachlässigung der Unterschied in σ ist das in der Doppelschicht erlaubt?

gewiss nicht denn es gilt:

$$\lambda = \frac{1}{\sigma} = \underline{\underline{e}}(n_1\beta_1 + n_2\beta_2)\,!$$

$$\frac{1}{\sigma}\nabla^2 U = -u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

$$\varepsilon = -\frac{1}{4\pi}\nabla^2 V = -\frac{1}{4\pi}\nabla^2(U+\varphi)$$

$$+\frac{4\pi\varepsilon}{\sigma} + \frac{\nabla^2\varphi}{\sigma} = u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

$$\varepsilon - \frac{\sigma}{4\pi}\left[v_n\frac{\partial\varepsilon}{\partial\zeta} + v_\xi\frac{\partial\varepsilon}{\partial\xi}\right] = -\frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\zeta^2}$$

I). Näherungswert:

$$\varepsilon = -\frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\zeta^2} - \frac{\sigma}{4\pi}\left[v_n\frac{\partial\varepsilon}{\partial\zeta} + v_\xi\frac{\partial\varepsilon}{\partial\xi}\right]$$

$$= -\frac{1}{4\pi}\left[\frac{\partial^2\varphi}{\partial\zeta^2} - \frac{\sigma}{4\pi}\left[v_n\frac{\partial^3\varphi}{\partial\zeta^3}\right]\right]$$

II) Näherung:

$$\varepsilon = -\frac{1}{4\pi}\left[\frac{\partial^2\varphi}{\partial\zeta^2} - \frac{\sigma}{4\pi}\left[v_n\left(\frac{\partial^3\varphi}{\partial\zeta^3} - \frac{\sigma}{4\pi}\frac{\partial}{\partial\zeta}\left(v_n\frac{\partial^3\varphi}{\partial\zeta^3}\right)\right) + v_\xi\cdot\frac{\sigma}{4\pi}\cdot\frac{\partial v_n}{\partial\xi}\frac{\partial^3\varphi}{\partial\zeta^3}\right]\right];$$

Ordnung: ξ^2 $\qquad \xi^3$ $\qquad \xi^2$

$$\therefore\quad \varepsilon = -\frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\zeta^2} + \frac{\sigma}{(4\pi)^2}\left[v_n\frac{\partial^3\varphi}{\partial\zeta^3} - \frac{\sigma}{4\pi}v_\xi\frac{\partial v_n}{\partial\xi}\frac{\partial^3\varphi}{\partial\zeta^3}\right]$$

$\frac{3}{2}c\cos\varphi\,\frac{\xi^2}{a^2}$ $\qquad \frac{3}{2}c\sin\varphi\cdot\frac{\xi}{a}$ $\qquad \frac{3}{2}c\cos\varphi\,\frac{\xi}{a^2}$

$$4\pi\varepsilon = -\frac{\partial^2 \varphi}{\partial\xi^2} + \frac{6}{4\pi}\,\frac{3c}{2}\,\frac{\cos\varphi}{a^2}\,\xi^2\frac{\partial^3\varphi}{\partial\xi^3} - \frac{6^2}{(4\pi)^2}\,\frac{9}{4}\,\frac{c^2\sin\varphi\cos\varphi}{a^3}\,\xi^2\frac{\partial^3\varphi}{\partial\xi^3}$$

$$\cancel{4\pi\int_0^\infty \varepsilon\, d\xi = \frac{6}{4\pi}\,\frac{3c\cos\varphi}{2a^2}(\varphi_i-\varphi_a) - \left(\frac{6}{4\pi}\right)^2 \frac{9c^2\sin\varphi\cos\varphi}{4a^3}\underbrace{\int_0^\infty \xi\,\frac{\partial\varphi}{\partial\xi}\,d\xi}_{=\int_0^\infty \varphi\, d\xi}}$$

~~wird ∞, also ist eine Entwicklung nach ξ nicht gestattet~~

~~Man muss die Form beibehalten:~~

$$\cancel{4\pi\int_0^\infty \varepsilon\, d\xi = \frac{6}{4\pi}\,\frac{3c\cos\varphi}{2a^2}(\varphi_i-\varphi_a) - \frac{6}{4\pi}^2\int_0^\infty v_\xi\,\frac{\partial v_r}{\partial\xi}\,\frac{\partial^2\varphi}{\partial\xi^2}\,d\xi}$$

$$4\pi\int_0^\infty \varepsilon\, d\xi = \frac{6}{4\pi}\cdot\frac{3c\cos\varphi}{a^2}\Big[1-\underbrace{\frac{6}{4\pi}\,\frac{3c\sin\varphi}{2a}}\Big](\varphi_i-\varphi_a) =$$

Korrektionsglied im Allgemeinen unbedeutend

Bei Vernachlässigung desselben

$$\frac{6}{4\pi}(\varphi_i-\varphi_a)\,\underbrace{\frac{3c\cos\varphi}{a^2}}_{\frac{1}{\mu}\frac{\partial p}{\partial\xi}\Big|_{r=a}} = \frac{\partial U}{\partial\xi}\Big|_{r=a} = \cancel{\frac{6}{4\pi}(\varphi_i-\varphi_a)\frac{3\,x}{2\,r^3}}\Big|_{r=a}$$

$$U = \frac{6}{4\pi}\,\frac{(\varphi_i-\varphi_a)}{\mu}\,p$$

Das alles unter Voraussetzung dass die Änderungen in der ursprünglichen Sinkgeschwindigkeit v verschwindend gering sind und dass Leitfähigkeits Unterschiede vernachlässigt werden.

Wenn Leitfähigkeits Unterschiede berücksichtigt werden sollen:

sie beschränken sich natürlich auf den Bereich der Doppelschicht, wo $\varepsilon \gtrless 0$

$$\left\{\begin{aligned} v_1 - \bar{v} &= -\beta_1 \nabla(\overbrace{V - \Phi}^{U = \text{Potential für Leitungsstrom}}) \\ \bar{v} - v_2 &= -\beta_2 \nabla(V - \Phi) \end{aligned}\right.$$

wobei Φ die aufgeprägten Kräfte angibt, welche die Doppelschichtladung bewirken

$$\varepsilon = (n_1 - n_2) e = -\frac{1}{4\pi} \nabla^2 V$$

Gesamtstrom: $$i = e(n_1 v_1 - n_2 v_2) = e(n_1 - n_2)\bar{v} + e n_1(v_1 - \bar{v}) + e n_2(\bar{v} - v_2)$$

$$= \varepsilon \bar{v} - \underbrace{e(n_1\beta_1 + n_2\beta_2)\nabla(V - \Phi)}_{\text{als } \frac{\partial U}{\partial x}}$$

$$\bar{v} = \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1 + \beta_2}$$

$$\boxed{\operatorname{div} i = 0}$$

$$\therefore \operatorname{div}(\varepsilon \bar{v}) = \operatorname{div}\left[e(n_1\beta_1 + n_2\beta_2)\nabla(V - \Phi)\right]$$

$$= \frac{1}{\beta_1 + \beta_2}\left[\beta_2 \operatorname{div} v_1 + \beta_1 \operatorname{div} v_2\right]$$

$$\cancel{= e[\operatorname{div} v_1 \bar{v}] = e \operatorname{div}[(n_1 - n_2)\bar{v}]}$$

$$\frac{\beta_1(v_1 - v_2)}{\beta_1 + \beta_2} = -\beta_1 \nabla(V - \Phi)$$

$$v_1 - v_2 = -(\beta_1 + \beta_2)\nabla(V - \Phi)$$

$\operatorname{div} \bar{v} = 0$ also $\cancel{\beta_1 \operatorname{div} v_1 = -\beta_1 \operatorname{div} v_2}$

$$\left\{\begin{aligned} \operatorname{div} v_1 &= -\beta_1 \nabla^2(V - \Phi) = \beta_1\left[4\pi\varepsilon + \nabla^2\Phi\right] \\ \operatorname{div} v_2 &= \beta_2 \nabla^2(V - \Phi) = \beta_2\left[4\pi\varepsilon - \nabla^2\Phi\right] \end{aligned}\right.$$

Annahme dass keine Ionen neu erzeugt werden, also: $\operatorname{div}(n_1 v_1) = \operatorname{div}(n_2 v_2) = 0$

$$\therefore \quad \operatorname{div}(n_1 \bar{v}) = \beta_1 \operatorname{div}\left[n_1 \nabla(V-\Phi)\right]$$

$$\operatorname{div}(n_2 \bar{v}) = -\beta_2 \operatorname{div}\left[n_2 \nabla(V-\Phi)\right]$$

$$\beta_2 \operatorname{div}(n_1 v_1) + \beta_1 \operatorname{div}(n_1 v_2) = \beta_1(\beta_1+\beta_2) \operatorname{div}\left[n_1 \nabla(V-\Phi)\right]$$

Dann gelten als unabhängige Gleichungen für Bestimmung der: $n_1\ n_2\ v_1\ v_2\ V$

$$\left\{\begin{array}{l} v_1 - \bar{v} = -\beta_1 \nabla(V-\Phi) \\ \bar{v} - v_2 = -\beta_2 \nabla(V-\Phi) \\ n_1 - n_2 = -\frac{1}{4\pi e}\nabla^2 V \\ \operatorname{div} n_1 v_1 = 0 \\ \operatorname{div} n_2 v_2 = 0 \end{array}\right.$$

falls $\bar{v}$ als gegeben angesehen wird

sonst kommt dann noch $\operatorname{div} \bar{v} = 0$

und die mechanische Druckgleichung:

$$\nabla p = (n_2 - n_1) e \nabla(V-\Phi) - \underbrace{\frac{\mu}{\beta_1+\beta_2}\left[\beta_1 \nabla^2 v_2 + \beta_2 \nabla^2 v_1\right]}_{\mu \nabla \bar{v}}$$

$$n_1 \operatorname{div} v_1 + u_1 \frac{\partial n_1}{\partial x} + v_1 \frac{\partial n_1}{\partial y} + w_1 \frac{\partial n_1}{\partial z} = 0$$

$$\left.\begin{array}{l} +\beta_1 \nabla^2 U = u_1 \frac{\partial \log n_1}{\partial x} + v_1 \frac{\partial \log n_1}{\partial y} + w_1 \frac{\partial \log n_1}{\partial z} \\ -\beta_2 \nabla^2 U = u_2 \frac{\partial \log n_2}{\partial x} + v_2 \frac{\partial \log n_2}{\partial y} + w_2 \frac{\partial \log n_2}{\partial z} \end{array}\right\}$$

jedenfalls ist $\left.\frac{\partial U}{\partial \xi}\right|_{r=a} = 0$

$$\cancel{u \frac{\partial}{\partial x}(\beta_2 \log n_1 + \beta_1 \log n_2) + v \frac{\partial}{\partial y}(\beta_2 \log n_1 + \beta_1 \log n_2) + w \frac{\partial}{\partial z} \cdots = 0}$$

$$\cancel{n_1 v_1 - n_2 v_2 =} -(\beta_1 n_1 + n_2 \beta_2) \nabla(V-\Phi) \cancel{+ (n_1 - n_2)\bar{v}}$$

$$\operatorname{div}(n_1 \bar{v}) = -\beta_1 \operatorname{div}\left[n_1 \nabla(\overbrace{V-\Phi}^{U})\right] = \bar{u} \frac{\partial n_1}{\partial x} + \bar{v} \frac{\partial n_1}{\partial y} + \bar{w} \frac{\partial n_1}{\partial z}$$

Grössenordnung von: $n_1 - n_2 = -\frac{K}{4\pi e}\frac{\partial^2\varphi}{\partial n^2} \;\#\; -\frac{K(\varphi_i-\varphi_a)}{4\pi e\, d^2}$

$$= \frac{4}{300\cdot 4\pi\cdot 4\cdot 7\cdot 10^{-10}\cdot(4\cdot 10^{-7})^2} = \frac{10^{22}}{3\cdot\pi\cdot 4\cdot 7\cdot 16} = \frac{4}{3}\,10^{21} \;!!$$

Also Ionenzahl in der Doppelschicht kolossal gross! pro cm²

$$D = \frac{\rho-\rho'}{\rho}\;\frac{16\pi^2\mu g}{6\cdot s}$$

$$\frac{19.8\cdot 16\cdot\pi^2\cdot 0.02\cdot 9\cdot 10^{11}}{23450\cdot 4.29\cdot 10^{-7}} \qquad \epsilon_{\text{elektrost.}} = \frac{4\cdot 29\cdot 10^{-7}}{9\cdot 10^{11}}$$

$$= \frac{1.98\cdot 1.6\cdot\pi^2\cdot 2}{2.345\cdot 4.29}\cdot 9 \mp 59$$

$A = 0.0063$ $\qquad E = \frac{59}{4\pi\cdot 0.0063} = 700$ (elektrost.) $= \frac{A}{d} = \frac{K(\varphi_i-\varphi_a)}{4\pi d}$

$$A = \frac{K(\varphi_i-\varphi_a)}{4\pi}$$

$$l = \frac{6.3^2\cdot 10^{-6}}{59} = 7.3\cdot 10^{-7}$$

Dagegen Anzahl d. Ionen pro cm² der Grenzschichte: $\frac{4}{3}\cdot 10^{21}\cdot 4\cdot 10^{-7} = 5.3\cdot 10^{14}$

Somit mittl. Abstand in der Ebene $\frac{10^{-7}}{\sqrt{5.3}}$ von derselben Grössenordnung wie Dicke der Dop-

Es ist eine wirklich „eine ionige" Schichte!

allerdings kolossal angereichert im Vergleich zum Flüssigkeits-Innern

Jedenfalls ist also $n_1 - n_2$ von höherer Ordnung als n_1 und n_2 im Innern d. Flüssigkeit

---

Im Falle lamellarer Strömung längs ebener Wand (im constanten Gefälle):

$n_1 = f_1(\xi) \quad n_2 = f_2(\xi)$

$V = ax + \varphi(\xi)$ $\qquad\Big\|\qquad$ $v_1 - \bar v = -\beta_1 a$

$\Phi = \varphi(\xi)$ $\qquad\Big\|\qquad$ $\bar v - v_2 = -\beta_2 a$

$p = 0$

$$(n_2 - n_1) e\, a = \frac{\mu}{\beta_1+\beta_2}\left[\beta_1 \frac{\partial^2 v_2}{\partial\xi^2} + \beta_2 \frac{\partial^2 v_1}{\partial \xi^2}\right]$$

$$= \mu \frac{\partial^2 \bar v}{\partial \xi^2}$$

$$n_1 - n_2 = -\frac{1}{4\pi e}\frac{\partial^2 V}{\partial\xi^2} = -\frac{1}{4\pi e}\frac{\partial \varphi}{\partial \xi^2} \qquad \bar v = -\frac{a}{4\pi\mu}(\varphi - \varphi_0)$$

$$v_1 = -\beta_1 a - \frac{(\varphi-\varphi_0)a}{4\pi\mu} \qquad \bar v_i = -\frac{a}{4\pi\mu}(\varphi_i - \varphi_0)$$

$$v_2 = \beta_2 a - \frac{(\varphi - \varphi_0)a}{4\pi\mu}$$

Alles in Übereinstimmung mit gewöhnl. Theorie

Also Komplikationen erst bei Strömung längs gekrümmter Flächen

---

$$\Psi = 2\pi r^2\int_0^\varphi \sin\varphi\, d\varphi \cdot \frac{\partial \Phi}{\partial r} = 2\pi r^2 c\left(1 - \frac{a^3}{r^3}\right)\int_0^\varphi \sin\varphi\cos\varphi\, d\varphi = 2\pi c\, r^2\left(1-\frac{a^3}{r^3}\right)\frac{\sin^2\varphi}{2}$$

$$= \pi c\, \omega^2\left(1 - \frac{a^3}{r^3}\right)$$

$$\cancel{\Delta\Psi_{r\,\mathrm{const}} = 2\omega\pi c\left(\frac{1}{r} - \frac{a^3}{r}\right)\Delta\omega = 2\pi c \sin\varphi\left(1 - \frac{a^3}{r^3}\right)\Delta\omega}$$

Für $\varphi = \frac{\pi}{2}$: $\quad \cancel{\Delta\psi = \pi c\left(1 + \frac{2a^3}{r^3}\right)\Delta r \Big|_{r=a} = \frac{3}{2}\pi c \Delta r}$

$$\psi = \pi c\left(r^2 - \frac{a^3}{r}\right)$$

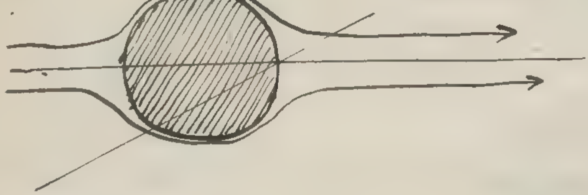
$$\Delta\psi = \pi c\left(2r + \frac{a^3}{r^2}\right)\Delta r\Big|_{r=a} = \frac{3}{2} a\pi c\, \Delta r \quad = \pi c \omega^2$$

Für r = const $\quad \Delta\psi = 2\pi c\,\omega\,\Delta\omega\left(1 - \frac{a^3}{r^3}\right)\Big|_{r=\infty} = 2\pi c\,\omega\,\Delta\omega$

also  $\quad \varphi_2 = \cancel{2a\pi\Delta r \frac{3}{2}} = \frac{3}{4}\varphi_1$

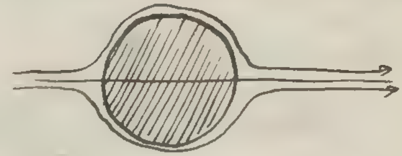
Im Fall der Elektrosmose:

Elektr. Stromlinien richtiger $\mathfrak{v}_1$ Linien



Unmöglich

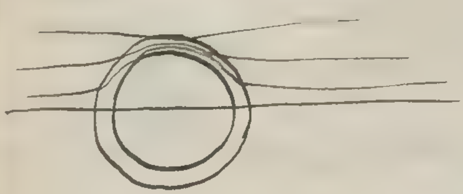
$\bar{\mathfrak{v}}$ Linien



In ∞ Abstand muss das Feld ungestört sein, also Querschnitt einer tangierenden Strömungsröhre von derselben Grössenordnung wie [illegible] Querschnitt durch Doppelschicht, daher müsste auch er in der ganzen Länge derselben von gleicher Grössenordnung sein!

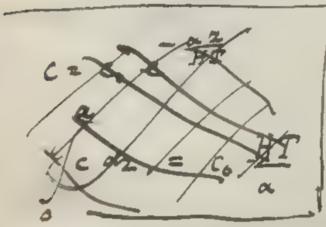
Wenn nicht so muss man $\mathrm{div}(n, \mathfrak{v}_1) = 0$ aufgeben!

Andere Möglichkeit: dass das Feld in der (Doppelschicht u. der) Randgegend entsprechend geschwächt ist. ~~st.~~ Dasselbe wäre der Fall, wenn ~~st.~~ anstatt Doppelsch. eine Schichte besser leitenden Materials vorhanden wäre. Auch ~~[illegible]~~ dann würde überall $\mathrm{div}(n, \mathfrak{v}_1) = 0$ sein.



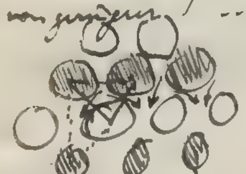
Daneben

φ	0.09	0.24	0.33	0.53	0.66	1.05	2.11
$\Delta\mu \cdot 10^5$	2	7 (22)	9	13 (24)	17 (38)	28	58 (148)
	2.2	2.9	2.7	2.5	2.6	2.7	2.7

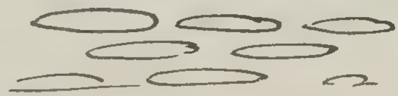


Idee zur Erklärung d. Viskositätsanomalie binärer Flüssigkeitsgemische im krit. Lösungspunkt:

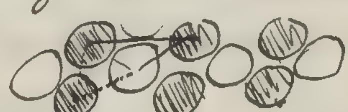
Bei Annäherung an krit. Lösungspunkt (von oben) erfolgt graduelle Trennung in 2 Phasen, so zwar dass diejenige von höheren Kohäsionsspannung kugelförmige Aggregate bildet, und die Zwischenräume von derjenigen von geringerer ↑ .. ausgefüllt werden, also ähnlich wie:



Würde Deformation der Flüssigkeit in homogener Weise vor sich gehen:



so würde Viskosität nur einen Mittelwert der beiden Visk. der Teile betragen. Infolge der Kohäsion setzen aber die inhomogenen Teilchen einer Formänderung Widerstand entgegen und suchen sich so zu bewegen wie Ensemble starrer Kugeln und intermediärer Flüssigkeit was natürlich eine kolossale Steigerung bedingt.



Vergleichsdaten:

Friedländer Zeitschr. ph. Ch. 38, 404, 1901
Rothmund 63, 54, 1908
Wo. Ostwald Koll. Z. 12, 219, 1913

μ_{H_2O} 1.12, $\mu_{\text{Isobutters.}}$ 1.46 } μ krit. Gemisch = 3.68!

prozentuale Visk. Zunahme ↑ pro 1°: 34.3%!

Dabei ist es wohl denkbar, dass bei langsamen Vorgängen die Kugeln sich starr verhalten, aber bei raschen sich deformieren. Also Abhängigkeit d. Visk. von Schergeschw. ist nicht ausgeschlossen.

$W(t)$ (in der Ebene)

Wahrsch. (dass vor Zeit t keine grössere Entfernung als a:

$$W(t) = \frac{1}{4\pi D t}\int_0^a e^{-\frac{r^2}{4Dt}}\, 2\pi r\, dr = \int_0^{\frac{a^2}{4Dt}} e^{-z}\, dz = \left[1 - e^{-\frac{a^2}{4Dt}}\right]$$

Wahrsch., dass war zur Zeit t .. $r<a$

aber zur Zeit $t+dt$ $r>a$ } = Wahrsch. eines Übertrittes (Eintrittes) zur Zeit t-$t+dt$

$$= \frac{dW}{dt}\,dt = -\frac{a^2}{4Dt^2}\, e^{-\frac{a^2}{4Dt}}\, dt$$

Erinnerungsfunktion: $e^{-\frac{t}{\tau}}$

~~Durchschn. remanenter Aufenthaltszeit~~

$$\cancel{\int_0^\infty e^{-\frac{t}{\tau}} \frac{\partial W}{\partial t}\, dt = W e^{-\frac{t}{\tau}} + \frac{1}{\tau}\int W e^{-\frac{t}{\tau}}\, dt}$$

Falls also seit t anwesend, so kommt dies mit $\int_0^t e^{-\frac{t}{\tau}}\, dt = \tau\left[1 - e^{-\frac{t}{\tau}}\right]$

in Rechnung; dies ist die äquivalente Zeitdauer

Also wird durchschnittl. äquivalente Zeitdauer:

$$-\int_0^\infty \frac{\partial W}{\partial t}\, dt \int_0^t e^{-\frac{t}{\tau}}\, dt = \cancel{-W\int_0^t e^{-\frac{t}{\tau}}\, dt\,\Big|_0^\infty} + \int_0^\infty e^{-\frac{t}{\tau}}\, W\, dt$$

$$= +\int_0^\infty e^{-\frac{t}{\tau}}\left[1 - e^{-\frac{a^2}{4Dt}}\right] dt = \tau - \underbrace{\int_0^\infty e^{-\left[\frac{t}{\tau} + \frac{a^2}{4Dt}\right]}\, dt}_{=\,?}$$

$$\frac{t}{\tau} + \frac{a^2}{4Dt} = z$$

$$t^2 - t\tau z + \frac{a^2\tau}{4D} = 0 \qquad t = \frac{\tau z}{2} \pm \sqrt{\left(\frac{\tau z}{2}\right)^2 - \frac{a^2\tau}{4D}}$$

$$dt = \frac{\tau}{2}\,dz \pm \frac{\frac{\tau^2}{4} z\,dz}{\sqrt{\frac{\tau^2 z^2}{4} - \frac{a^2\tau}{4D}}}$$

~~Im Grenzfall kleiner~~ $\cancel{\frac{a^2}{D}\tau}$ $\qquad \sum \left(e^{-\frac{t_1}{\tau}}\Delta t_1 + e^{-\frac{t_2}{\tau}}\Delta t_2 + \dots\right)$

$$\cancel{\int_0^\infty e^{-\frac{t}{\tau}}\left[\frac{a^2}{4Dt} - \;\right]dt = \frac{a^2}{4D}\int_0^\infty e^{-z}\frac{dz}{z}}$$

Kriterium für Verschwinden d. Teilchen infolge Br. B.:

Falls Teilchen an gewisser Stelle: Wahrsch. dass eine äquivalente Aufenthaltsdauer derselben innerhalb gewisser Grenzen ~~…~~ liege = ?

Integral dieser Wahrsch. bis zur Sichtbarkeitsdauer = Wahrsch. der Unsichtbarkeit

0·01279	0·01714	0·03702	0·01083
1004	1004	1004	1004
275	710	2698	

$79 = \mu_0[1 + 0·0787]$

$\sim 73\varphi$

$\frac{5\,\mu}{100\,\mathrm{m}^3}$

$\varphi\delta + (1-\varphi)\delta_0 = \bar{\delta}$

$\varphi\delta = 0·05$

$\varphi = \frac{0·05}{2} = 0·025$

oder

$\varphi = \frac{0·05}{1·90} = 0·0263$

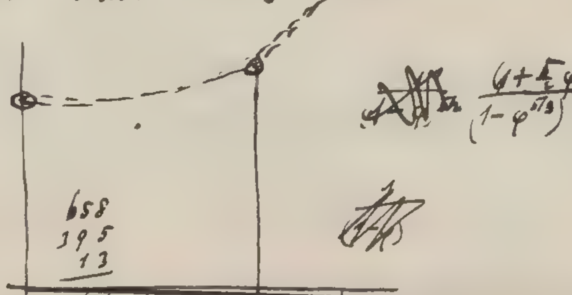
$\frac{275 \cdot 2}{12·51} = \frac{11 \cdot 2}{5} = 4·4 \qquad \varphi = 0·0625$

$\frac{710 \cdot 2}{25·02} = 5·7 \qquad 0·125$

$\frac{2688 \cdot 2}{5003} = 10·8 \qquad 0·25$

$\mu = \mu_0(1 + 2\varphi)$

Schalt :	5	12·5	25	50
$\varphi =$	0·0263	0·06583	0·13165	0·26333
$\varepsilon =$	~~3·16~~	~~4·9~~	~~5·7~~	~~10·8~~
	1·99	4·16	5·37	10·2
$\frac{a}{b} =$	0·297	~~0·396~~ 0·404	0·509	0·641
$\left(\frac{a}{b}\right)^2 =$	0·0885	0·163	0·259	0·411



658
395
13
1066
533

$+ \varepsilon\varphi + \frac{\frac{\varepsilon}{2}\frac{3}{2}}{1 \cdot 2}\varphi^2$

$\frac{35}{8}$

$\frac{(1 + \frac{\varepsilon}{2}\varphi)}{(1 - \varphi^{5/3})^2}$

8960
−4202
4758

6990
2788

$\lg\varphi \quad 4·202 - 3$

7378
−0184
6194

4200
0·3670 −3
0·4734 −1

2·3969
0·699

0·9468 −2

0972
2788
18184

0·0305 −2
0·0061
1·2122

2·3979
0·7993

8494
−1194
7300

3982
2788
2·1194

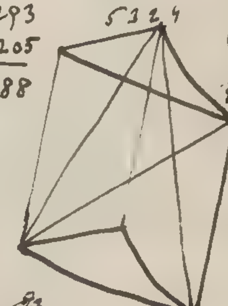
0993
2788
2·4205

2687

7065
4130
5324

8068
6136
0341
1761
2102

4293
−4205
0088



0·033

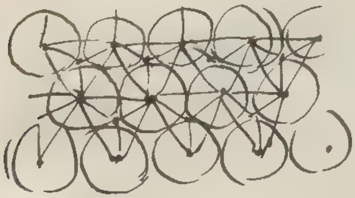
1052
26
3
108

$\varphi^{5/3} = \left(\frac{a}{b}\right)^5 = \varphi\left(\frac{a}{b}\right)^2$

Der Kugelstein
Diese gerade ist wohl von Steffen

∞ ∞
∞

ganz anders als o o o !
o o o



$$\sqrt{a^2 - \left(\frac{2}{3}\frac{a}{2}\sqrt{3}\right)^2} = a\sqrt{1-\frac{1}{3}} = a\sqrt{\frac{2}{3}}$$

$$a \cdot \frac{a}{2}\sqrt{3} \cdot a\sqrt{\frac{2}{3}}$$

$$\frac{x}{a} \quad \frac{y}{\frac{a}{2}\sqrt{3}} \quad \frac{z}{a\sqrt{\frac{2}{3}}} \cdot \frac{4}{3}\left(\frac{a}{2}\right)^3\pi : xyz =$$

~~0.49715~~
~~− 0.3891~~
~~0.10805~~
~~0.89195~~

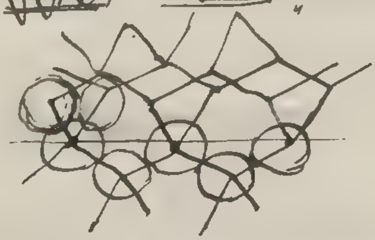
4771
1505
0.6276

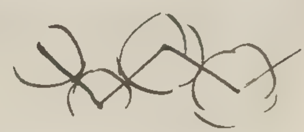
0.49715
0.6276
0.86955

$\boxed{0.13045}$

$$\frac{\pi}{3\sqrt{2}} : 1 = \frac{\pi}{3\sqrt{2}} : 1$$

~~0.7406~~ $\underline{\underline{0.7406}}$ „closest packing“ $\frac{3\sqrt{2}}{\pi} = 1.3505$





$$h = a\sqrt{\frac{2}{3}} \;\Big\|\; \mathrm{Vol}_{\text{Tetr.}} = \frac{a}{2}\,\frac{a}{2}\sqrt{3}\cdot\frac{a}{3}\sqrt{\frac{2}{3}} = \frac{a^3}{6\sqrt{2}}$$

$$\mathrm{Vol}_{\text{eingeschr. Kugel}} = \frac{4}{3}\pi\left(\frac{1}{4}a\sqrt{\frac{2}{3}}\right)^3 = a^3\pi\cdot\frac{4}{3}\,\frac{2\sqrt{2}}{3\sqrt{3}}\,\frac{1}{64} = \frac{a^3\pi}{9\sqrt{3}\cdot 4\sqrt{2}}$$

$$\frac{\mathrm{Vol}_{\text{Kugel}}}{\mathrm{Vol}_{\text{Tetr.}}} = \frac{\pi\cdot 6\sqrt{2}}{9\sqrt{3}\cdot 4\sqrt{2}} = \frac{\pi}{6\sqrt{3}} = \frac{\pi}{2\sqrt{27}}$$

0.49715
0.7157
0.48145 −1

~~= 0.3036~~

$\underline{= 0.3023}$ „loosest packing“

Erörterung d. Effektes der Ungleichförmigkeit d. Geschwindigkeitsgefälles

$$\mu \frac{\partial}{\partial r}\left(2\pi r \frac{\partial u}{\partial r}\right) = -2\pi r \frac{\partial p}{\partial x} = \mu \frac{\partial u}{\partial r} + r \frac{\partial^2 u}{\partial r^2}$$

$$-\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = -c$$

$u = \alpha(R^2 - r^2)$

$$\mu(-2\alpha - 2\alpha) = -c$$

$$\alpha = \frac{c}{4\mu}$$

$$= \frac{1}{4\mu}\frac{\partial p}{\partial x}\cdot(R^2 - r^2)$$

$$u_{Max} = \frac{R^2}{4\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial r} = -\frac{2r}{4\mu}\frac{\partial p}{\partial x} = -\frac{2r}{R^2}u_{Max}$$

$$\frac{\partial^2 u}{\partial r^2} = -\frac{2}{R^2}u_{Max}$$

Grössenordnung d. relat. Translationsgeschw. $a^2\frac{\partial^2 u}{\partial r^2}$

Widerstandsarbeit pro sek. pro cm³: $6\pi a\mu\left(a^2\frac{\partial^2 u}{\partial r^2}\right)^2 N$ Anzahl d. Kugeln pro cm³

~~Normale Widerstandsarbeit:~~ $\frac{\partial p}{\partial x}$

Zeitdauer d. Durchgangs durch die Röhre. $\tau = \frac{l}{u}$

∴ Widerstandsarbeit bei Durchgang durch die Röhre: $\int \frac{6\pi a\mu\left(a^2\frac{\partial^2 u}{\partial r^2}\right)^2 N l}{\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2-r^2)}\cdot Vol$

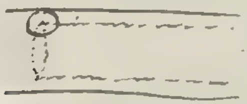
Gesamte

Normale Widerstandsarbeit: $Vol\cdot\frac{\partial p}{\partial x}\cdot l$

$$\frac{W_1}{W_0} = \frac{6\pi a\mu a^4 \left(\frac{\partial p}{\partial x}\right)^2 N l}{\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)^2 l\,(R^2-r^2)} \quad \left(\frac{1}{\pi R^2}\int_0^R \frac{2\pi r\,dr}{R^2-r^2} = -\frac{1}{R^2}\lg(R^2-r^2)\Big|_0^{R-a} = \frac{1}{R^2}\log\left(\frac{2aR}{R^2}\right)\right.$$

$$\frac{W_1}{W_0} = \frac{6\pi a^5 N}{R^2} \lg \frac{R}{2a} = \frac{9}{2}\varphi \cdot \frac{a^2}{R^2} \log\left(\frac{R}{2a}\right)$$

Somit verschwindend gering im Vergleich zur elastischen Korrektion



Einengung d. Lösung

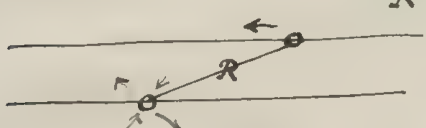
$$\varphi' = \varphi\left(\frac{R^2\pi - 2aR\pi}{R^2\pi}\right) = \varphi\frac{R}{R-2a} = \varphi\frac{1}{1-\frac{2a}{R}}$$

Ad Einsteins Rechnung:

Komponenten der durch Gegenwart der Kugel bedingten Geschwindigkeit:

$$\left.\begin{matrix} u \\ v \\ w \end{matrix}\right\} \frac{5}{3} P^3 \cdot \begin{matrix} A \\ B \\ C \end{matrix} \left\{\begin{matrix} \frac{\xi}{\rho^3} \\ \frac{\eta}{\rho^3} \\ \frac{\zeta}{\rho^3} \end{matrix}\right. - \ldots \qquad (\text{in Richtung} \pm 45^\circ) \qquad \mathfrak{A} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \text{ etc.}$$

diese muss also auf die in Entfernung R befindliche Kugel P einen Druck ausüben: $\sim 6\pi P\mu \cdot \frac{5}{3}\frac{P^3}{R^3}\xi A$ und pro Zeiteinheit eine Arbeit leisten: $6\pi P\mu\left(\frac{5}{3}\frac{P^3}{R^3}\xi A\right)^2$



$$\int \frac{1}{R^2}\cos\varphi \; 2\pi R^2 \sin\varphi \, d\varphi \, dR$$

falls aber Kugeln überall gleich dicht verteilt wären würden sich alle jene Induktions-Komp. kompensieren. In Wirklichkeit kommen die zufälligen Überschüsse zur Geltung.

Wahrsch., dass nur eine Kugel in der Schichte: $\underbrace{4R^2\pi \, dR}_{dV}$: $W(R.) = N \cdot dV \, e^{\cdots}$ // $N = \frac{1}{\delta^3}$

$$N\int_{R=P} P^7 \frac{A^2 \xi^2}{R^6} R^2 \, dR = \frac{P^6}{\delta^3}\cdots A^2$$

auf grössere Distanzen wird die Zeitgrenze an sich immer kleiner und überdies wird es immer unwahrscheinlicher dass keine Kompensation eintritt

wenn dies für alle Kugeln summiert wird, gibt es einen Ausdruck von Ordnung φ^2 für $\Delta\mu$

Freundlich Z. Kolc. 12, 230, 1913

49 52

Al_2O_3 54 48 102

$Al_2O_3 + 3H_2O$ 102 6 48 156

$Al(OH)_3$ Dichte 2'42 | Zandolt 170

$$\frac{0'187}{2'42} \cdot \frac{156}{102} =$$

2718 1931 4649 3924 0725

3838 0086

0'1182 $\frac{cm^3}{Liter}$ || $\varphi = 0'000118$

$$\frac{\Delta\mu}{\mu} = \frac{8}{52} = 0'154$$

$$k = \frac{0'154}{0'118} \cdot 10^3 = \underline{1'3 \cdot 10^3}$$

$$h = \frac{32}{3} \pi^3 \frac{(n-1)^2}{N\lambda^4} = \frac{32}{3} \pi^3 \frac{(2'944)^2 \cdot 10^{-8}}{3 \cdot 10^{19} \cdot 5^4 \cdot 10^{-20}} = \frac{32 \cdot \pi^3 \, 2'944}{25 \cdot 225} \cdot 10^{-7}$$

'49715 : 3
1'49145
'9378
1'5051
3'9344
3'7501
0'1843

4689 | 1'3979
2'3522
3'7501

$= 1'53 \cdot 10^{-7}$

$$\varepsilon' = \frac{\varepsilon q \omega \cdot x}{2\pi x \cdot 4\pi} h = \frac{\varepsilon r^2 \pi \omega}{8\pi\pi} h = \frac{\varepsilon r \omega}{8} h$$

$$\omega = \frac{3^2 \pi}{4 \cdot 4 \cdot 29^2 \pi} = \left(\frac{3}{4 \cdot 29}\right)^2 =$$

$\varepsilon = 15.000$

$$= \frac{1'5 \cdot 10^4 \cdot 0'2}{8} \cdot \left(\frac{3}{4 \cdot 29}\right)^2 \cdot 1'5 \cdot 10^{-7}$$

$$= \frac{2'4}{8 \cdot (40)^2} \cdot 10^{-4} = \frac{0'3}{1'6} \cdot 10^{-7} = 2 \cdot 10^{-8}$$

$\varphi = 4\pi\varepsilon' \cdot 2rx = 8\pi \cdot 0'2 \cdot \varepsilon'$
$= 3'14 \cdot 1'6 \cdot \varepsilon'$
188
502 $= 5 \cdot \varepsilon'$
$= 10^{-7}$

mur blanc = le plus bien = $45 \cdot 10^{-6}$

~~$\varphi + 1 = 20^2 : (12 \cdot 10^5)^2$~~

$$\alpha = \frac{(12 \cdot 10^5)^2}{20^2} 10^{-7} = \frac{1}{4} 10^3$$

$H = \dfrac{13{\cdot}6 \cdot [illegible] \cdot 76}{0{\cdot}001293 \cdot [illegible]}$

~~= [illegible]~~

$= 8$ km

$kH = 0{\cdot}1222$

$e^{-kH} = 0{\cdot}8856$

1·1335
1·9912
1·8808
6·0055
− 1116 +3
8·8939
0·1843 −7
2·0782

0·4343
8686
869
87
− ·053072
0·94693

1·1335
1·8808
3·0143
− 0·1116 +3
5·9027
0·1843 −7
0·0870 −1
~~−0·9120.~~ 0·4343
3648
2236
365
27
− 0·3966 ~~7~~
= 0·6039 −1

$\frac{1}{e}$

Woudtia k:

$\frac{13}{93}$	$\frac{21}{190}$	$\frac{45}{288}$	$\frac{57}{337}$	$\frac{98}{385}$	$\frac{457}{490}$	$\cdot \frac{10^{-4}}{10^{-6}} \cdot 10^{5}$
1139	3222	6532	7559	9912	6599	
9685	2788	4594	5276	5855	6902	
1454−1	0434−1	1938	2283	4057	9697	

Koagulationsproblem:

Irgend eines der Teilchen kann man herausfassen und die relativen Bewegungen der übrigen in Bezug auf jenes ins Auge fassen

$$\overline{\Delta\xi^2} = \overline{(\Delta x_1 - \Delta x_2)^2} = \overline{\Delta x_1}^2 + \overline{\Delta x_2}^2 - 2\cancel{\overline{\Delta x_1}\,\overline{\Delta x_2}} = 2\,\overline{\Delta x^2}$$

Also finden die relativen Bewegungen so statt als ob die Diff-Konstante verdoppelt wäre.

Nun Frage, wie gross ist die W., dass eines der beweglichen T. an das ruhende zum ersten Male in der Zeit t in dt stösst?

Wahrsch., dass bis zur Zeit t keines angeklebt sei, berechnet sich nach Diffusionstheorie, wie früher dargelegt, falls dabei nur das Ankleben an den herausgefassten und nicht der beweglichen untereinander berücksichtigt wird.

(1) $W = \ldots e^{-\ldots t}$

In Wirklichkeit vermindert sich sowohl die Anzahl wie die Beweglichkeit der bewegl. T. durch die ~~etc~~ Doppelteilchenbildung etc.

Also gibt Formel (1) einen zu raschen Koagulationsverlauf für die späteren Stadien

(und zwar desto mehr wenn nur Bildung von Doppelteilchen gemessen wird)

Wenn also ~~...~~ ausschliesslich die Zusammenstösse der ursprünglichen Teilchen ins Auge gefasst werden so

vielleicht so: Anzahl n ~~der~~ beweglichen verringert sich im gleichen Masse wie W

also $W = \ldots e^{-\ldots n_0 W t}$

Das sollte aber nicht im Zeitpunkt sondern in der Diff Gleichung geschehen!

ob das wahr ist?

$$W(\xi)\,d\xi = \int W(x_1)\,dx_1\,W(x_2)\,dx_2 = \int_{x_1=-\infty}^{+\infty} W(x_1)\,dx_1 \int W(\xi - x_1)\,d\xi$$

$$= \frac{1}{(\sqrt{2\pi Dt})^2}\int e^{-\frac{(\xi-x)^2}{2Dt} - \frac{x^2}{2Dt}}\,d\xi\,dx = \frac{d\xi}{2\pi Dt}\cdot e^{-\frac{\xi^2}{2Dt}}\cdot\int e^{-\frac{x^2}{Dt}+\frac{x\xi}{Dt}}\,dx$$

$$= \frac{e^{-\frac{\xi^2}{2Dt}}\,d\xi}{2\pi Dt}\int_{-\infty}^{\infty} e^{-\frac{(x-\frac{\xi}{2})^2}{Dt}+\frac{\xi^2}{4Dt}}\,dx = \frac{e^{-\frac{\xi^2}{2Dt}+\frac{\xi^2}{4Dt}}}{2\sqrt{\pi Dt}}\,d\xi = \frac{e^{-\frac{\xi^2}{4Dt}}}{\sqrt{4\pi Dt}}\,d\xi$$

also wirklich so als ob $2D$ anstatt D

Eindimensionale Diffusion gegen feste Wand

Wahrsch. dass bis t nicht angeklebt: falls ein Teilchen auf Schichte $0 \ldots H$

$$W' = \cancel{\ldots} = 1 - \frac{2}{H}\sqrt{\frac{Dt}{\pi}}$$

Falls n Teilchen auf jener Schichte, Wahrsch. dass bis t keines derselben angeklebt:

$$W_0 = (W')^n = \cancel{\ldots} = e^{-2\nu\sqrt{\frac{Dt}{\pi}}}$$

$$\lg W = -\cancel{\ldots} - 2\nu\sqrt{\frac{Dt}{\pi}}$$

$$\frac{1}{W}\frac{dW}{dt} = \cancel{\ldots} = -\nu\sqrt{\frac{D}{t\pi}}$$

Wahrsch. dass bis t $(n-1)$ nicht angeklebt aber eines (irgend unter den n) angeklebt

$$W_1 = (W')^{n-1}\,\frac{2}{H}\sqrt{\frac{Dt}{\pi}}\binom{n}{1} = 2\nu\sqrt{\frac{Dt}{\pi}}\,e^{-2\nu\sqrt{\frac{Dt}{\pi}}}$$

Wahrsch. dass bis t irgend welche zwei unter den n angeklebt, die übrigen aber nicht

$$W_2 = \frac{1}{2}\left(2\nu\sqrt{\frac{Dt}{\pi}}\right)^2 e^{-2\nu\sqrt{\frac{Dt}{\pi}}}$$

Tatsächlich ist dann $\Sigma(W_0 + W_1 + W_2 + \ldots) = 1$

und durchschnittl. Anzahl der bis t angeklebten Teilchen ist:

$$2\nu\sqrt{\frac{Dt}{\pi}} = \beta$$

$$0\,e^{-\beta} + 1\beta e^{-\beta} + 2\frac{\beta^2}{2}e^{-\beta} + 3\frac{\beta^3}{3!}e^{-\beta} + 4\frac{\beta^4}{4!}e^{-\beta} + \dots\ ]$$

$$= e^{-\beta}\beta\left[1 + \beta + \frac{\beta^2}{2} + \frac{\beta^3}{3!} + \dots\right] = \beta = 2\nu\sqrt{\frac{Dt}{\pi}}$$

Allgemeine Form

$$W_{(n)} = \frac{e^{-\nu\varphi}(\nu\varphi)^n}{n!}$$

Falls: $\overline{n\,W_{(n)}} = \nu\varphi$ und falls Unabhängigkeit der Einzelereignisse (für jedes Teilchen)

Falls Abgang der Teilchen durch Diff. sukzessive Änderung:

$$\frac{\partial}{\partial t}(We^{\nu\varphi}) = D\frac{\partial^2}{\partial x^2}(We^{\nu\varphi})$$

$$W e^{\nu\varphi}\cdot\nu\frac{\partial\varphi}{\partial t} + e^{\nu\varphi}\frac{\partial W}{\partial t} = D e^{\nu\varphi}\frac{\partial^2 W}{\partial x^2}$$

$$\frac{\partial W}{\partial t} + W\nu\frac{\partial\varphi}{\partial t} = D\frac{\partial^2 W}{\partial x^2}$$

Andere Annäherung: $W_0 = e^{-\nu\varphi}$

$$\frac{\partial W}{\partial t} = e^{-\nu\varphi}\cdot\nu\frac{\partial\varphi}{\partial t}$$ $\nu = \nu_0 W_0$

$$\frac{\partial W}{\partial t} = e^{-\nu_0 W\varphi}\cdot\nu_0 W\cdot\frac{\partial\varphi}{\partial t}$$

Oder auch:

$$v = e^{-\nu\varphi}$$

$$\lg v = -\nu\varphi$$

$$\varphi = -\frac{\lg v}{\nu}$$

Voraussetzungen:

$t=0$: überall $=0$ mit Ausnahme $r=b$

$$v = \lim \alpha(r+R)\, e^{-\frac{(r+R-b)^2}{\varepsilon}}$$

$$= \lim \frac{r+R}{\varepsilon b^2\sqrt{\pi}}\, e^{-\frac{(r+R-b)^2}{\varepsilon^2}}$$

$$= \Phi(r) = \lim \frac{r+R}{\varepsilon(b+R)^2\sqrt{\pi}}\, e^{-\frac{(r-b)^2}{\varepsilon^2}}$$

$$\int v\,(r+R)\,dr = \int_0^\infty \rho^2 e^{-\frac{(\rho-b)^2}{\varepsilon^2}}\,d\rho$$

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$$= \varepsilon b^2 \underbrace{\int e^{-\frac{\xi^2}{\varepsilon^2}}\,d\xi}_{\sqrt{\pi}} = 1$$

$$\alpha\,\varepsilon\, b^2\sqrt{\pi} = 1$$

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial r^2}$$

$r=0$: $v=0$

$r=A$: $\dfrac{\partial v}{\partial r} = \dfrac{v}{R+A}$

$$v = \sum B_n\, e^{-a^2\beta_n^2 t} \sin\beta_n r \qquad B_n = 2\,\frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R}\int_0^A \Phi(r)\sin\beta_n r\,dr$$

$$\lim \int_0^A (r+R)\,\frac{e^{-\frac{(r+R-b)^2}{\varepsilon^2}}}{\varepsilon b^2\sqrt{\pi}} \sin\beta_n r\,dr = \cancel{\frac{1}{b}\sin\beta_n(b-R)}$$

$$= \frac{1}{b+R}\sin\beta_n b \qquad \beta_n = \frac{2n+1}{2}\pi\frac{1}{A} = x$$

$$v_n = \frac{2}{b+R}\sum \frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2-R}\sin\beta_n r\,\sin\beta_n b\; e^{-D^2\beta_n^2 t}$$

$$\underbrace{\frac{1}{A}\sum_{\Delta x = \frac{\pi}{A}} \sin rx\,\sin bx\; e^{-Dx^2 t}}_{= \frac{1}{\pi}\int_0^\infty e^{-Dx^2 t}\sin rx\,\sin bx\,dx}$$

$$\lim B_0 = \frac{2}{b+R}\,\frac{(A+R)^2\beta_0^2+1}{A(A+R)^2\beta_0^2 - R}\sin\beta_0 b \qquad \beta_0 = \frac{1}{A}\sqrt{\frac{3R}{A}}$$

$$= \cancel{\frac{2}{b+R}\,\frac{1}{2R}\sin\beta_0 b} = \frac{2b}{b+R}\,\frac{1}{A}\sqrt{\frac{3R}{A}}\;\frac{(A+R)^2\cdot 3R + A^3}{A(A+R)^2\cdot 3R - A^3 R} = 0$$

$$\int_0^\infty e^{-Dx^2t}\underbrace{\sin bx \sin rx}_{\frac{1}{2}\left[\cos(b-r)x-\cos(b+r)x\right]}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{Dt}}\left[e^{-\frac{(b-r)^2}{4Dt}}-e^{-\frac{(b+r)^2}{4Dt}}\right]$$

$$v=\frac{\alpha\sqrt{\frac{1}{4Dt\pi}}}{b+R}\left[e^{-\frac{(b-r)^2}{4Dt}}-e^{-\frac{(b+r)^2}{4Dt}}\right]$$

Wenn nun gebildet wird:

$$V=c\int_{b=0}^\infty 4\pi(b+R)^2 u\,db = c\,4\pi\int_{b=0}^\infty (b+R)\,v\,db$$

so sollte dies dem früheren Beispiel entsprechen, wo $\Big|_{t=0}\; v=c(r+R)$ war

$$V=\frac{4\pi c}{\sqrt{\pi}}\int_0^\infty\underbrace{\frac{(b+R)}{\sqrt{4Dt}}\left[e^{-\frac{(b-r)^2}{4Dt}}-e^{-\frac{(b+r)^2}{4Dt}}\right]\frac{db}{\sqrt{4Dt}}}\sqrt{4Dt}$$

~~$b=r+R=r$~~

$$=\frac{4c}{\sqrt{\pi}}\sqrt{4Dt}\int_0^\infty(\beta+P)\left[e^{-(\beta-\rho)^2}-e^{-(\beta+\rho)^2}\right]d\beta$$

$$\left\{\left[\beta-\rho+P+\rho\right]e^{-(\beta-\rho)^2}-\left[\beta+\rho+P-\rho\right]e^{-(\beta+\rho)^2}\right\}d\beta$$

$$=\cancel{\frac{1}{2}\left[-e^{-(\beta-\rho)^2}+e^{-(\beta+\rho)^2}\right]_0^\infty}+\cancel{(P-\rho)\int e^{-(\beta-\rho)^2}d\beta}$$

$$+(P+\rho)\int_0^\infty e^{-(\beta-\rho)^2}d\beta-(P-\rho)\int_0^\infty e^{-(\beta+\rho)^2}d\beta$$

$$=(P+\rho)\left[\frac{\sqrt{\pi}}{2}+\int_0^\rho e^{-z^2}dz\right]-(P-\rho)\left[\frac{\sqrt{\pi}}{2}-\int_0^\rho e^{-z^2}dz\right]=\rho\sqrt{\pi}+2P\int_0^\rho e^{-z^2}dz$$

Kontaktsbestimmung

$$\alpha\int_{r=0}^\infty\underbrace{(R+r)^2u\,dr}=\cancel{\ldots}$$

$$=4\pi(R+b)^2db\cdot c$$

$$=\alpha(R+b)$$

$$=\alpha\int(R+r)\,v\,dr$$

$$\alpha=(R+b)\,db\cdot c\cdot 4\pi$$

$$v=\frac{(R+b)\,db\cdot c}{\sqrt{4Dt\pi}}\left[e^{-}-\ldots\right]4\pi$$

$$V=\sum_{b=0}^\infty v$$

$$=\frac{c\cdot 4\pi}{\sqrt{4\pi Dt}}\int(b+R)\left[e^{-\frac{(b-r)^2}{4Dt}}-\ldots\right]db$$

$$V = \frac{4c\pi}{\sqrt{\pi}}\frac{\sqrt{4Dt}}{}\left[\rho\sqrt{\pi} + 2\rho\int_0^{\rho} e^{-z^2}dz\right] = 4c\pi\left[r + 2\frac{R}{\sqrt{\pi}}\int_0^{\frac{r}{2\sqrt{Dt}}} e^{-z^2}dz\right]$$

$$u = \frac{V}{r+R}$$ stimmt mit früheren Resultat mit Ausnahme des Faktor 4π

---

also ist u für einen in Entfernung $b+R$ befindlichen Quellpunkt:

$$u = \frac{v}{(R+b)} = \frac{c}{R+b}\;\cancel{\frac{}{\sqrt{4Dt\pi}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]}$$

und für einen in Entfernung $b+R$ befindlichen Quellpunkt:

$$u = \cancel{\left(\frac{v}{4\pi(R+b)^3\,db}\right)} = \frac{c}{4\pi(R+b)^2\sqrt{4Dt\pi}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$

$$\left.\frac{\partial u}{\partial r}\right|_{r=0} = \frac{c}{4\pi(R+b)^2\sqrt{4Dt\pi}}\left[\frac{b-r}{2Dt}\cdot e^{-\frac{(b-r)^2}{4Dt}} + \frac{b+r}{2Dt}\cdot e^{-\frac{(b+r)^2}{4Dt}}\right]_{r=0} = \frac{c}{(R+b)^2\sqrt{4Dt\pi}}\,\frac{b}{Dt}\,\frac{e^{-\frac{b^2}{4Dt}}}{4\pi}$$

(Konstante γ so bestimmen, dass $\int 4\pi u(R+b)^2 db = c = 4\pi c\gamma$ somit $\gamma = \frac{1}{4\pi}$)

$$\frac{J}{R} = 4\pi\left.\frac{\partial u}{\partial r}\right|_{r=0} R^2 D = \frac{c\,R^2 D}{(R+b)^2\sqrt{4Dt\pi}}\,\frac{b}{Dt}\,e^{-\frac{b^2}{4Dt}}$$

$$\frac{b}{2\sqrt{Dt}} = z \qquad -\frac{b\,dt}{4\sqrt{Dt^3}} = dz$$

Gesammte mit $t=\infty$ eingeflossene Menge:

$$\varphi = D\frac{cR^2}{(R+b)^2\sqrt{\pi}}\int_0^t \frac{b}{Dt\sqrt{4Dt}}\,e^{-\frac{b^2}{4Dt}}dt = D\frac{cR^2}{(R+b)^2\sqrt{\pi}}\cdot\frac{2}{D}\cdot\int_{\infty}^{\frac{b}{2\sqrt{Dt}}} e^{-z^2}dz$$

bis $t=\infty$ würde also einfliessen: $\varphi_\infty = \frac{cR^2}{(R+b)^2}$

Daraus für den Fall einer bis $R=\infty$ homogenen Raumerfüllung

$$J = \frac{cR^2\,4\pi}{t\sqrt{\pi}}\int_{b=0}^{\infty}\frac{b}{\sqrt{4Dt}}\,e^{-\frac{b^2}{4Dt}}db = \frac{4cR^2\sqrt{\pi}}{\sqrt{t}}$$

$$v = \frac{\alpha}{\sqrt{4Dt\pi}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$ für Quellsphärenschale

~~Also für in Entfernung b+R befindlichen Quellpunkt:~~

$$\cancel{u = \frac{1}{r+R}\,\frac{\alpha}{4\pi(R+b)^2 db}\,\frac{1}{\sqrt{4Dt\pi}}\left[e^{\cdots} - \right]}$$

~~Bedingung für α:~~

$$\cancel{\int_0^\infty 4\pi(R+b)^2 db\cdot u = \frac{1}{r+R}\left[r+\frac{2R}{\sqrt{\pi}}\int_0^{\frac{r}{2\sqrt{Dt}}} e^{-z^2}dz\right]}$$

$$\cancel{= \frac{\alpha}{r+R}\int_0^\infty \frac{(R+b)}{\sqrt{4Dt\pi}}\left[e^{\cdots} - \cdots\right]db} \qquad (\alpha = 1)$$

Bedingung für α: $\qquad u = \frac{v}{r+R}$

$$\lim_{t=0}\left[\int_0^\infty 4\pi(r+R)^2\,u\,dr = 1 = \int_0^\infty 4\pi(r+R)\,v\,dr\right.$$

$$= 4\pi(R+b)\,\frac{\alpha}{\cdots}$$

Also: für Quellsphärenschale:

$$v = \frac{1}{4\pi(R+b)}\,\frac{1}{\sqrt{4\pi Dt}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$

$\boxed{\text{stimmt vollständig}}$

für Quellpunkt dasselbe!

Also für homogene Raumerfüllung:

$$v = \int_0^\infty 4\pi c(R+b)^2 db\,\frac{1}{4\pi(R+b)}\,\frac{1}{\sqrt{4\pi Dt}}\left[e^{\cdots}\right] = c\underbrace{\int_0^\infty \frac{(R+b)}{\sqrt{4\pi Dt}}\left[e^{\cdots}\right]db}_{= r+\frac{2R}{\sqrt{\pi}}\int e^{-z^2}dz}$$

Für Quellpunkt:

$$J_R = +4\pi D R^2 \frac{\partial u}{\partial r}\Big|_{r=0} = \ldots = \cancel{4\pi D \left(r \frac{\partial v}{\partial r} + v\right)\Big|_{r=0} = -4\pi D v_{r=0}}$$

$$= +4\pi D R^2 \frac{\partial}{\partial r}\left[\frac{v}{(r+R)}\right]\Big|_{r=0} = +4\pi D R^2\left[\frac{1}{(r+R)}\frac{\partial v}{\partial r} + v\right] = +4\pi D R^2\left[\frac{1}{R}\frac{\partial v}{\partial r}\right]_{r=0}$$

$$\frac{\partial v}{\partial r}\Big| = \frac{1}{4\pi(R+b)}\,\frac{1}{\sqrt{4\pi D t}}\left[\frac{b-r}{2Dt}\,e^{-\frac{(b-r)^2}{4Dt}} + \frac{b+r}{2Dt}\,e^{-\frac{(b+r)^2}{4Dt}}\right]_{r=0} = \frac{1}{4\pi(R+b)}\,\frac{1}{\sqrt{4\pi D t}}\,\frac{b}{Dt}\,e^{-\frac{b^2}{4Dt}}$$

$$v\Big| = \frac{1}{4\pi(R+b)}\,\frac{1}{\sqrt{4\pi D t}} = 0$$

$$J_R = \frac{R}{R+b}\,\frac{b}{\sqrt{4\pi D t^3}}\,e^{-\frac{b^2}{4Dt}}$$

Für homogene Raumerfüllung:

$$J = 4\pi c\int_{b=0}^{\infty} J_R\,(R+b)^2\,db = 4\pi c R\int_0^{\infty}\frac{(R+b)\,b}{\sqrt{4\pi D t^3}}\,e^{-\frac{b^2}{4Dt}}\,db$$

$$= \frac{4\pi c}{\sqrt{\pi}}\,R\left[\frac{R}{\sqrt{t}}\,\frac{b\,e^{-\frac{b^2}{4Dt}}\,db}{4Dt}\cdot\sqrt{4D} + \frac{b^2 e^{-\frac{b^2}{4Dt}}\,db}{(4Dt)^{3/2}}\cdot 4D\right]$$

$$= \frac{4\pi c}{\sqrt{\pi}}\left[\frac{R^2}{\sqrt{t}}\,\frac{\sqrt{4D}}{2} + 4DR\underbrace{\int_0^{\infty} z^2 e^{-z^2}\,dz}_{\frac{\sqrt{\pi}}{4}}\right]$$

$$= 4\pi c D R\left[1 + \frac{R}{\sqrt{Dt\pi}}\right] \qquad \boxed{\text{stimmt}}$$

$$\frac{b^2}{4Dt} = z^2$$

$$-\frac{b^2}{4Dt^2}\,dt = 2z\,dz$$

$$\frac{dt}{t} = \frac{2\,dz}{z}$$

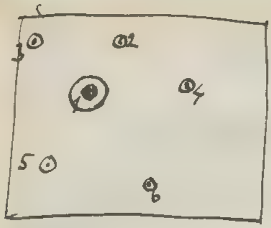
Gesamte seit $t=0$ eingeflossene Quantität:

$$Q = \int J_R\,dt = \frac{R}{R+b}\,\frac{1}{\sqrt{\pi}}\int_0^t \frac{b}{\sqrt{4Dt}}\,\frac{e^{-\frac{b^2}{4Dt}}}{t}\,dt = -\frac{R}{R+b}\,\frac{1}{\sqrt{\pi}}\cdot 2\int_{\infty}^{\frac{b}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

und lange Zeit fließt ein:

$$\boxed{Q_\infty = \frac{R}{R+b}}\,!$$

also Unterschied gegen 2 dimensionalen Fall, wo für $t=\infty$; $Q_\infty = 1$



In der früheren Rechnung war

Wahrsch. dass bis zur Zeit t keine Anlagerung stattgefunden habe

W_t = Wahrsch. dass $\underbrace{\text{weder (2) noch (3) noch (4)}\ldots}_{\text{wobei Anzahl der Partner } (n_0-1)}$ an (1) angelagert seien

$= W_{t2} \cdot W_{t3} \cdot W_{t4} \cdots \qquad = (W)^{n_0-1}$

Dagegen in Wirklichkeit, falls es sich um die Bildung von Doppelteilchen handelt, ist

W_{t2} = Wahrsch. dass sich (2) nicht angelagert habe und zwar weder falls es einfach ist noch

falls es

Man muss dann unterscheiden: ~~die Wa~~ die Bildungsgeschw. der Doppelteilchen d.h. den Teilbetrag der Abnahme der Zahl der Einzelteilchen welcher von Bildung von Doppelteilchen herrührt: erhält man aus

$\frac{d W_{(1)}}{dt}$ wobei $W_{(1)} = W^{n-1}$

dabei ist W = Wahrsch. dass

$$n_0 = n_{(1)} + 2 n_{(2)} + 3 n_{(3)} + \cdots$$

$$\frac{d}{dt} n_0 = -2 \frac{d n_{(2)}}{dt} - 3 \frac{d n_{(3)}}{dt} - \cdots$$

Damit sich ein Doppelteilchen bildet, müssen die zusammentreffenden einfach sein, ~~also~~

~~Wahrsch. dass~~ Wahrsch. für eine Doppelbildung in Zeitmoment dt:

$W_{(1+1)}\, dt$

$$-\frac{d n_{(1)}}{dt} = n_{(1)} \left[W_{(1+1)} + W_{(1+2)} + W_{(1+3)} + \cdots \right] =$$

$$\frac{d n_{(2)}}{dt} = \frac{1}{2} n_{(1)} W_{(1+1)} - n_{(2)} \left[W_{(2+1)} + W_{(2+2)} + W_{(2+3)} + \cdots \right]$$

$$\frac{d n_{(3)}}{dt} = n_{(1)} W_{(1+2)} - n_{(3)} \left[W_{(3+1)} + W_{(3+2)} + \cdots \right]$$

Dabei ist allgemein $n_{k} W_{(k+i)} = n_{i} W_{(i+k)}$

$$-\frac{dn_1}{dt} = n_1\left[W_{(1+1)} + W_{(1+2)} + W_{(1+3)} + \cdots\right] = n_1 W_{(1+1)} - 2n_2\left[W_{(2+1)} + W_{(2+2)} + \cdots\right]$$
$$+ 3n_1 W_{(1+2)} - 3n_3\left[W_{(3+1)} + W_{(3+2)} + \cdots\right]$$
$$+ 4n_1 W_{(1+3)} + 2n_2 W_{(2+2)} -$$
$$- 4n_4\left[W_{(4+1)} + W_{(4+2)} + \cdots\right]$$

$$= n_1 W_{(1+1)} + 3n_1 W_{(1+2)} - 2n_2 W_{(2+1)}$$
$$+ 4n_1 W_{(1+3)} + \cancel{2n_2 W_{(2+2)}} - 3n_3 W_{(3+1)} - \cancel{2n_2 W_{(2+2)}}$$

$$= n_1 W_{1+1} + n_1 W_{1+2} + n_1 W_{1+3} + \cdots \qquad \boxed{\text{stimmt}}$$

Bei bimolekularen chemischen Reaktionen ist

$\frac{dn_1}{dt} = n_1 W_{(1+1)}$ und: $W_{(1+1)} = (n_1 - 1)\alpha$ falls Voraussetzung, dass auf viele Zusammenstöße nur eine gewisse Anzahl günstiger stattfindet, so dass jeden Moment die Flüssigkeit als vollständig durchgemischt angesehen werden kann, dass also momentaner Vorgang unabhängig von der Art der Vorgeschichte.

Charakteristikum der üblichen chemischen Kinetik: jeder Folgezustand kann als momentaner Anfangszustand angesehen werden.

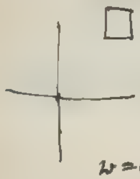
Bei Prozessen, für welche das wahr ist, müsste Intensität der Durchmischung ohne Einfluss sein auf den Reaktionsverlauf.

Versuche bei rasch reagierenden Substanzen in sehr großer Verdünnung!

Im Gegenteil dazu müsste Koagulationsverlauf in hohem Grad abhängen von Rührgeschwindigkeit

Diffusion in einem in Scherungsbewegung befindlichen Medium

$$\frac{\partial c}{\partial t} = D\frac{\partial c}{\partial x}\Big|_{x+\Delta x} + (uc)_{x+\Delta x} - D\frac{\partial c}{\partial x}\Big|_{x} + (uc)_{x} + \cdots$$



$$= D\nabla^2 c - \left[\frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial z}(wc)\right]$$

$w=0$

$v=0$

$u=\beta y$

$$\frac{\partial c}{\partial t} + \beta y\frac{\partial c}{\partial x} = D\nabla^2 c$$

Allgemeine Ähnlichkeitsgesetze

1). Abhängigkeit von D resp. μ:

Koagulationsgeschw. in beliebigen Stadium $\sim \frac{\theta}{\mu}$

Koagulationszeit (für beliebigen Teilprozess) $\sim \frac{\mu}{\theta}$

2). Abhängigkeit von Lineardimensionen

Falls alle Lineardim. im Verh. α vergrößert, wird $R \dots \rightarrow \alpha R_0$, $n \dots \rightarrow \frac{n_0}{\alpha^3}$

$D \dots \frac{x^2}{t}$ dann ist $\underline{t \sim \alpha^2}$

z.B. $t = \frac{1}{4\pi n D R} = \frac{\alpha^3}{4\pi n_0 D \alpha R_0} = \alpha^2 t_0$ stimmt

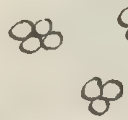
für kl. t

$n\sqrt{D}.R^2\sqrt{t} = \dots$

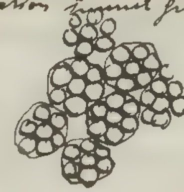
$t = \frac{1}{Dn^2R^4}$ $\frac{\alpha^6}{n_0^2 R_0^4 \alpha^4} = \alpha^2 t_0$ stimmt

Raumerfüllung bei fortschreitender Koagulation immer geringer:

Anfang



später



Denn die Zwischenräume bleiben immer einen gewissen Prozentsatz des entstehenden Aggregats

Zwischenraum von Größenordnung der koagulierenden Teilchen

Darum dürfte Raumerfüllung auch abhängen von Konzentr. d. koagulierenden Subst.

Wenn man so argumentiert:

$$W = e^{-4\pi n D R t}$$

$$\frac{dW}{dt} = -4\pi n DR\, W$$ = relative Anzahl der # zum ersten Mal sich anlagernden

darin sollte aber nicht die Anzahl n_0 der ursprünglich vorhandenen Teilchen, sondern nur mehr die Anzahl $\underbrace{n_0 W}_{n}$ der zur Zeit t noch freien Teilchen stehen,

also:

$$\frac{dW}{dt} = -4\pi n_0 W^2 DR$$ oder auch $$\frac{dn}{dt} = -4\pi n^2 DR$$

$$\frac{dW}{W^2} = -4\pi n_0 DR\, dt$$ $$\frac{dn}{n^2} = -4\pi DR\, dt$$

also entsprechend der chemischen Kinetik einer Reaktion ~~...~~ zweiter Ordnung

$$n = \frac{n_0}{1+4\pi n_0 DRt}$$

infolge Einflusses der Relativbewegung ist aber D zu verdoppeln, also

$$n = \frac{n_0}{1+8\pi n_0 DRt}$$

Dies wäre folgendermaßen zu begründen: Für die Anzahl der noch zur Zeit t anlagernden Teilchen kann man die ursprüngliche Formel verwenden, wenn darin die ~~#~~ Zahl n_0 durch n ersetzt wird, denn es wird auf dasselbe hinauskommen ob unter den n_0 ein gewisser Prozentsatz infolge

vorheriger Anlagerung an andere Teilchen angeschaltet wird oder aber ob von vornherein für die zur Zeit t anlagernden nur die zu jener Zeit bestehende Dichte eingesetzt (und die übrigen von vornherein von der Betrachtung ausgeschlossen wurden) und ihre weitere Verminderung ignoriert wird. Dabei ist allerdings vorausgesetzt, dass die Verteilung jener n im Raume gleichmäßig ist. Ob das genau richtig?

Aber außerdem: Verminderung erfolgt nicht nur durch Anlagerung einfacher sondern auch mehrfacher Teilchen, also ist die D. Gl. zu ergänzen durch Berücksichtigung derselben

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Unter den früheren ~~[illegible]~~ vereinfachten Annahmen (nur Ankleben an die herausgehobene Kugel)

Wahrsch. dass bis t gerade	0 Teilchen angeklebt und die übrigen	n Teilchen nicht angeklebt sind	$= e^{-\nu\varphi}$
	1	$n-1$	$\nu\varphi\, e^{-\nu\varphi}$
	2	$n-2$	$\frac{(\nu\varphi)^2}{2} e^{-\nu\varphi}$

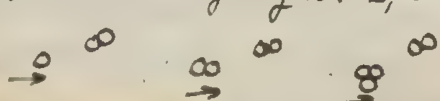
also Zuwachsgeschw. der Doppelteilchen:

$$= \frac{1}{2}\frac{d}{dt}\left(\nu\varphi\, e^{-\nu\varphi}\right) = \underbrace{\nu \frac{d\varphi}{dt} e^{-\nu\varphi}}_{\text{sich vereinigende Paare}} - \underbrace{\nu^2\varphi \frac{d\varphi}{dt} e^{-\nu\varphi}}_{\text{entstehende Dreiergruppe}} = \nu \frac{d\varphi}{dt}\left(1-\nu\varphi\right) e^{-\nu\varphi}$$

$$= -\frac{d}{dt}\left(e^{-\nu\varphi}\right)$$

In Wirklichkeit ist nun der erste Teil (Zuwachs durch Vereinigung einzelner T.) ebenso wie früher zu ersetzen durch ~~[illegible]~~ $-\frac{d}{dt}\left[e^{-n_0 W\varphi}\right] . n_0 W$

dagegen erfolgt Abnahme der Doppelteilchen nicht nur infolge Übergang in Dreier sondern auch durch Anlagerung von 2, 3 .... an ein Doppelteilchen



etc.

und besser sind verschiedene D Werte gültig

Wenn also v_0 v_1 v_2 v_3 --- die augenblicklichen Anzahlen von einfachen|doppelten|dreifachen|.... Teilchen bedeuten, so ist

$$\frac{dv_1}{dt} = -4\pi v_1^2 D_{11} R_{1} - 4\pi v_1 v_2 D_{12} R_{12} - 4\pi v_1 v_3 D_{13} R_1 - \cdots$$

$$\frac{dv_2}{dt} = \frac{1}{2}(4\pi v_1^2 D_{11} R_1) - 4\pi v_2 v_1 D_{21} R_{2,1} - (4\pi v_2^2 D_{22} R_{22}) - 4\pi v_2 v_3 D_{23} R_{23} - - -$$

$$\frac{dv_3}{dt} = 4\pi v_1 v_2 D_{12} R_{12} - 4\pi v_3 v_1 D_{13} R_{13} - 4\pi v_3 v_2 D_{23} R_{23} - (4\pi v_3^2 D_{33} R_{33}) - -$$

$$\frac{dv_4}{dt} = 4\pi v_1 v_3 D_{13} R_{13} + \frac{1}{2}(4\pi v_2^2 D_{22} R_{22}) - 4\pi v_4 v_1 D_{14} R_{14} - -$$

Relative Bewegung von Teilchen ungleicher Grösse:

$$W(\xi)\,d\xi = \frac{1}{2\pi t\sqrt{D_1 D_2}} \int e^{-\frac{(\xi+x)^2}{4D_1 t} - \frac{x^2}{4D_2 t}}\, d\xi\, dx = \frac{e^{-\frac{\xi^2}{4D_1 t}}\, d\xi}{\cdot -} \int e^{-\frac{x^2}{4t}\left(\frac{1}{D_1}+\frac{1}{D_2}\right) - \frac{\xi x}{2D_1 t}}\, dx$$

$$= \cdots \int e^{-\frac{x^2(D_1+D_2) + 2x\xi D_2 \pm \frac{\xi^2 D_2^2}{D_1+D_2}}{4D_1 D_2 t}} = \frac{e^{-\frac{\xi^2}{4D_1 t} + \frac{\xi^2 D_2}{4D_1(D_1+D_2)t}}}{2\pi t\sqrt{D_1 D_2}}\, d\xi \int e^{-(x-\cdots)^2 \frac{D_1+D_2}{4D_1 D_2 t}}\, dx$$

$$= \frac{e^{+\frac{\xi^2 D_2 - (D_1+D_2)\xi^2}{4D_1(D_1+D_2)t}}}{2\sqrt{\pi t}\,\sqrt{D_1 D_2}} \cdot \sqrt{\frac{D_1 D_2}{D_1+D_2}} = \frac{e^{-\frac{\xi^2}{4(D_1+D_2)t}}}{2\sqrt{\pi(D_1+D_2)t}}$$

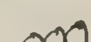
also gilt für die relativen Bewegungen die Summe der Diff Konstanten

$$D_{12} = D_1 + D_2 = DR\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$D_{12} R_{12} = D_{12}\frac{(R_1+R_2)}{2} = \frac{D_1 R_1}{2}$$

$$= DR\frac{(R_1+R_2)^2}{2 R_1 R_2}$$

z.B. $D_{11} R_{11} = 4D\frac{R}{2}$



$D_{(12)} R_{(12)} = \frac{3}{2} D \cdot 3 = \frac{9}{2} D\frac{R}{2}$

$D_{(22)} R_{22} = D \cdot \frac{4R}{2} = 4\frac{DR}{2}$

$D_{(13)} R_{(13)} = \frac{4}{3} D \cdot 4 = \frac{16}{3} \frac{DR}{2}$

also unwesentlicher Unterschied

Wenn also $4\pi D_{11} R_{11} = \alpha$ gesetzt wird, ist annähernd:

$$\frac{d\nu_1}{dt} = -\alpha(\nu_1^2 + \nu_1\nu_2 + \nu_1\nu_3) + \nu_1\nu_4 + \cdots \quad = -\alpha\nu_1(\nu_1+\nu_2+\nu_3+\cdots)$$

$$\frac{d\nu_2}{dt} = -\alpha\left(-\frac{\nu_1^2}{2} + \nu_1\nu_2 + \nu_2^2\right) + \nu_2\nu_3 + \cdots \quad = \alpha\frac{\nu_1^2}{2} - \alpha\nu_2(\nu_1+\nu_2+\nu_3+\cdots)$$

$$\frac{d\nu_3}{dt} = -\alpha(-\nu_1\nu_2 + \nu_1\nu_3) + \nu_2\nu_3 + \nu_3^2 + \cdots = \alpha\nu_1\nu_2 - \alpha\nu_3(\nu_1+\nu_2+\nu_3+\cdots)$$

$$\frac{d\nu_4}{dt} = \alpha\left(\nu_1\nu_3 + \frac{1}{2}\nu_2^2\right) - \alpha\nu_4(\nu_1+\nu_2+\nu_3+\cdots)$$

$$\frac{d\nu_5}{dt} = \alpha(\nu_1\nu_4 + \nu_2\nu_3) - \alpha\nu_5(\nu \cdots)$$

$$\frac{d\Sigma\nu}{dt} = \alpha\left[\frac{\nu_1^2}{2} + \nu_1\nu_2 + \frac{\nu_2^2}{2} + \nu_1\nu_3 + \nu_1\nu_4 + \nu_2\nu_3 + \nu_1\nu_5 + \nu_2\nu_4 + \frac{\nu_3^2}{2} + \cdots\right]$$

$$-\alpha(\Sigma\nu)^2$$

$$= -\frac{\alpha}{2}(\Sigma\nu)^2$$

$$\frac{d\Sigma}{\Sigma^2} = -\frac{\alpha}{2}.$$

$D_{11} \neq 2D$

$$\frac{\alpha\nu_0}{2} = \beta = 4\pi\nu_0 D R_{11}$$

$$\frac{1}{\Sigma(\nu)} = +\frac{\alpha}{2}t + \frac{1}{\nu_0}$$

$$\Sigma(\nu) = \frac{1}{\frac{1}{\nu_0} + \frac{\alpha t}{2}} = \frac{\nu_0}{1 + \frac{\alpha t \nu_0}{2}} =$$

~~[illegible]~~

$$\frac{d\nu_1}{dt} = -\alpha\nu_1\Sigma =$$

$$\frac{1}{\nu_1}\frac{d\nu_1}{dt} = -\alpha\Sigma = -\frac{\alpha\nu_0}{1+\alpha t\frac{\nu_0}{2}}$$

$$\log \nu_1 = -\int \frac{\alpha \nu_0 \, dt}{1+\frac{\alpha}{2}\nu_0 t} = -2 \log\left(1+\frac{\alpha}{2}\nu_0 t\right) + \log \nu_0$$

für $\lim t = 0$
$\lim \nu_1 = \nu_0(1-\alpha\nu_0 t)$
$= \nu_0 - \alpha\nu_0^2 t$ (wie früher)

$$\cancel{\nu_1 = \left(1+\frac{\alpha}{2}\nu_0 t\right)^{\alpha}}$$

$$\boxed{\nu_1 = \cancel{\left[1+\frac{\alpha}{2}\nu_0 t\right]^{2}} = \frac{\nu_0}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^2} = \frac{\left(\Sigma^1\right)^2}{\nu_0}}$$

$$\underbrace{\frac{d\nu_2}{dt} + \alpha \nu_2 \Sigma^1 = \alpha \frac{\nu_1^2}{2}}$$

$$\nu_2 + \nu_3 + \nu_4 + \cdots = \Sigma^1 - \nu_1 = \frac{\nu_0}{1+\frac{\alpha\nu_0 t}{2}}\left[1 - \frac{1}{1+\frac{\alpha\nu_0 t}{2}}\right]$$

$$= \frac{\frac{\alpha\nu_0^2 t}{2}}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^2}$$

$$\frac{d\nu_2}{dt} + \frac{\alpha\nu_0\nu_2}{1+\frac{\alpha\nu_0 t}{2}}$$

$$\frac{d}{dt}\left[\nu_2\left(1+\frac{\alpha\nu_0 t}{2}\right)^2\right] = \frac{d\nu_2}{dt}\left(1+\frac{\alpha\nu_0 t}{2}\right)^2 + \cancel{2}\frac{\alpha\nu_0}{\cancel{2}}\left(1+\frac{\alpha\nu_0 t}{2}\right)\nu_2$$

$$= \left(1+\frac{\alpha\nu_0 t}{2}\right)^2 \cdot \alpha\frac{\nu_1^2}{2} = \frac{\alpha\nu_0^2}{2\left[1+\frac{\alpha\nu_0 t}{2}\right]^2}$$

$$\nu_2\left(1+\frac{\alpha\nu_0 t}{2}\right)^2 = -\frac{\nu_0}{1+\frac{\alpha\nu_0 t}{2}} + \text{const} \quad \longrightarrow = \nu_0\left(1-\frac{1}{1+\frac{\alpha\nu_0 t}{2}}\right)$$

$$0 = -\nu_0 + \text{const} \qquad = \frac{\frac{\alpha\nu_0^2 t}{2}}{1+\frac{\alpha\nu_0 t}{2}}$$

$$\boxed{\therefore \quad \nu_2 = \frac{\frac{\alpha\nu_0^2}{2}t}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^3}}$$

$$\nu_3 + \nu_4 + \nu_5 + \cdots = \frac{\alpha\nu_0^2 t}{2}\left[\frac{1}{(1+\varepsilon)^2} - \frac{1}{(1+\varepsilon)^3}\right] = \nu_0\frac{\left(\frac{\alpha\nu_0 t}{2}\right)^2}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^3}$$

Maximum für ν_2:

$$\frac{d}{dt}\left[\frac{\varepsilon}{(1+\varepsilon)^3}\right] = \frac{1}{(1+\varepsilon)^3}\frac{d\varepsilon}{dt} - \frac{3\varepsilon\frac{d\varepsilon}{dt}}{(1+\varepsilon)^4} = 0$$

$$3\varepsilon = 1+\varepsilon$$

$$\varepsilon_M = \frac{1}{2} \longrightarrow t_M = \frac{1}{\alpha\nu_0}$$

$$\nu_{2\,max} = \frac{\frac{\nu_0}{2}}{\left(\frac{3}{2}\right)^3} = \frac{4\nu_0}{27}$$

$$\frac{dv_3}{dt} + \frac{\alpha v_0 v_3}{1+\frac{\alpha v_0 t}{2}} = \frac{\alpha v_0 \frac{\alpha v_0^2 t}{2}}{\left(1+\frac{\alpha v_0 t}{2}\right)^5}$$

$$\frac{d}{dt}\left[v_3\left(1+\frac{\alpha v_0 t}{2}\right)^2\right] = \frac{\alpha v_0 \cdot \frac{\alpha v_0^2 t}{2}}{\left(1+\frac{\alpha v_0 t}{2}\right)^3} = \frac{\alpha v_0^2}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}\left[1 - \frac{1}{1+\frac{\alpha v_0 t}{2}}\right]$$

$$v_3\left(1+\frac{\alpha v_0 t}{2}\right)^2 = \int \dots dt = \frac{v_0}{\left(1+\frac{\alpha v_0 t}{2}\right)^2} - \frac{2 v_0}{1+\frac{\alpha v_0 t}{2}}$$

$$= \frac{v_0}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}\left[1 - 2\left(1+\frac{\alpha v_0 t}{2}\right)\right] = -v_0 \frac{(1+\alpha v_0 t)}{\left(1+\frac{\alpha v_0 t}{2}\right)^2} + c$$

$\cancel{v_3 = v_0}$

$$= v_0\left\{\frac{\left(1+\frac{\alpha v_0 t}{2}\right)^2 - (1+\alpha v_0 t)}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}\right\}$$

$$= \frac{v_0\left(\frac{\alpha v_0 t}{2}\right)^2}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}$$

$$\therefore v_3 = \frac{v_0\left(\frac{\alpha v_0 t}{2}\right)^2}{\left(1+\frac{\alpha v_0 t}{2}\right)^4}$$

$$\overset{\Sigma v}{\frac{1}{1+\varepsilon}} = \overset{v_1}{\frac{1}{(1+\varepsilon)^2}} + \overset{v_2}{\frac{\varepsilon}{(1+\varepsilon)^3}} + \overset{v_3}{\frac{\varepsilon^2}{(1+\varepsilon)^4}} = \frac{1}{(1+\varepsilon)^2}\left[1 + \frac{\varepsilon}{1+\varepsilon} + \left(\frac{\varepsilon}{1+\varepsilon}\right)^2 + \dots\right]$$

$\cancel{1+\varepsilon = x}$

$$\cancel{\frac{1}{x} = \frac{1}{x^2} + \frac{x-1}{x^3}} + \underbrace{\frac{(x-1)^2}{x^4} + \dots}$$

$$\cancel{\frac{\varepsilon}{(1+\varepsilon)^2} = \frac{\varepsilon}{(1+\varepsilon)^3} \dots \varepsilon^2}$$

$$\cancel{1 = \frac{1}{1+\varepsilon} + \frac{\varepsilon}{(1+\varepsilon)^2}} + \dots$$

$$\frac{1}{1-\frac{\varepsilon}{1+\varepsilon}} = 1+\varepsilon$$

$\boxed{\text{Stimmt}}$

Also wahrscheinlich ist

$$\boxed{v_n = \frac{v_0\left(\frac{\alpha v_0 t}{2}\right)^{n-1}}{\left[1+\frac{\alpha v_0 t}{2}\right]^{n+1}}}$$

Maximum für v_3: $\frac{2\varepsilon}{(1+\varepsilon)^4} = \frac{4\varepsilon^2}{(1+\varepsilon)^5}$

$1 = \frac{2\varepsilon}{1+\varepsilon}$

$\varepsilon = 1$

$$\boxed{t_M = \frac{2}{\alpha v_0} \qquad v_{3\,Max} = \frac{v_0}{16}}$$

$$\frac{(n-1)\varepsilon^{n-2}}{(1+\varepsilon)^{n+1}} = \frac{(n+1)\varepsilon^{n-1}}{(1+\varepsilon)^{n+2}} \quad (1+\varepsilon)(n-1) = (n+1)\varepsilon$$

$$\varepsilon_M = \frac{n-1}{2}$$

$$v_{n\,Max} = 4 v_0 \frac{(n-1)^{n-1}}{(n+1)^{n+1}}$$

Wie rasch ändern sich die Amplituden der Normalkoordinaten ?

Raschheit der Schwankungen ?

Wie schnell ändert sich (durchschnittlich) die kinet. Energie eines (Gas-)Moleküls ?

abhängig nur von mittl. Weglänge. (resp. Stosszahl)

$$\nu_i = 2c\sin\frac{i\pi}{2(n+1)} = 2c\sin\theta = 2\sqrt{\frac{\alpha}{m}}\sin\frac{\varphi}{2}$$

$$\nu_n = \sqrt{\frac{\alpha}{m}\,p_n} \qquad p_n \to 4\sin^2\frac{\varphi}{2}$$

$$\nu_n = 2\sqrt{\frac{\alpha}{m}}\sin\frac{\varphi}{2} = \nu_0\sin\frac{\varphi}{2}$$

„man kann zeigen, dass:

$$N(\varphi)\,d\varphi = C\,d\varphi$$

$$C = \frac{N\bar{L}}{2\pi}$$

([illegible])

$$p = 4\sin^2\frac{\varphi}{2} = \nu^2\frac{m}{\alpha} = \frac{4\nu^2}{\nu_0^2} \qquad \Big\| \ \nu_0 = 2\sqrt{\frac{\alpha}{m}}$$

$$dp = 4\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\,d\varphi = 2\sin\varphi\,d\varphi$$

$$\therefore N(p)\,dp = \frac{N\bar{L}}{4\pi\sin\varphi}\,dp = N(\nu)\,d\nu$$

oder auch im Massstabe ν : $\cancel{N(\nu)\,d\nu = \frac{N\bar{L}}{4\pi\sin\varphi}\cdot\frac{\nu_0}{2}\cos\frac{\varphi}{2}\,d\varphi = N\bar{L}}$

$$N(\nu)\,d\nu = \frac{N\bar{L}}{4\pi\sin\varphi}\,2\sin\varphi\,d\varphi = \frac{N\bar{L}}{2\pi}\,d\varphi$$

Das ist wohl auch unmittelbar ersichtlich aus obiger Form

indem $\sum\nu = n$

$$\Delta\theta = \frac{\pi}{2(n+1)}$$

$$x + x^2 + x^3 + \dots = \frac{x}{1-x}$$

$$1 + 2x + 3x^2 + \dots = \frac{1}{1-x} + \frac{x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$1 + 1.2x + 2.3x^2 + \dots = \frac{2x}{(1-x)^3} + \frac{1}{(1-x)^2} = \frac{1+x}{(1-x)^3}$$

$$\nu_1 + 2\nu_2 + 3\nu_3 \dots = \frac{1}{(1+\varepsilon)^2}\left[1 + 2\frac{\varepsilon}{1+\varepsilon} + 3\left(\frac{\varepsilon}{1+\varepsilon}\right)^2 + \dots\right] = \frac{1}{(1+\varepsilon)^2}\frac{1}{\left(1-\frac{\varepsilon}{1+\varepsilon}\right)^2} = 1$$

$$\nu_1 + 1.2\nu_2 + 2.3\nu_3 + \dots = \frac{1}{(1+\varepsilon)^2}\left[1 + 1.2\frac{\varepsilon}{1+\varepsilon} + \dots\right] = \frac{1}{(1+\varepsilon)^2}\frac{1+\frac{\varepsilon}{1+\varepsilon}}{\left(1-\frac{\varepsilon}{1+\varepsilon}\right)^3} = 1 + 2\varepsilon$$

$$\begin{vmatrix} \cancel{1} & 0 & -(2+\frac{\gamma}{\alpha}) \\ (2+\frac{\gamma}{\alpha}) & 0 & -1 \\ 1 & -\frac{\gamma}{\alpha} & 1 \end{vmatrix} = 0$$

$$\begin{matrix} & 0 & \dots \\ (\quad) & 0 & -1 \end{matrix}$$

$$\cancel{X + \frac{\gamma}{\alpha}\left(2+\frac{\gamma}{\alpha}\right)^2 - X + (2+\frac{\gamma}{\alpha}) - \frac{\gamma}{\alpha} - (2+\frac{\gamma}{\alpha}) = 0}$$

$$\cancel{(2+\frac{\gamma}{\alpha})^2 = 1}$$

$$\cancel{2+\frac{\gamma}{\alpha} = \pm 1}$$

$$\cancel{\begin{cases} \frac{\gamma_1}{\alpha} = 0 \\ \frac{\gamma_2}{\alpha} = -1 \\ \frac{\gamma_3}{\alpha} = -3 \end{cases}}$$

$$\cancel{\begin{aligned} \xi_3 &= x_2 - x_1 \\ \xi_1 &= x_3 - x_2 \\ \xi_2 &= x_1 - x_3 \end{aligned}}$$

$$x_1 = A_1 \sin(\sqrt{\alpha\gamma}\, t + \varepsilon_1)$$

$$\begin{vmatrix} (1+\gamma)A_1 & -A_2 & 0 \\ -A_1 & (2+\gamma)A_2 - A_3 & \\ 0 & -A_2 & A_3(1+\gamma) \end{vmatrix} = 0$$

$$\begin{vmatrix} 1+\gamma & -1 & 0 \\ 1 & -(2+\gamma) & 1 \\ 0 & -1 & 1+\gamma \end{vmatrix}$$

$$\begin{matrix} 1+\gamma & -1 & 0 \\ 1 & -(2+\gamma) & 1 \end{matrix}$$

$$-(1+\gamma)^2(2+\gamma) + 2(1+\gamma) = 0$$

$$(1+\gamma)(2+\gamma) = 2$$

$$\cancel{2} + 3\gamma + \gamma^2 = \cancel{2}$$

$$\gamma = -1$$

$$\gamma = 0$$

$$\gamma = -3$$

Normale Vorgang:

$0 < k < \frac{n}{2}$

$$y_k = \sum_{i=1}^{\frac{n-1}{2}} \varphi_i \sin \frac{ik\pi}{\frac{n+1}{2}} \qquad \Bigg\| \qquad \frac{n}{2} < k < n+1 \qquad y_k = \sum_{i=1}^{\frac{n-1}{2}} \psi_i \sin \frac{ik\pi}{\frac{n+1}{2}}$$

$$\Phi_i = \frac{2}{n+1} \left\{ \sum_{k=1}^{\frac{n-1}{2}} \sin \frac{ik\pi}{n+1} \sum_{i=1}^{\frac{n-1}{2}} \varphi_i \sin \frac{ik\pi}{\frac{n+1}{2}} + \sum_{k=\frac{n+1}{2}}^{n} \sin \frac{ik\pi}{n+1} \sum_{i=1}^{\frac{n-1}{2}} \psi_i \ldots \right\}$$

$$\frac{d^2 x_1}{dt^2} = \alpha (x_1 - x_2)$$

$$\frac{d^2 x_2}{dt^2} = \alpha (x_2 - x_1) + x_2 - x_3 = \alpha (2x_2 - x_1 - x_3)$$

$$\frac{d^2 x_3}{dt^2} = \alpha (x_3 - x_2)$$

$$\frac{d^2(x_2 - x_1)}{dt^2} = \alpha \left[x_2 - x_3 + 2 (x_2 - x_1)\right]$$

$$\frac{d^2(x_3 - x_2)}{dt^2} = \alpha \left[2(x_3 - x_2) + (x_1 - x_2)\right]$$

$$\frac{d^2(x_1 - x_3)}{dt^2} = \alpha \left[(x_1 - x_2) - (x_3 - x_2)\right]$$

$$\ddot{\xi}_3 = \alpha \left[2 \xi_3 - \xi_1\right] = -\gamma \xi_3$$

$$\ddot{\xi}_1 = \alpha \left[2 \xi_1 - \xi_3\right] = -\gamma \xi_1$$

$$\ddot{\xi}_2 = \alpha \left[-\xi_3 - \xi_1\right] = -\gamma \xi_2$$

$$\cancel{(\ddot{\xi}_1 + \ddot{\xi}_3) = \alpha (\xi_1 + \xi_3) = -\ddot{\xi}_2}$$

$$\cancel{\xi_1 - \frac{\gamma}{\alpha} \xi_3 = 0}$$

$$\cancel{\xi_1}$$

$$\xi_1 - \left(\frac{\gamma}{\alpha} + 2\right) \xi_3 = 0$$

$$\xi_1 \left(2 + \frac{\gamma}{\alpha}\right) - \xi_3 = 0$$

$$\xi_1 - \frac{\gamma}{\alpha} \xi_2 + \xi_3 = 0$$

oder bei Annahme der Quantentheorie:

$$W(U_k, \dot U_k)\, dU_k\, d\dot U_k = A\, e^{-\left[\frac{\mu}{2}\dot U_k^2 + \frac{\alpha p_k}{2} U_k^2\right]\cdot\frac{1-e^{-\frac{h\nu_k}{kT}}}{h\nu_k}}\, dU_k\, d\dot U_k$$

Also Wahrscheinlichkeit eines ~~…~~ Wertes der Normalkoordinate U_k bei beliebigem $\dot U_k$:

$$W(U_k)\, dU_k = \int_{-\infty}^{+\infty} \dots\, d\dot U_k = C\, e^{-\frac{\alpha}{2} p_k U_k^2 \frac{N}{RT}}\, dU_k$$

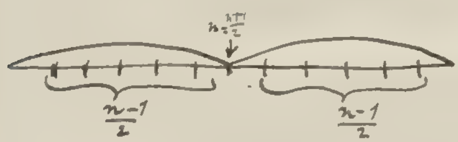
Somit mittlerer Betrag der Arbeit pro Zeiteinheit:

$$\left[\overline{\frac{\alpha}{2}\, y_k\, (y_{k+1} - y_{k-1})}\right] = \frac{\alpha}{2} \sum \sin\frac{ik\pi}{n+1} \cos\frac{ik\pi}{n+1} \sin\frac{i\pi}{n+1} \int_{-\infty}^{\infty} \underbrace{\Phi_i^2\, W(\Phi_i)\, d\Phi_i}_{\frac{1}{2}\frac{2}{\alpha}\frac{1}{p_k}\frac{RT}{N}}$$

Falls aber Temperaturdifferenz zu beiden Seiten von k?

Wie hängen die U_k des Partialsystems mit den U_k des ganzen Systems zusammen?

Annahme zwei gleich lange Punktreihen



$$y_m = \sum \Phi_i \sin\frac{i\pi\frac{n+1}{2}}{n+1} = \sum_{i=1}^{n} \Phi_i \sin\frac{i\pi}{2}$$
$$= \Phi_1 - \Phi_2 + \Phi_3 \dots\dots$$

$$\Phi_i = \frac{2}{n+1} \sum_{k=1}^{n} y_k \sin\frac{ik\pi}{n+1}$$

$$\Phi_{n-i+1} = \frac{2}{n+1} \sum_{k=1}^{n} y_k \underbrace{\sin\left[\frac{(n+1-i)k\pi}{n+1}\right]}_{\sin\left(k\pi - \frac{ik\pi}{n+1}\right)} = \frac{2}{n+1} \sum_{k=1}^{n} y_k \sin\frac{ik\pi}{n+1} (-1)^{k+1}$$

$\cancel{\Phi_{2i} + \Phi_{2i} = \Phi_i}$ $\quad \Phi_{2m} = \varphi_m + \psi_m \qquad \varphi_m = \frac{2}{\frac{n-1}{2}+1} \sum_{k=1}^{\frac{n-1}{2}} y_k \sin\frac{mk\pi}{\frac{n+1}{2}}$

~~$\sin\left(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}\right) = \sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + \sin\frac{\alpha-\beta}{2}\cos$~~

~~Wenn Elektron eindringt in Elektrizitätsträgern, Betrag der gesammten Arbeit:~~

~~$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$~~

~~$W = \frac{D-1}{4\pi}\int E^2\,dv = \frac{D-1}{4\pi}\,4\pi e^2\int_0^R \frac{r^2\,dr}{r^4} = \infty$~~

$\sin\alpha - \sin\beta = \cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$$T = \frac{m}{2}\left[\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2 \cdots\right] = \frac{m}{2}\left[\dot{U}_1^2 + \dot{U}_2^2 + \dot{U}_3^2 + \cdots\right.$$

$$V = \frac{\alpha}{2}\left[(u_1-u_0)^2 + (u_2-u_1)^2 + \cdots\right] = \frac{\alpha}{2}\left[p_1 U_1^2 + p_2 U_2^2 + \cdots\right]$$

$$F_{01} = \alpha(u_1 - u_0)$$

$$\overline{W}_{ik}\,dt = \dot{u}_i F_{ik}\,dt = \alpha\, u_i (u_k - u_i)$$

~~$\alpha\, u_i (u_k - u_i)$~~

vielleicht besser den symmetrischen Ausdruck

$$\alpha\left[\frac{u_i(u_{i+1}-u_i) + u_i(u_i - u_{i-1})}{2}\right] = \frac{\alpha}{2}\, u_i(u_{i+1} - u_{i-1})$$

$$y_k = \sum_{i=1}^{n} \Phi_i \sin\frac{ik\pi}{n+1} = \int \Phi$$

$$y_{k+1} - y_{k-1} = \sum \Phi_i \cos\frac{ik\pi}{n+1}\sin\frac{i\pi}{n+1}$$

$$\alpha\cdot\frac{y_k(y_{k+1} - y_{k-1})}{2} = \frac{\alpha}{2}\cdot\sum_{i=1}^{n}\Phi_i \sin\frac{ik\pi}{n+1}\cdot\sum_i \dot{\Phi}_i \cos\frac{ik\pi}{n+1}\sin\frac{i\pi}{n+1}$$

Wahrscheinlichkeit eines Wertes $y \cdots y+dy$ — $\qquad y = \sum_1^n \Phi_i \sin\frac{ik\pi}{n+1}$

$$W(y)\,dy = A e^{-}$$

Wahrscheinlichkeit eines Amplitudenwertes einer Normalkoordinate $\qquad \Phi_i = A_i \sin(\nu_i t + \varepsilon_i)$

$$W(A_i) = \mathfrak{D} e^{-A}$$

Wahrscheinlichkeit der Werte $\left\{\begin{matrix} U_k \ldots U_k + dU_k \\ \dot{U}_k \cdots \dot{U}_k + d\dot{U}_k \end{matrix}\right\}$ $\qquad W(U_k, \dot{U}_k)\,dU_k\,d\dot{U}_k =$

$$= A\, e^{-\left[\frac{m}{2}\dot{U}_k^2 + \frac{\alpha}{2}p_k U_k^2\right]\frac{N}{HT}}\,dU_k\,d\dot{U}_k$$

$$y_k = \sum_{i=1}^{n} \alpha_{ik} \cdot C_i \sin(\nu_i t + \varepsilon_i) = \sum_{i=1}^{n} \alpha_{ik} \Phi_i = \sum_{i=1}^{n} \Phi_i \sin\frac{ik\pi}{n+1}$$

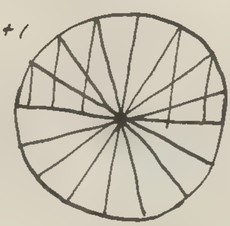
$$y_1 = \sin\frac{\pi}{n+1} C_1 \sin(\nu_1 t+\varepsilon_1) + \sin\frac{2\pi}{n+1} C_2 \sin(\nu_2 t+\varepsilon_2) + \cdots \sin\frac{i\pi}{n+1} C_i \sin(\nu_i t+\varepsilon_i)$$

$$y_2 = \sin\frac{2\pi}{n+1} C_1 \sin(\nu_1 t+\varepsilon_1) + \sin\frac{2.2\pi}{n+1} C_2 \sin(\nu_2 t+\varepsilon_2) + \cdots \sin\frac{2i\pi}{n+1} C_i \sin(\nu_i t+\varepsilon_i)$$

$$y_3 = \sin\frac{3\pi}{n+1} C_1 \sin(\nu_1 t+\varepsilon_1) + \sin\frac{3.2\pi}{n+1} C_2 \sin(\nu_2 t+\varepsilon_2) + \cdots \sin\frac{3i\pi}{n+1} C_i \sin(\nu_i t+\varepsilon_i)$$

$$2\sum_{k=1}^{n} \sin\frac{km\pi}{n+1} \sin\frac{ki\pi}{n+1} = \sum_{k=1}^{n}\left[\cos\frac{k(m-i)\pi}{n+1} - \cos\frac{k(m+i)\pi}{n+1}\right] = 0$$

$$2\sum_{k=1}^{n}\left(\sin\frac{ki\pi}{n+1}\right)^2 = \sum_{k=1}^{n}\left(1-\cos\frac{2ki\pi}{n+1}\right) = \sum_{k=0}^{n+1} = n+1$$



$$\sum y_k \sin\frac{ki\pi}{n+1} = \frac{n+1}{2} C_i \sin(\nu_i t+\varepsilon_i) = \frac{n+1}{2}\Phi_i$$

$$\Phi_i = \frac{2}{n+1}\sum_{k=1}^{n} y_k \sin\frac{ki\pi}{n+1}$$

$$\frac{E}{p} = c^2\frac{m}{2}\sum_{k=1}^{n}(y_k')^2 = \frac{m}{2}c^2\frac{n+1}{2}\sum_{k=1}^{n} C_k^2 \sin^2(\nu_k t+\varepsilon_k) = c^2\frac{m}{2}\frac{n+1}{2}\sum_{k=1}^{n}\Phi_k^2 \quad (?)$$

$$T = \frac{m}{2}\sum {y_k'}^2 = \frac{m}{2}\frac{n+1}{2}\sum_{k=1}^{n}(\Phi_k')^2$$

$$\Phi_i = C_i \cos\varepsilon_i \sin\nu_i t + C_i \sin\varepsilon_i \cos\nu_i t = (\Phi_i)_0 \cos\nu_i t + \frac{1}{\nu_i}(\Phi_i')_0 \sin\nu_i t$$

$$\Phi_i' = (\Phi_i')_0 \cos\nu_i t - \nu_i(\Phi_i)_0 \sin\nu_i t$$

$\alpha_{ik} = \beta_{ik} = \sin\frac{ik\pi}{n+1}$

$$y_k = \alpha_{1k} U_1 + \alpha_{2k} U_2 + \alpha_{3k} \cdot U_k + \dots \quad \alpha_{nk} U_n + \beta_{1k} V_1 + \beta_{2k} V_2 + \dots \quad \beta_{nk} V_n$$

$$y_k = \alpha_{1k} A_1 \sin\nu_1 t + \alpha_{2k} A_2 \sin\nu_2 t + \dots \quad + \beta_{1k} B_1 \cos\nu_1 t + \beta_{2k} B_2 \cos\nu_2 t + \dots$$

$$y'_k = \alpha_{1k}\nu_1 A_1 \cos\nu_1 t + \alpha_{2k}\nu_2 A_2 \cos\nu_2 t + \dots \quad - \beta_{1k}\nu_1 B_1 \sin\nu_1 t - \beta_{2k}\nu_2 B_2 \sin\nu_2 t \dots$$

$$\sum y_k \alpha_{ik} = \sin\nu_1 t \cdot A_1 \sum \alpha_{1k}\alpha_{ik} + \cos\nu_1 t \cdot A_1 \nu_1 \sum \alpha_{1k}\alpha_{ik} + \dots$$

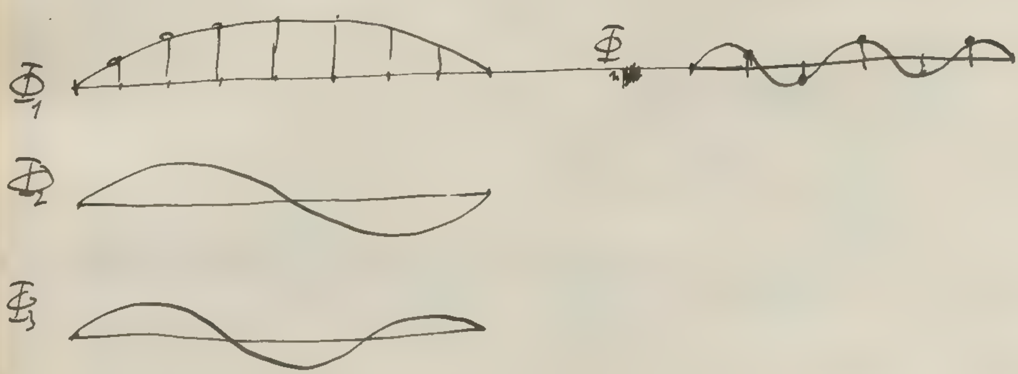
$$\sin\nu_i t \cdot A_i \underbrace{\sum(\alpha_{ik})^2}_{\frac{1}{2}(n+1)} + \dots$$

$$\sum y_k \alpha_{ik} = \frac{1}{2}(n+1)\left[A_i (\sin\nu_i t) + B_i (\cos\nu_i t)\right]$$

$$\sum y'_k \alpha_{ik} = \frac{1}{2}(n+1)\nu_i \left[A_i \cos\nu_i t \quad - B_i \sin\nu_i t\right]$$

$$\sum y_k \alpha_{ik} = \frac{n+1}{2}(U_i + V_i)$$

$$\sum y'_k \alpha_{ik} = \frac{n+1}{2}(U'_i -$$



Wie wird infolge der elektrostatischen Druckkräfte die Gestalt eines Flüssigkeitströpfchens durch ein angehängtes Elektron geändert 

$$y_k = \sum_{i=1}^{i=n} E_i \sin 2k\theta \cos(2ct\sin\theta) + \sum F_i \sin 2k\theta \sin\overbrace{(2ct\sin\theta)}$$

$$\boxed{\theta = \frac{i\pi}{2(n+1)}}$$

Allgemeine Form:

$$U_i = A_i \sin \nu_i t$$
$$V_i = B_i \cos \nu_i t$$

$$y_k = \alpha_{1k} \cdot U_1 + \alpha_{2k} \cdot U_2 + \alpha_{3k} \cdot U_3 + \dots \alpha_{nk} \cdot U_n + \beta_{1k} V_1 + \dots \beta_{nk} V_n$$

$$\nu_i = 2c\sin\theta = 2c \sin\frac{i\pi}{2(n+1)}$$

$$A_i = F_i$$

$$B_i = E_i$$

$$\alpha_{ik} = \sin 2k\theta = \sin\left(\frac{ik\pi}{n+1}\right) = \beta_{ik}$$

Dabei ist

$$E_i = \frac{2}{n+1} \sum_{k=1}^{k=n} (y_k)_{t=0} \sin 2k\theta$$

$$F_i = \frac{1}{(n+1)c\sin\theta} \sum_{k=1}^{k=n} (y'_k)_{t=0} \sin 2k\theta$$

$$\Phi_j = \frac{2}{n+1} \sum_{k=1}^{n} \sin kj\pi \left\{ \sum_{i=1}^{n} \frac{2}{n+1} \sin 2k\theta_i \left[\cos(2ct\sin\theta_i) \sum_{m=1}^{n} (y_m)_0 \sin 2m\theta_i + \frac{\sin(2ct\sin\theta_i)}{2c\sin\theta_i} \sum (y'_m)_0 \sin 2m\theta_i \right] \right\}$$

$$= \frac{2}{n+1}$$

$$\frac{d}{d\vartheta}\int_{x=-\infty}^{x} W(x\,x_0)_\vartheta\, dx$$

$$\lim \frac{1}{\vartheta}\left\{ \int_{x_0=X_0}^{\infty}(A + Bx_0 + Cx_0^2)\,dx_0 \cdot \int_{x=-\infty}^{X_0} W(x\,x_0)_\vartheta\, dx - \int_{x_0=-\infty}^{X_0}(A+Bx_0)\,dx_0 \int_{x=X_0}^{\infty} W(x\,x_0)\,dx \right.$$

$$= \frac{d}{d\vartheta}\left\{ \underbrace{A\left[\int_{x_0=X_0}^{\infty} dx_0 \int_{x=-\infty}^{X_0} - \int_{x_0=-\infty}^{X_0} dx_0 \int_{X_0}^{\infty}\right]} + B\left[\int x_0\, dx_0 \quad - \quad\right]\right\}$$

$$= A\frac{d}{d\vartheta}\int_{-\infty}^{\infty} dx_0 \int_{x=-\infty}^{X_0} W(x\,x_0)_\vartheta\, dx + B\,\frac{d}{d\vartheta}\int_{-\infty}^{\infty} x_0\, dx_0 \int W \ldots$$

$$\cancel{\int_{x=X_0}^{\infty} f(x_0)\,dx_0 \int_{x=-\infty}^{X_0} W(x\,x_0)\,dx} \qquad \int_{x_0=+X_0}^{\infty} dx_0 \int_{-\infty}^{X_0} W(x\,x_0)_\vartheta\, dx = \frac{1}{\sqrt{\pi}}\int_{x_0=X_0}^{\infty} dx_0 \int_{-\infty}^{\frac{X_0-x_0-f(x_0)}{2\sqrt{D\vartheta}}} e^{-z^2}\, dz$$

$$\frac{\left[x-x_0-\beta\vartheta f(x_0)\right]^2}{4D\vartheta} = z^2$$

$$x = x_0 + \beta\vartheta f(x_0) + 2z\sqrt{D\vartheta}$$

$$dx = 2\sqrt{D\vartheta}\, dz$$

$$= -\frac{1}{2\sqrt{D\vartheta}}\underbrace{\frac{1}{\sqrt{\pi}}\int_{-\frac{f(x_0)}{2\sqrt{D\vartheta}}}^{-\infty} dy \int_{-\infty}^{y} e^{-z^2}\, dz}$$

$$y\cdot\int_{-\infty}^{y} e^{-z^2}\, dz - \int y\, e^{-y^2}\, dy$$

$= 2 \cdot 4 \ldots 10^7$

$= 2500 \ldots$ im!

Abst: $10^{10} \cdot 5 \cdot 10^{-5} = 5 \cdot 10^{5}$

$3 \cdot 4 \cdot 10^{7} \ldots 10^{-12}$ = $2 \cdot 10^{-5}$

$6 \cdot 10^{14} \cdot 10^{\ldots}$

$\ldots 10^{5}$

$10\frac{}{2}$

Mit welcher „Dichtigkeit" [Wahrscheinlichkeit] ~~war~~ hat zur Zeit $(-t)$ das Teilchen die Stelle O ~~überdeckt~~ überdeckt?

falls sehr oft von derselben Anfangslage ausgegangen, wie oft hatte es zu jener Zeit eine Entfernung $< a$

$$= \frac{1}{2Dt}\int_0^a e^{-\frac{r^2}{4Dt}}\, r\, dr = \left[1 - e^{-\frac{a^2}{4Dt}}\right] = \text{Wahrsch., dass } r < a$$

Wie lange Zeit hindurch wird durchschnittlich jede Stelle überdeckt?

Was ist die Wahrsch., dass die gesammte „Überdeckungsdauer" $\tau \ldots \tau + d\tau$ beträgt?

" " die „relative Überdeckungsdauer" zur Zeit $(-t)$ den Wert z gehabt habe.

Grössenordnung: $\overline{\Delta r^2} = 4Dt = a^2$ $\quad$ $D = \frac{H\theta}{N}\frac{1}{6\pi\mu a}$

$$t = \frac{a^2}{4D} = \underbrace{\frac{3\pi\mu a^3}{2}}\frac{N}{H\theta}$$

dies ist massgebend für die photographische Helligkeit

Während welchen Bruchteiles des Zeitraumes $-t$ bis $-(t+dt)$ war der O Punkt durchschnittlich überdeckt?

Was ist die ~~Wahrsch~~ Wahrscheinlichkeit einer Überdeckungsdichtigkeit $z \ldots z+dz$ zur Zeit $(-t)$?

Andrerseits kann man die Sache auffassen, als ob die Dauer des Sichtbarbleibens einer Integrierung nach Massgabe einer Sichtbarkeitsfunktion $e^{-\gamma t}$ gleichkommt. Infolge dessen wäre statt der eigentlichen Zickzack Kurve eine teilweise ausgeglichene Curve zu sehen.

$$c\int_0^\infty \frac{1}{2\sqrt{\pi D t}}\, e^{-\frac{x^2}{4Dt}}\, dt \qquad \frac{x}{2\sqrt{Dt}} = z \qquad \frac{x\, dt}{4\sqrt{Dt^3}} = dz$$

$$\frac{dt}{2\sqrt{Dt}} = \frac{2\,dz \cdot t}{x}$$

$$= \int \frac{2t}{x\sqrt{\pi}}\, e^{-z^2}\, dz \qquad t = \frac{x^2}{4Dz^2}$$

$$= \int_0^\infty \frac{1}{2D\sqrt{\pi}}\, \frac{e^{-z^2}\, dz}{z^2} = \infty$$

Aber trotzdem kann die früher erwähnte Wahrsch. endlich sein, weil zweidimensionaler Fall

Superposition von Verteilungen:

$$\sum_t \frac{1}{4\pi D t}\, e^{-\frac{r^2}{4Dt}}\, r\, dr\, d\varphi \qquad \text{in Zeitintervallen } \tau \neq \frac{2a}{c_{xy}}$$

$$c\int \frac{1}{2Dt}\, e^{-\frac{r^2}{4Dt}}\, 2a\, dt$$

10/10 15

An O. Wie gross ist die Wahrsch., dass, falls ein Teilchen sich momentan im Punkte o befindet, dasselbe sich während des (verflossenen) Zeitraumes $-t \ldots -(t+dt)$ (ununterbrochen?) in einer Entfernung $r \ldots r+dr$ aufgehalten habe?

$$\int_{r=0}^{\infty} W(-t, r)\, dr = 1 \qquad \int_0^\infty\int_0^\infty W\, dr\, dt = 1$$

$= W(-t, r)\,.\, dr\, dt$

Wenn man annimmt, dass der (physiologische) Gesichtseindruck nach Massgabe einer (Erinnerungs) Funktion φ sich zusammensetzt aus den verflossenen Eindrücken, so wird die (durchschnittl.) resultierende Stärke desselben in einem gewissen Moment:

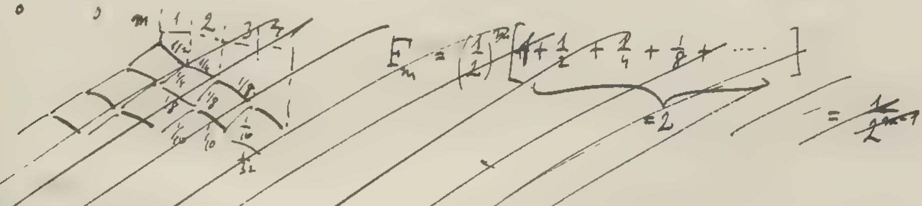
$$S = \int_0^\infty \varphi(t)\, dt \int_0^a W(-t, r)\, dr$$

wobei a = Teilchenradius

$\varphi(t)$ wahrscheinlich von der Form $\varphi(t) = e^{-\gamma t}$

Dagegen wäre die durchschnittl. photographische Stärke (Summenwirkung) nach sehr langer Zeit

$$F = \int_0^\infty dt \int_0^a W(-t, r)\, dr \quad (= \text{Wahrsch. von der Belichtung})$$



$$F_m = \left(\frac{1}{2}\right)^m \Big[1 + \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots}_{=2}\Big] = \frac{1}{2^{m-1}}$$

$$2\sum_1^N F_m = 2\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \frac{1}{2^{N-1}}\right]$$

(für ungerade m)

$$\sum_m F_m = \left(\frac{1}{2}\right)^m 2\left[\binom{m-1}{0} + \binom{m-1}{1} + \binom{m-1}{2} + \binom{m-1}{3} + \cdots\right] = 1$$

$$\sum_m F_{mn} = \left(\frac{1}{2}\right)^m \left[1 + \frac{1}{2} + \frac{1}{2^2}\binom{m+1}{1} + \frac{1}{2^3}\binom{m+2}{1} + \frac{1}{2^4}\binom{m+3}{2} + \frac{1}{2^5}\binom{m+4}{2} + \cdots\right] = F$$

Wenn Oberfläche = Ω

Stosszahl in der Zeit $t = \nu \frac{c}{\sqrt{6\pi}} \Omega t$ $\left.\begin{array}{l}= \text{eintretende} \\ = \text{austretende}\end{array}\right\}$ Mol B

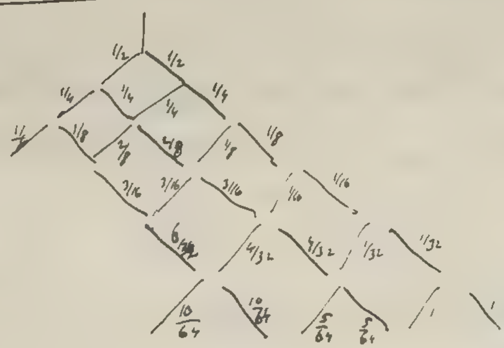
Dann wird die zufällige Schwankung

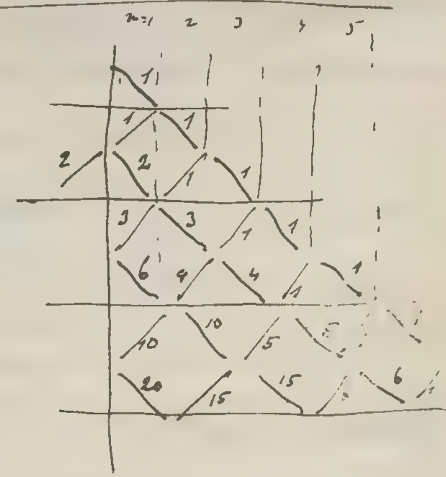
$$\overline{\Delta_t^2} = \frac{2\nu c}{\sqrt{6\pi}} \Omega t$$

Im Falle planparallelen Platte von Fläche F:

$$\overline{\Delta_t^2} = \frac{4\nu c}{\sqrt{6\pi}} F t$$

gilt für sehr kurze Zeit t
so dass nur äusserste Schichte in Betracht kommt





Über gewisse
Mängel im Aufbau d. klassischen Thermodynamik
und d. Boltzmann'schen Gastheorie

Automat! nicht vom Menschen unterstützte oder nachhelfende, unveränderliche Einrichtung

Angeblich aufgebaut auf Unmöglichkeit d. Perp. mob. II — dies soll aber auch nach Schwankungsth. gelten

De facto noch eine Reihe weiterer Voraussetzungen:

Determinationspostulat : Vorgang eindeutig abhängig von d. makroskopischen Variabeln || dies ist Hauptfehler

Klass. Th. lässt willkürl. Eingreifen d. Menschen zu

Im Stosszahlansatz gibt es Schwankungen, entsprechend nicht der ganzen Zahl ($n\,d\omega$) des betreffenden Volumelementes, sondern der hervorgehobenen Art von Molekülpaaren $n n' f f_1 d\omega d\omega \ldots$
(also viel grösser als wenn ersteres gälte) wäre).

$\iint (n n_1 - n' n_1') \ldots$ enthält Differenz zweier Glieder welche unabhängige Schwankungen ausführen.
Solange die Differenz gross im Verhältnis zu den Schwankungen, macht dies nichts aus, aber wenn $^{ein}/_{gegen}$ 0 konvergiert, kommen eben die Schwankungen in Betracht

So wie wenn es sich um Zusammenstösse von blauen, roten Kugeln untereinander oder aller untereinander handeln würde.

Stossgleichung in der Boltzmann Form ist ja streng gültig für d. Mittelwert über ein vielgleichartige Gase, aber nicht für ein individuelles Gas.

Allgemeine Form:

Schwankungsglied

$$\frac{\partial f}{\partial t} + \int v_x \ldots + F\frac{\partial f}{\partial \ldots} = \iint \quad + \Phi_t$$

Wenn Integration über alle Geschwind.

$$\frac{\partial n}{\partial t} = -\int v_x \frac{\partial n}{\partial x}\, d\omega$$

falls alle örtlichen Ungleichheiten unabhängig, aber gerade darauf kommt es an

$$\overline{\left[\frac{\partial}{\partial t}(n-\nu)\right]^2} = ?$$
$$= \overline{\Delta_t^2}$$

Die physikalische Wahrsch. eines Ereignisses, muss ganz unabhängig davon sein, was wir über dasselbe wissen

---

Andere Fassung der Zufälligkeit:

$y = f(x)$ ~~Dabei~~

y sind zufällig, wenn einer (diskreten beliebigen Verteilung der Punktmenge, mit Ausnahme prozentuell unendlich wenig zahlreicher singulärer Ausnahmsfälle) der x, eine und dieselbe immer [illegible] (Verteilung der Punktmenge y entspricht.

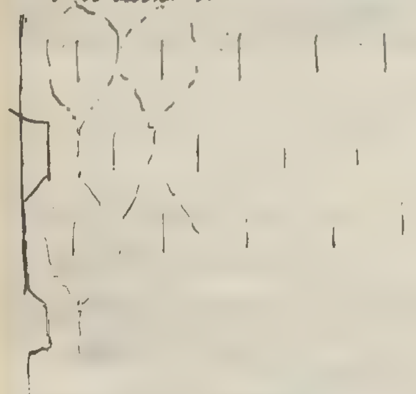
(und zwar mit wachsendem n immer genauer Annäherung)

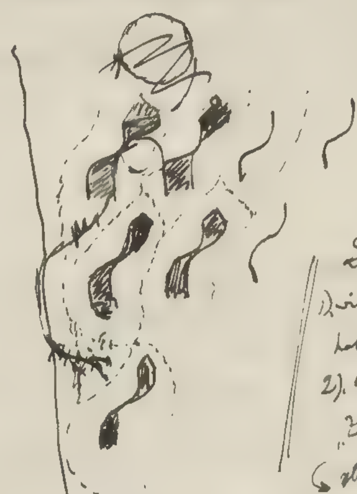
Allgemeine Definition eines Diffusionsexponenten bei jeder Zufälligkeit

---

Konstruktion d. Galton'schen Brettes: es müssen die Nagelentfernung d nur sehr wenig grösser sein als der Kugeldurchmesser, damit keine persistency der horizontalen Komponenten verbleibe.

Oder auch so:





zwei Probleme:
~~Das eine Teil~~:
1) wie Zufall regelm. Wirkung haben können
2). wie regelm. Änderung den „Zufall" erzeugen können
z.B. Ein Galton'sches Brett, Zersplitterung d. Entwicklungsbahn (Vorl.)

Wodurch ist der regelmässige Effekt d. Zufalls.

(untergeordnet
(Beweis des φ z.B. Schiessen mit Kugel, Sandkorn = höheres Stadium der Abstraktion von $\varphi(x)$)

NOE Münzenwurf = Beispiel objektiver Wahrsch.

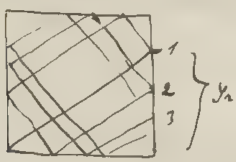
~~Bedingungen~~; [illegible]

„Es fehlt ein ganz wesentliches Moment.... ohne welches man allenfalls eine Statistik aber nicht eine Wahrscheinlichkeitsrechnung begründen könnte".

„Zufall" wenn solcher Zusammenhang zwischen x und y dass zwar im Allg. für auch wenig verschiedene x ganz andere y auftreten, jedoch so dass immer einer beliebigen / x Punktmenge (mit hinreichend grosser Punktzahl n) ein solcher y Punkt Komplex entspricht, dass er sich für $\lim n = \infty$ einer entsprechend beliebig genau Häufigkeits Funktion $\varphi(y)$ beliebig nähert — unabhängig davon wie die x Punkte gewählt werden

---

Nehmen wir ein Quadrat und werfen von einem Eckpunkt unter $\measuredangle$ 30° eine elastische Kugel hinein, so wird die Fläche des Quadrates ~~mit der~~ im Laufe der Zeit von der Bahn derselben gleichförmig überdeckt, also kann man sagen „dass im Laufe der Zeit jeder ~~[illegible]~~ Aufenthalts Bereich innerhalb jenes Quadrates gleich wahrscheinlich wird?"



Nur in der Bedeutung, dass eine lange Zeit verstreicht und dann innerhalb eines nicht näher definierten willkürlichen Intervalles der Aufenthaltsort untersucht wird. Also auch hier gibt es ein variables Element: diesen Moment der Beobachtung.

~~Also~~ Die Ordinaten y_n sind definiert durch ~~[illegible]~~

$(\frac{n}{2}\sqrt{3} - m)$ wo m die größte ganze Zahl bedeutet welche $< \frac{n}{2}\sqrt{3}$ ist.

in ~~$\frac{n}{2}\sqrt{3}$ [illegible]~~ Dass dieselben gleichförmig verteilt sein dürften ~~[illegible]~~ scheint ebenso wahrscheinlich wie die gleichförmige Verteilung in Poincaré's Beispiel (der ~~[illegible]~~ Logarithmen)

d.h. wenn ich denselben Versuch mit einer gegebenen Zeit T ~~[illegible]~~ hundertmal anstelle, so werde ich jedesmal die Kugel ~~[illegible]~~ an derselben Stelle finden

an welcher? das weiss ich nicht, wenn ich die Rechnung nicht angestellt habe, und die subjektive Wahrsch., sie in einem bestimmten Gebiete vorzufinden, ist gleich der Größe des Gebietes : Größe d. Quadrates

der „Vermutungsgrad"

aber objektiv physikalisch ist dies wertlos: entweder ist sie drin oder nicht.

Doch wenn ich jenen Versuch hundertmal ~~[illegible]~~ in verschiedenen jedesmal sehr langen Intervallen T wiederhole (also wenn T variabel ist) so erhalte ich die objektive ~~[illegible]~~ Häufigkeit des Auftretens der Kugel

(Kleine Ursache – grosse Wirkung. Aber das genügt nicht!)

Würfel auf einer Ecke aufgestellt wird umfallen und (wenn weicher Stoss) eine der drei Zahlen 1, 2, 3 ergeben

daher Prinzip 4.!

Daher kann von Zufall nicht die Rede sein; der Effekt hängt nur von der Anfangslage ab und zwar ob Schwerpunkt in einer der drei Richtungen (wenn auch ~~[illegible]~~ unmerklich wenig) überhängt.

Dagegen wenn der Würfel aus einer Höhe von 1 m herabfallen gelassen wird, ändert sich die Sache vollständig, denn er wird wieder aufspringen dabei rotieren, wieder auffallen u.s.w. und es wird einer allerkleinsten Änderung der Anfangslage ein rascher Wechsel der möglichen ~~[illegible]~~ erreichten Endlage entsprechen, so dass letztere alle ungefähr gleich wahrscheinlich werden, (falls der Variations-Bereich der Anfangslage nicht gar zu beschränkt ist!)

Aus den Beispielen der vorigen Seite

ist 1). keine Erklärung der „objektiven Wahrscheinlichkeit" weil eben eine solche schon bezüglich der Ursache vorausgesetzt wird

wohl aber 2).

Analog wäre: bei Anwendung des seitlich begrenzten Galtonschen Brettes:

wenn ~~die~~ eine Kugel rollen gelassen wird, und die sukzessive bei Passieren der Stiftreihen auftretenden x notiert werden, so wird bei genügend grosser Zahl n der Stiftreihen eine annähernd gleichförmige Verteilung der x statthaben

---

Letztere Art (2) nur dort möglich, wo eben derselbe Vorgang automatisch sich wiederholt, jedesmal mit Anfangsbedingungen welche von d. vorhergehenden abhängen und den Bedingungen der ... unterliegen.

In jedem Falle aber kann für jedes $x, \ldots x+\Delta x$ ein so grosses α_n gefunden werden dass

$$\frac{x}{\alpha_n} = n\pi \qquad n = \text{ganze Zahl} \qquad \left(\alpha_n = \frac{x}{n\pi}\right.$$

und gleichzeitig . . .

$$\frac{x+\Delta x}{\alpha_n} = n\pi + \delta \qquad \delta < \vartheta\pi \qquad \vartheta = \text{beliebig kleiner echter Bruch}$$

$$\frac{\Delta x}{\alpha_n} = \delta \qquad \frac{\Delta x}{\alpha_n} < \vartheta\pi$$

$$\frac{\Delta x}{x} n < \vartheta$$

Wenn in ein Gefäss von mathematisch genau bekannter Gestalt (und mit genau reflektierenden Wänden) ein Punkt von bestimmter Stelle ($x y z$) in bestimmter Richtung ($u v w$) mit gegebener Anfangsgeschw. hineingeschleudert wird, so kann man dessen Lage und Geschw. richtung für einen gegebenen Zeitpunkt T vorausberechnen und zwar eindeutig.

1). Wird in (x y z) oder in (u v w) eine Variation zugelassen, so entsteht schon ein Wahrscheinlichkeitsproblem.) 2). Ebenso falls ein ganzer Zeitraum T .. $T+t$ betrachtet wird. Im letzteren Falle kommt eine Mannigfaltigkeit diskreter Werte von u v w etc in Betracht, welche mit Wachsen von t sich einer kontinuierl. Mannigfalt. nähert, so dass für jedes Bereich $u+\Delta u, v+\Delta v, \ldots$ eine Zeit t angebbar ist . . . .

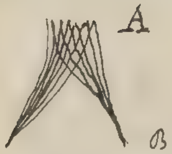
→ So wohl nicht, denn sonst wäre jedes $y\,dy = f(x)\,dx$ ein Wahrscheinl. problem, sondern falls t so gross ist, und der Verschiebt (Ungenauigkeit) bereich $\Delta y\ \Delta x\ \Delta z$.. so gross dass Endverteilung (annähernd) unabhängig von der Grösse $\Delta x \Delta y \Delta z$.

Im Falle 1). stützt man sich schon auf den Begriff d. kontinuierlichen Variationsbereiches, also wird schon eine Wahrscheinlichkeitsverteilung der Ursache vorausgesetzt

" " 2). ist dies nicht der Fall; da kommt der Wahrsch. begriff durch Grenzübergang einer diskreten Wertreihe zustande.

(Dabei vorausgesetzt, dass die im Variabilitätsbereich herrschende Funktion eine „reguläre" ist, also Rekursion auf Wahrscheinlichkeitsbegriff

~~Apo~~



(Begriff d. Zufalls) Bedingungen

unbekannte (oder absichtlich ignorierte) Teilursache

1). Kleine Änderung d. A bewirkt grosse Änderung d. B. (Das ist der populäre Begriff d. Zufalls! Aber dies allein ermöglicht keine „Wahrscheinlichkeit")

2). Innerhalb d. Variabilitätsbereiches von A gibt es ~~[illegible]~~ für jeden (ihn hervorbringende) diskreten Endzustand B (eine Menge B+ΔB) Anfangszustände A

Dieser Punkt (2) schliesst aber … in sich ein!

Vielleicht besser so: innerhalb d. Variabilitätsbereiches von A gibt es kein zusammenhängendes endliches Gebiet keinen Teil der so beschaffen wäre, dass nicht ~~[illegible]~~

3). Die Häufigkeit der ~~[illegible]~~ B – B+ΔB hervorbringenden Zustände ist im ganzen Variabilitätsbereich von A überall dieselbe.

ΔB das ist eine ~~ungeheure~~ Frage, im Allgem. wird nur annähernd dieselbe Dichte bestehen.

(selbstverständlich) Beim Galton'schen Brett:

für jeden noch so kleinen Bereich der Anfangsbedingungen A – A+ΔA ist eine Zahl n der Nagelreihen angebbar, für welche die grösste Schwankung der B (bei homogener Verteilung ~~[illegible]~~ innerhalb ΔA) kleiner gemacht werden kann als eine beliebig kleine vorgegebene Zahl ε

Ebenso bei Gastheorie: Einzelne Variab.bereich ... (Zersplitterung der Zustandsbereiche, Boltz) alle möglichen Zustände mit Schwankungsdichte

Natürlich ist $n = f.(\Delta A, \varepsilon)$

Grenzfall v. B.:

$$y = \lim_{\alpha=0} \sin\left(\frac{x}{\alpha}\right)$$

dies ergibt eine ungleichförmige Verteilung; hängt mit gleichf. direkt zusammen,

dass $\left(\frac{x}{\alpha}\right) = n\pi + z$ | n = grösst mögliche ganze Zahl

$\lim_{\alpha=0}\left(\frac{x}{\alpha}\right)$ eine gleichförmige Verteilung von z ergibt

falls $x, \ldots x+\Delta x$ commensurabel sind kann (für jedes $x, x+\Delta x$) ~~[illegible]~~ immer ein solches α_n gefunden werden dass ~~[illegible]~~

$\frac{x}{\alpha} = n\pi + z$ $\quad \frac{x}{\alpha_n} = n\pi$ $\quad$ n, m = ganze Zahl

$\frac{x+\Delta x}{\alpha} = n\pi + z + \Delta z$ $\quad \frac{x+\Delta x}{\alpha_n} = (n+m)\pi$

$\frac{\Delta x}{\alpha} = \Delta z$

$D + \xi = \ldots$

$x = (2n\pi + f(\xi))\alpha$ arcsin

g = Wärmekapazität eines Moleküls =

gn = " $\quad$ eines cm³ Gas $= c\rho_0$

$\therefore \quad \theta_1 - \theta_0 = w \sqrt{\frac{3\pi}{8}} \frac{1}{C c \rho_0}$

Verhältnis zum Eigengewicht:

$$\frac{F}{\varphi} = \frac{\cancel{2\pi}\,\cancel{\lambda}\ \frac{a^2\lambda}{\cancel{2}}\ \frac{w}{c\theta_0}\ \frac{1}{C}\sqrt{\frac{3\pi}{8}}}{\frac{4}{3}a^3\pi\,\rho_1\,\cancel{\lambda}} = \frac{3}{4}\sqrt{\frac{3\pi}{8}}\ \frac{\lambda}{a}\ \frac{w}{c\rho_1\theta_0}\ \frac{1}{C}$$

Annahme (Jäger) $w = 1 \frac{\text{Kalgr.}}{\text{cm}^2\ \text{sek}}$ $\qquad \rho_1 = 10 \qquad c = 0{\cdot}27 \qquad \theta_0 = 300^\circ$

$C = 4.10^4$

Somit ganz verschwindend!

Andre Schätzungsart, für entgegengesetzten Extrem, d. i. unter Voraussetzung d. gewöhnl. Wärmeleitung:

$$-r^2 \frac{\partial\theta}{\partial r} = \text{const} = \alpha$$

$$\left.\begin{aligned} \theta &= \frac{\alpha}{r} + \theta_0 \\ \theta_1 &= \frac{\alpha}{a} + \theta_0 \end{aligned}\right\} \qquad \theta - \theta_0 = (\theta_1 - \theta_0)\frac{a}{r}$$

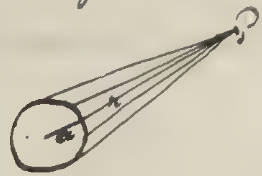
$$F = \rho_0 g \int_{r=a}^{\infty} \frac{\theta_1 - \theta_0}{\theta_0}\ \frac{4 r^2 \pi\, a}{\cancel{r}}\, dr = \cancel{\infty} \ldots$$

$$\cancel{= \rho_0 g\, \infty\, \frac{\theta_1 - \theta_0}{\theta_0^2}}$$

~~also muss die Abweichung von der eigen Verteilung ganz wesentlich in Betracht kommen!~~

Erwärmtes Kügelchen in Luft; Auftrieb infolge Konvektionsströmung?

Voraussetzung so klein dass $\frac{a}{\lambda}$ klein



Temperatur im Punkte P?

$$\theta = \frac{2r^2\pi\left(1-\frac{\sqrt{r^2-a^2}}{r}\right)\theta_1 + \left[4r^2\pi - 2r^2\pi\left(1-\frac{\sqrt{r^2-a^2}}{r}\right)\right]\theta_0}{4r^2\pi}$$

$$2\pi a^2 \underbrace{\int_0^{\varphi} d\varphi \sin\varphi}_{1-\cos\varphi}$$

$$= \theta_0 + \frac{1}{2}\left[\theta_1-\theta_0\right]\left[1-\sqrt{1-\left(\frac{a}{r}\right)^2}\right]$$

Auftrieb im gesammten Raum

$$F = \rho g \int_{r=a}^{\infty} \frac{\theta-\theta_0}{\theta_0}\, 4r^2\pi\, dr = \rho_0 g\, \frac{1}{2}\, \frac{\theta_1-\theta_0}{\theta_0}\, 4\pi \int_a^{\infty} \underbrace{\left[1-\sqrt{1-\left(\frac{a}{r}\right)^2}\right]}_{= \frac{1}{2}\frac{a^2}{r^2}} r^2\, dr$$

wäre $= \infty$!

Rationeller Weise ist aber die obere Grenze nicht ∞ sondern $a+\lambda$ zu machen (womit der Effekt wohl unterschätzt wird)

Somit:

$$F = 2\pi \rho_0 g\, \frac{\theta_1-\theta_0}{\theta_0}\, \frac{a^2\lambda}{2}$$

Berechnung von θ_1:

Zugeführte Wärmemenge (durch Strahlung $w \frac{cal}{cm^2}$) $a^2\pi w$

Abgeführte " (durch Molekularstösse) $4a^2\pi \frac{nC}{\sqrt{6\pi}}(\theta_1-\theta_0)g$

Somit: $\theta_1-\theta_0 = w\frac{\sqrt{6\pi}}{4}\frac{1}{nCg}$

Also tritt die maximale Wahrsch. [illegible] $\sim \xi^2$

(aber nicht $t = \frac{\xi^2}{2D}$ wie nach gewöhnlicher Ok. F. zu erwarten wäre)

Also Wahrsch. dass bis zur Zeit t die Elongation ξ noch nicht (kein einziges Mal) erreicht worden ist:

$$W = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2} dz$$

Wahrsch. dass bis zur verstrichenen Zeit t die (mindestens) festgehaltene Elongation ξ erreicht wurde:

$$W(t) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2} dz$$

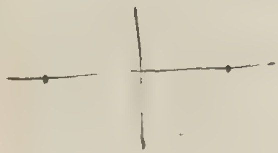
Wahrsch. dass in der (festen) Zeit t die (variable) Elongation (0 bis ξ) nicht überschritten wird

$$W(\xi) = \int_0^{\xi} \frac{1}{2\sqrt{D\pi t}} e^{-\frac{\xi^2}{4Dt}} d\xi \qquad \frac{\xi^2}{4Dt} = z^2 \qquad \xi = z\, 2\sqrt{Dt}$$

$$= \frac{1}{\sqrt{\pi}} \int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2} dz$$

$$\nu \Delta = \frac{3\pi c_0^2}{\frac{e^2}{m} \cdot \rho} \qquad \rho = \frac{3\pi \cdot (3 \cdot 10^{10})^2}{4'77 \cdot 10^{-10} \cdot 1'77 \cdot 10^{7} \cdot \cancel{2 \cdot 10^{8}}}$$

Kann man nicht dasselbe machen nach Diffusionsmethode?



$$\Phi(\xi)\,d\xi = \frac{1}{2\sqrt{D\pi t}}\left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{(x+\xi)^2}{4Dt}}\right]d\xi$$

$$U = \int_0^\infty \Phi(\xi)\,d\xi$$

$$\frac{x-\xi}{2\sqrt{Dt}} = z$$

$$\frac{dx}{2\sqrt{Dt}} = dz$$

$$W(t)\,dt = -\frac{\partial U}{\partial t}\,dt$$

$$U = \frac{1}{\sqrt{\pi}}\left[\int_{-\frac{\xi}{2\sqrt{Dt}}}^{\infty} e^{-z^2}dz - \int_{\frac{\xi}{\cdot}} \right]$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2}dz$$

$$W(t)\,dt = +\frac{2}{\sqrt{\pi}}\, e^{-\frac{\xi^2}{4Dt}} \cdot \frac{\xi}{4\sqrt{Dt^3}}$$ stimmt ./.

Wann ist ein Maximum d. Wahrscheinlichkeit?

$$\frac{1}{\sqrt{t^3}}\, e^{-\frac{\xi^2}{4Dt}}$$

$$\frac{1}{\sqrt{t^3}}\,\frac{\xi^2}{4Dt^2} - \frac{3}{2\sqrt{t^5}} = 0$$

$$\frac{\xi^2}{4Dt^2} = \frac{3}{2t}$$

$$t_m = \frac{\xi^2}{6D}$$

Falls genaue Schwerbegrenzung herrscht, Wahrsch. der Überschreitung von ξ in vorgegebener Zeit

Wahrsch. eines Überschusses n beim m ten Wurf

$$W(n) = \frac{1}{2^m}\binom{m}{\frac{m-n}{2}} \mp \sqrt{\frac{2}{\pi m}}\, e^{-\frac{n^2}{2m}}$$

Wahrsch. eines <u>erstmaligen</u> Übersch. n beim m ten Wurf

$$W'(n) = \frac{n}{m}\cdot\frac{1}{2^m}\binom{m}{\frac{m-n}{2}}$$

$$m = \frac{t}{\tau} \qquad D = \frac{\delta^2}{2\tau} \qquad N = \frac{1}{\delta}$$

$$n = \frac{\xi}{\delta}$$

$$W(\xi)\,d\xi = \frac{1}{2\sqrt{D\pi t}}\, e^{-\frac{\xi^2}{4Dt}}\, d\xi$$

$$\underbrace{\sqrt{\frac{2\tau}{\pi t}}\, e^{-\frac{\xi^2\tau}{2\delta^2 t}}} = e^{-\frac{\xi^2}{4Dt}}\cdot\frac{\delta}{\sqrt{D\pi t}}$$

$$\frac{n}{m} = \frac{\xi\tau}{\delta t} = \frac{\xi}{Vt} \quad (?)$$

$$d\xi = \delta\cdot dn$$

ξ' viel zu ungenau

Somit wäre

$$\lim W'(\xi,t)\,dt = \frac{\xi}{Vt}\,\frac{\tau}{2\sqrt{D\pi t}}\, e^{-\frac{\xi^2}{4Dt}}$$

$$\frac{\delta}{\tau} = V$$

$$\frac{\xi^2}{4Dt} = z$$

$$t = \frac{\xi^2}{4Dz}$$

$$dt = -\frac{\xi^2}{4Dz^2}\,dz$$

$$\frac{\xi\,dt}{2t\sqrt{D\pi t}}\, e^{-\frac{\xi^2}{4Dt}} = \sqrt{\frac{z}{\pi}}\cdot\frac{4Dz}{\xi^2}\cdot e^{-z}\cdot\frac{\xi^2}{4Dz^2}\,dz$$

$$= \frac{1}{\sqrt{\pi}}\,\frac{e^{-z}}{\sqrt{z}}\,dz$$

$$\int_0^\infty \frac{e^{-z}}{\sqrt{z}}\,dz = \int_0^\infty 2\frac{e^{-x^2}}{x}\,x\,dx = \sqrt{\pi}$$

Stimmt

$$\text{also: } \int_{t=0}^{\infty} \frac{\xi\, e^{-\frac{\xi^2}{4Dt}}\,dt}{2t\sqrt{D\pi t}} = 1$$

Wie rasch rotieren solche Teilchen? (Wirkliche Rot.geschw.)

Ungefähr $\bar{K}\;F\; m\frac{a^2}{2} = \frac{4}{3}\frac{\pi a^5\rho}{2} = 2a^5\rho$ für $a = 10^{-5}$ $\rho = 1$

$$\bar{K}\frac{(2\pi n)^2}{2} = 10^{-14} = 2\cdot 10^{-25}\cdot 2\pi^2 n^2$$

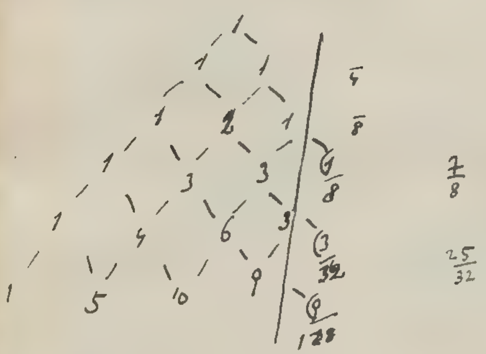
$$n = 10^5$$

~~$A_{\text{ext}} : A_{\text{...}} =$~~

~~$4\pi^2 n^2 a\,\frac{a}{2}\,m : 30\;4a^2 n$~~

~~$\frac{m}{2}\pi n^2 = 30$~~

10^{-15}



Anzahl der Zusammenstöße im Volumen ω:

$$\frac{n^2}{\omega}\cdot\sigma\sqrt{\frac{2\pi}{hm}} = \frac{\nu^2(1+\delta)^2}{\omega}\dots$$

?

$$\overline{n^2} = \nu^2(1+\overline{\delta^2}) = \nu^2+\nu$$

$$\sum\frac{n^2}{\omega} = \sum\frac{\nu^2+\nu}{\omega} = \left[N^2 + \frac{N}{\omega}\right]$$

Otázka v tom že pořádně dosttadu být:

$\frac{n(n-1)}{\omega}\dots$ stačí jest $\sum\frac{n^2-n}{\omega} = N^2+\frac{N}{\omega}-\frac{N}{\omega} = N^2$

$h = 6{\cdot}4 . 10^{-27}$

ε (Rutherf s Ribbons) $= 0{\cdot}4284 . 10^{13} e \qquad = h\nu = 6{\cdot}4 . 10^{-27} \frac{3 . 10^{10}}{\lambda}$

$\frac{4{\cdot}78 . 10^{-10}}{3 . 10^{10}}$

$\lambda = \cancel{0{\cdot}428 . 4{\cdot}78 . 10^{3}} \quad \frac{3 . 6{\cdot}4 . 10^{-17}}{0{\cdot}428 . 4{\cdot}78 . 10^{3}} . 3 . 10^{10}$

$4{\cdot}78 . 0{\cdot}1428 . 10^{-7}$
191
14
$0{\cdot}683 . 10^{-7}$

$= \frac{9 . 6{\cdot}4}{0{\cdot}428 . 4{\cdot}78} . 10^{-10} = \frac{576}{4{\cdot}78 . 0{\cdot}428} . 10^{-9}$

$\lambda = 3 . 10^{-9}$

$\nu = \frac{0{\cdot}683 . 10^{-7}}{6{\cdot}4 . 10^{-27}} = \frac{c}{\lambda}$.

Reibungseinfluss hängt ab von der Grösse von

$a = 10^{-4}$ cm $= 1 \mu$

$$\left(\frac{\beta}{\alpha n}\right)^2 = \left(\frac{6\pi\mu a}{2\pi n . \frac{4}{3}\pi a^3 \rho}\right)^2$$

$$A = \frac{4{\cdot}7 . 10^{-10}}{\frac{4}{3}\pi\rho . 10^{-12}} \quad \frac{\frac{V}{300}}{4\pi^2 n^2}$$

$$= 10^{-2} \frac{V}{n^2}$$

($\rho = 1$)

$\|$ V, n von Grössenordnung 1–10

$a = 0{\cdot}1\mu = 10^{-5}$

$A = 10 . \frac{V}{n^2}$

$$= \frac{9}{4} \frac{\mu}{a^2 \rho \pi} \frac{1}{n}$$

$$= \frac{9}{4} \frac{10^{-4} . 2}{\pi . 10^{-8}} \frac{1}{n}$$

$$\left(= \frac{10^4}{n}\right)^2$$

also ist Reibungseinfluss kolossal

Auch so Allgemein: $\Big\}$ $v = \frac{mg}{\beta}$ = Fallgeschwind.

$$\frac{\beta}{\alpha n} = \frac{g}{v\, 2\pi n} = \frac{10^3}{10^{-2} . 2\pi n}$$

~~Soweit unverglichen~~

Wie wird also Kirchhoffs Formel?

Trägheitswirkung (prozentuelle) $\quad 1 + \frac{\rho_{Luft}}{\rho_{Kugel}}\left(\frac{1}{2} + \frac{9}{4 . 10^{-4} \sqrt{\frac{1}{2}\frac{2\pi n}{\mu}\rho_{Luft}}}\right)$

$$= 1 + \frac{\rho'}{\rho_0}\left(\frac{1}{2} + \frac{3 . 10^3}{\sqrt{n}}\right)$$

Falls die Messung etwas Brauchbares liefern soll, muss $\frac{\beta}{\alpha n}$ klein sein $= \frac{9}{4} \frac{10^{-4} . 2}{\pi . a^2 \rho n} = \frac{3}{2} \frac{10^{-4}}{a^2 . \rho . n}$

für $a = 10^{-5}$ $\quad \rho = 10 \quad n = 10^5$!!!

Praktisch nicht ausführbar!

Bei so kleinen Teilchen ist Trägheitseinfluss ganz verschwindend!

Falls bei Millikan-Ehrenhaft Versuchen ein Wechselstromfeld angewendet wird mit horizontaler Axe:

$$m\frac{d^2x}{dt^2} = e\,E\cos\alpha t$$

$$x = -A\cos\alpha t$$

$$m A\alpha^2 = eE$$

$$A = \frac{e}{m}\frac{E}{\alpha^2}$$

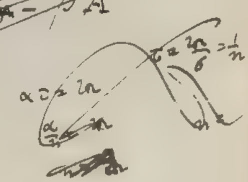
$$= \frac{e}{m}\frac{E}{(2\pi n)^2}$$

Dies gibt also eine Relation, welche an und für sich durch Messung von A das Verhältnis $\frac{e}{m}$ ergibt

(unter Voraussetzung dass die Hemmung zu vernachlässigen ist)

$$m\frac{d^2x}{dt^2} - \overset{\beta}{6\pi\mu a}\,\frac{dx}{dt} = eE\cos\alpha t$$

$$m A\alpha^2\cos\varepsilon t - \beta A$$



$$x = -A\cos(\alpha t+\varepsilon)$$

$$A\alpha^2 m\cos\varepsilon + A\beta\alpha\sin\varepsilon = eE$$

$$\cancel{A}\,\alpha^2 m\sin\varepsilon + \cancel{A}\beta\cancel{\alpha}\cos\varepsilon = 0$$

$$\mathrm{tg}\,\varepsilon = -\frac{\beta}{\alpha m}$$

$$A\left(\alpha^2+\frac{\beta^2}{m^2}\right)m\cos\varepsilon = eE$$

$$A = \frac{eE}{m\left(\alpha^2+\frac{\beta^2}{m^2}\right)}\sqrt{1+\frac{\beta^2}{\alpha^2 m^2}} = \frac{eE}{\alpha\sqrt{\alpha^2+\frac{\beta^2}{m^2}}\,m} \mp \frac{eE}{m\alpha^2}\left(1-\frac{\beta^2}{2\alpha^2 m^2}\right)$$

$\cancel{\tau = \frac{x}{v}} \quad \cancel{t' = t +}$

$\cancel{}\; c\tau = x_0 + u\tau$

$t = \frac{x_0}{c-u}$

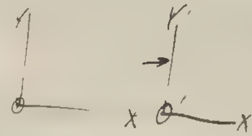
$\frac{x}{c} - \frac{x\sqrt{1-\frac{v^2}{c^2}}}{c+v}$

$= \frac{x}{c}\left[1 - \frac{\sqrt{c^2-v^2}}{c+v}\right] = \frac{x}{c}\left[1 - \sqrt{\frac{c-v}{c+v}}\right]$

$t_1' = t_1 + \frac{x_0}{c-u}$

$t_2' = t_2 + \frac{x_0 + ut}{c-u}$

$t_1 + t$



$t_2' - t_1' = t\left(1 + \frac{u}{c-u}\right) = t\,\frac{c}{c-u}$

$$\left\{\begin{aligned} t' &= \beta\left(t - \frac{v}{c^2}x\right) \\ x' &= \beta(x - vt) \\ y' &= y \\ z' &= z \end{aligned}\right.$$

$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Anfangspunkt: O'. $x' = 0$; $x = vt$; $x = \beta v t'$

$\left.\begin{matrix} O \\ x = 0 \end{matrix}\right\}$ $x' = -\beta v t$; $x' = -vt'$

$$\left\{\begin{aligned} t &= \beta\left(t' + \frac{v}{c^2}x'\right) \\ x &= \beta(x' + vt') \\ y &= y' \\ z &= z' \end{aligned}\right.$$

Zur Zeit $t = t' = 0$ $x = x'$

Nach Ablauf von t wird von O ein Zeichen nach O' geschickt, dort reflektiert

$vt + v\tau = c\tau \qquad \tau = \frac{vt}{c-v}$

$T = 2\tau \qquad t + \tau = t\left(1 + \frac{v}{c-v}\right) = t\,\frac{c}{c-v}$

$=$

$2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = 2\pi n\left(t - \frac{x}{\lambda}\right) = \cancel{n\beta t' + n\beta\frac{v}{c^2}x' - \frac{\beta x'}{\lambda} - \frac{\beta v t'}{\lambda}}$

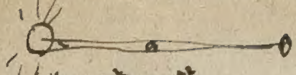
$= \left(n - \frac{v}{\lambda}\right)\beta t' + \beta x'\left(\frac{nv}{c^2} - \frac{1}{\lambda}\right)$

$n\left(t - \frac{x}{c}\right) = n\beta t' - n\beta\frac{v}{c^2}x' - \frac{n}{c}\beta x' - \frac{n\beta v t'}{c} = n\beta t'\left(1 - \frac{v}{c}\right) - \frac{n\beta x'}{c}\left(1 - \frac{v}{c}\right)$

$= \left(1 - \frac{v}{c}\right) n\beta\left(t' - \frac{x'}{c}\right)$

$= \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\; n\left(t' - \frac{x'}{c}\right)$

$\mp\left(1 - \frac{v}{c}\right) \ldots$



$d\omega = \frac{r^2\pi}{4\pi D^2} = \frac{r^2}{4D^2}$

$dS_1 = D^2 \alpha^2 \pi$

$F = \mathcal{E}_1 \cdot \frac{r^2 \alpha^2 \pi}{4}$

$\frac{1{,}5^2}{4}\pi \cdot \left(\frac{1}{120}\right)^2 \cdot 300000 \,\Big\|\, \frac{10^4}{200} \cdot 0{,}04$

$= \cancel{4}\, 3.10 = 30 \quad \| \; 2$

$\odot$

$\mathcal{E} \rightarrow \boxed{\;\;|\;\;} d \qquad \mathcal{E}' = \frac{\mathcal{E}\, h\, \Delta x}{4\pi\, d\, \Delta x} = \mathcal{E}\, \frac{h}{4\pi d} = \mathcal{E}'$

$F = \mathcal{E}_1\, dS_1\, d\omega_1$

$\mathcal{E}_2 = \frac{\mathcal{E}_1\, dS_1\, d\omega_1\, h\, \cancel{\Delta x}}{4\pi\, d.\, \cancel{\Delta x}}$

$dS_1 = \frac{d^2}{4}$

$d\omega_1 = \frac{1}{200}$

$h = 10^{-7}$

$d =$

$= \mathcal{E}_1\, \frac{d}{4}\, \frac{d\omega_1\, h}{4\pi}$

$= 10^4 \cdot \frac{0{,}1}{200 \cdot 10} \cdot 10^{-7} = \frac{10^{-4}}{2.10^3} = \frac{1}{2} \cdot 10^{-7}$

mur blanc à pleine lune à 45° : 10^{-6}

$h = \frac{32}{3}\, \frac{\pi^3}{n \lambda_F^4}\, (n_0 - 1)^2$

$0{,}000273$

$\lambda_F = 0{,}486 \cdot 10^{-4}$

$n = 3{,}10^{19}$

$0{,}6866 \cdot 10^{-5}$

$0{,}7464 - 18$

~~1,4914~~

1,49145

1,5051

0,8724 − 8

0,8690 − 5

−2,7006

0,1684 − 7

0,954249

0,7464 − 18

2,7006

$h = 1{,}47 \cdot 10^{-7}$

$2x + \Delta t\, v = \tau c$

$x + 2\tau v = \tau c$

$\tau = \frac{2x}{c - 2v}$

$t_0 \quad t$

$t_1 \quad t + \frac{x'}{c+v}$

$t_2 \quad t + \frac{x'}{c+v} + \frac{x'}{c-v}$

$t_1 = \frac{t_0 + t_2}{2}$

$= t + \frac{1}{2}\left[\frac{x'}{c+v} + \frac{x'}{c-v}\right]$

$= t + \frac{x' c}{c^2 - v^2}$

$t + \frac{x}{c} + \frac{x}{c - 2v}$

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…nificant Emblems

…HY NOT FOLLOW …TTER EXAMPLES?

…body has been saying once more …r British postage-stamps are …e most insignificant and un… the world.

…N. has often said it; and we are …d that, if we have regard to the …ence of the nation that issues …they must be awarded the palm …verty. Their design is of the lowest …r of art, and they amount to the …rest advertisement of our country. …hat a pity it is! This is not a matter … stamp-collector alone; it is …l and imperial importance. …f people outside these islands … see our … by

A Brave Bengali

How To Hold a Tiger

The courage with which natives of Africa go lion-hunting, armed only with spears, is well known.

Now comes a story concerning the bravery of a young man in India who tackled a tiger with his hands.

In the district of Bankura a man-eating tiger had become a constant terror, until at last the whole manhood of the village turned out to hunt it with clubs and poles.

They found the tiger, which fled before the crowd as a bird of prey flies before a flock of small ones; but when the beast had climbed a palmyra tree it was a different matter, because only one man could tackle it there.

Such a man was found in a young Bengali named Banshi Mukherji, who went to the tree, took steady aim, and hurled a spear.

With one bound the tiger was upon him. But the unarmed youth did not lose his head. Instantly he thrust his …d into the tiger's open mouth, and …d its tongue. He held on till the …rs had rushed in and killed it …dgels.

…sured sev… …alf

Tracking Down … Influenza

The Ferret To … of M…

The first step to… human beings from influen… successfully made.

Three doctors in the research… tories of the National Institute … Hill have proved that the ferret catc… influenza from man and hands it on … contact to other ferrets. A seru… obtained from infected ferrets … tralises the virus, or filter-passing … which has for some years been held … at any rate one of the causes of influ…

The cause having now been fou… discoverers are hoping to find a …

The Return of the Plague Rat

The black rat, dreaded carrier … bubonic plague, is incr…ng its numbe… in England.

Perhaps our succ… …erminating its natural enemy, … rat of the sewers and basem… …n it fresh oppor… …ck rat … …erse to … …gerous … …rts …